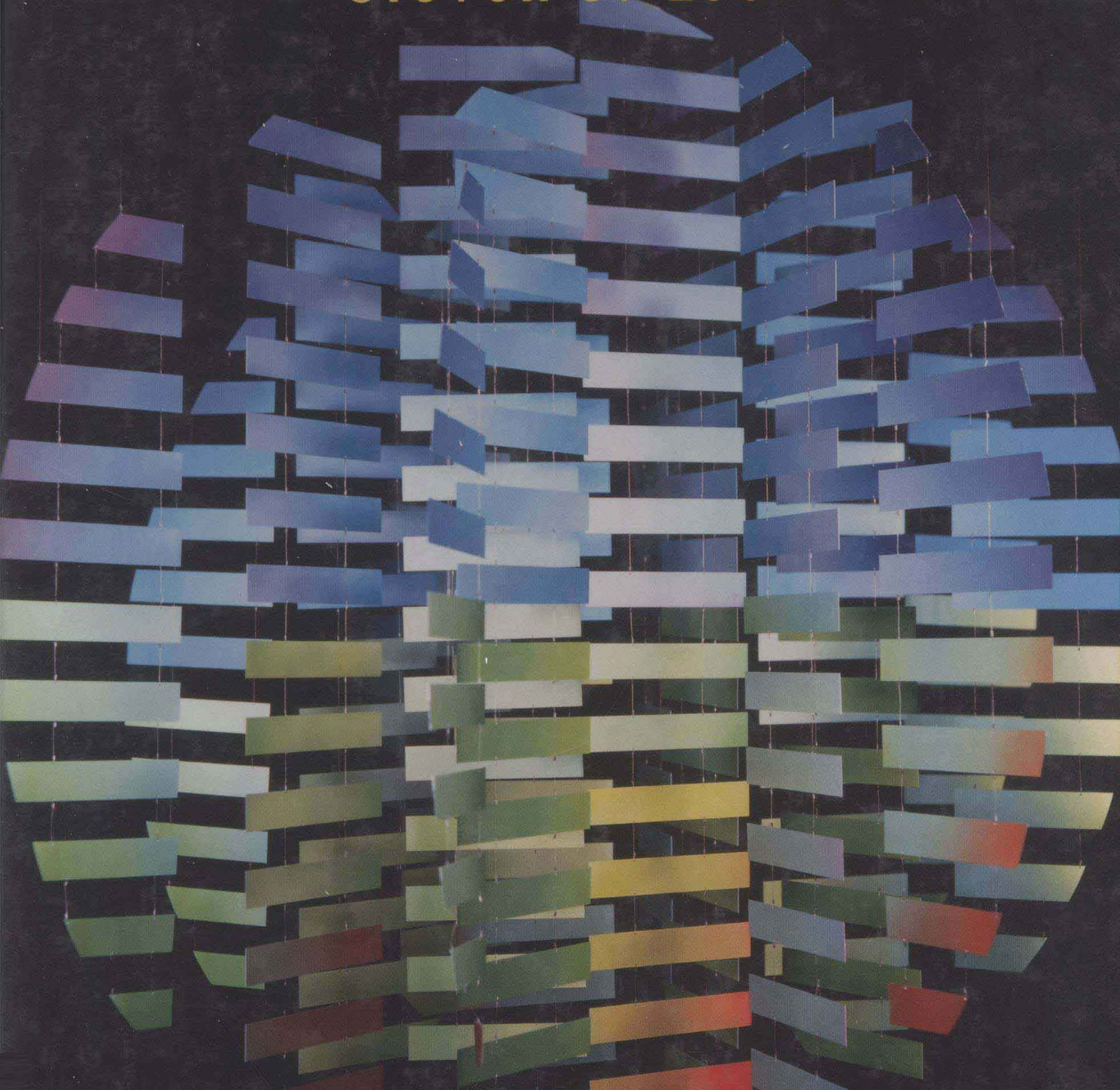
Steven J. Leon



LINEAR ALGEBRA

WITH APPLICATIONS

Sixth Edition

LINEAR ALGEBRA WITH APPLICATIONS

SIXTH EDITION

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Prentice Hall

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To Judith Russ Leon

PREFACE

The author is pleased to see the text reach its sixth edition. While the continued support and enthusiasm of the many users has been most gratifying, this does not mean that a mild revision is in order. Linear algebra is more exciting now than at almost any time in the past. Its applications continue to spread to more and more fields. Largely due to the computer revolution of the last half century, linear algebra has risen to a role of prominence in the mathematical curriculum rivaling that of calculus. Modern software has also made it possible to dramatically improve the way the course is taught. The author teaches linear algebra every semester and continues to seek new ways to optimize student understanding. For this edition every chapter has been carefully scrutinized and enhanced. Additionally, many of the revisions in this edition are due to the helpful suggestions received from users and reviewers. Consequently, this new edition, while retaining the essence of previous editions, incorporates a wide array of substantive improvements.

WHAT'S NEW IN THE SIXTH EDITION?

1. Chapter Tests

New to this edition are chapter tests. At the end of each chapter there is a true-false exam testing the basic concepts covered in the chapter. Students are asked to prove or explain all of their answers.

2. Earlier Presentation of the Singular Value Decomposition

The singular value decomposition (SVD) has emerged as one of the most important tools in matrix applications. Unfortunately, the topic is often omitted from linear algebra textbooks. When covered, it usually appears near the end of the book and classes rarely have time to get that far. To remedy this, we have moved the singular value decomposition approximately 100 pages forward in the book. It is now covered in Section 5 of Chapter 6. In this section we also show the applications of the singular value decomposition to least squares problems, principal component analysis, information retrieval, numerical rank of a matrix, and digital imaging. The SVD section nicely ties together some of the major topics, such as fundamental subspaces, orthogonality, and eigenvalues. It provides an ideal climax to a linear algebra course.

3. New and Improved Applications

Eight applications were added to the previous edition. Some of these have been revised and improved in the current edition. A number of new applications have also been added. In Chapter 1 we show how matrices are used for search engines and information retrieval applications. This application is revisited in Chapters 5 and 6 after students have learned about orthogonality and singular values. Similarly, the statistical applications in Chapter 5 are revisited later in Chapter 6 after students have learned about the singular value decomposition.

4. New Computer Exercises Emphasizing Visualization

Chapter 6 has ten new MATLAB exercises to help students to visualize eigenvalues and singular values and to help them gain geometric insight into these subjects.

5. New and Improved Examples

Worked out examples make the textbook seem less abstract and more user friendly. Often students don't understand what a theorem says until they see a worked out example that illustrates the theorem. The impressive collection of examples was often cited as one of the strong points of the first edition of this book. This collection has continued to grow and improve with each new edition. More examples have been added throughout the sixth edition, and many of the previous examples have been revised and improved. Now, for example, the numbered of worked out examples in Chapter 1 has increased from 32 to 34. In a number of cases color shading is now used to emphasize how rows and columns are paired off in matrix computations.

6. New Theorem and Improved Nomenclature

Throughout this edition we have made a special effort to assign names to theorems so as to emphasize the importance of the results. Also, it is easier to refer back to a theorem if it has a name. We have added a new theorem to Chapter 6. This theorem does have a name, *The Principal Axes Theorem.*

7. Revised Organization of Chapter 5

In Chapter 5 the order of two of the sections has been reversed. Least squares problems are now covered before the section on general inner product spaces. To facilitate this change, some new material was added to Section 1 of the chapter. With this new ordering it is possible for classes that only treat Euclidean vector spaces to skip most of Section 4. These classes need only introduce the inner product notation in Section 4 and then move on to the next section or, if pressed for time, skip ahead to the next chapter.

8. New Subsection on Outer Products

A new subsection on outer product expansions has been added to Chapter 1. Outer product expansions are used in later chapters applications such as digital imaging.

9. Special Web Site and Supplemental Web Materials

Prentice Hall has developed a special Web site to accompany this book. This site includes a host of materials for both students and instructors. The Web pages are being extensively revised for the sixth edition and an exciting collection of new interactive course materials is currently being developed as we go to press. Some of the other features to be included on the Web pages are a collection of links with downloadable materials relating to each of the chapters in the book and a collection of application projects that are related to the topics covered in the book. You can also download two supplemental chapters for this book from the Prentice Hall site. The new chapters are:

- Chapter 8. Iterative Methods
- Chapter 9. Canonical Forms

The URL for the site is listed in the Supplementary Materials section of this Preface.

10. The ATLAST Companion Computer Manual Revision

ATLAST (Augmenting the Teaching of Linear Algebra through the use of Software Tools) is an NSF sponsored project to encourage and facilitate the use of software in the teaching of linear algebra. During a five year period, 1992–1997, the ATLAST Project conducted 18 faculty workshops using the MATLAB software package. Participants in these workshops designed computer exercises, projects, and lesson plans for software-based teaching of linear algebra. A selection of these materials has been published as a manual. *ATLAST Computer Exercises for Linear Algebra* (Prentice Hall, 1997). The ATLAST book is available as a *free* companion volume to this textbook when the two books are wrapped together for class orders. The ISBN for ordering the two-book bundle is given in the Supplementary Materials section of this Preface. The collection of software tools (M-files) developed to accompany the ATLAST book may be downloaded from the ATLAST Web site. You can link to the ATLAST site from the Prentice Hall Web page for this book. A second edition of the ATLAST book is in preparation and publication is expected in the fall of 2002. New developments related to the ATLAST Project and manual will be posted on the ATLAST Web site.

11. Mathematica Computer Exercises and Projects

A collection of ATLAST Mathematica Notebooks has been developed by Richard Neidinger of Davidson College. The collection contains Mathematical versions of the ATLAST projects and exercises. It can be downloaded for free from the ATLAST Web pages.

12. Maple Companion Manual

A new manual, *Visualizing Linear Algebra Using Maple*, by Sandra Keith, is now available as a companion volume to this book. The manual provides an ideal vehicle for those wishing to teach the course using Maple. The Keith manual is offered as a bundle with this book at a special discount price. The ISBN for ordering the two-book bundle is given in the Supplementary Materials section of this Preface.

13. MATLAB Companion Manual

A new manual, *Understanding Linear Algebra Using MATLAB*, by Irwin and Margaret Kleinfeld, is now available as a companion volume to this book. The book has MATLAB problems and projects suitable for a first course in linear algebra. The manual is offered as a bundle with this book at a special discount price. The ISBN for ordering the two-book bundle is given in the Supplementary Materials section of this Preface.

14. Student Guide to Linear Algebra with Applications

A new student study guide has been developed to accompany this edition. The guide is described in the Supplementary Materials section of this preface.

15. Other Changes

In preparing the sixth edition, the author has carefully reviewed every section of the book. In addition to the major changes that have been listed, many new exercises have been added and numerous minor improvements have been made throughout the text.

COMPUTER EXERCISES

This edition contains a section of computing exercises at the end of each chapter. These exercises are based on the software package MATLAB. The MATLAB Appendix in the book explains the basics of using the software. MATLAB has the advantage that it is a powerful tool for matrix computations and yet it is easy to learn. After reading the Appendix, students should be able to do the computing exercises without having to refer to any other software books or manuals. To help students get started we recommend one 50 minute classroom demonstration of the software. The assignments can be done either as ordinary homework assignments or as part of a formally scheduled computer laboratory course.

As mentioned previously, the ATLAST book is available as a companion volume to supplement the computer exercises in this book. Each of the eight chapters of the ATLAST book contains a section of short exercises and a section of longer projects.

While the course can be taught without any reference to the computer, we believe that computer exercises can greatly enhance student learning and provide a new dimension to linear algebra education. The Linear Algebra Curriculum Study Group has recommended that technology be used for a first course in linear algebra, and this view is generally accepted throughout the greater mathematics community.

OVERVIEW OF TEXT

This book is suitable for either a sophomore-level course or for a junior/senior-level course. The student should have some familiarity with the basics of differential and integral calculus. This prerequisite can be met by either one semester or two quarters of elementary calculus.

If the text is used for a sophomore-level course, the instructor should probably spend more time on the early chapters and omit many of the sections in the later chapters. For more advanced courses a quick review of many of the topics in the first two chapters and then a more complete coverage of the later chapters would be appropriate. The explanations in the text are given in sufficient detail so that beginning students should have little trouble reading and understanding the material. To further aid the student, a large number of examples have been worked out completely. Additionally, computer exercises at the end of each chapter give students the opportunity to perform numerical experiments and try to generalize the results. Applications are presented throughout the book. These applications can be used to motivate new material or to illustrate the relevance of material that has already been covered.

The text contains all the topics recommended by the National Science Foundation (NSF) sponsored Linear Algebra Curriculum Study Group (LACSG) and much more. Although there is more material than can be covered in a one-quarter or one-semester course, it is the author's feeling that it is easier for an instructor to leave out or skip material than it is to supplement a book with outside material. Even if many topics are omitted, the book should still provide students with a feeling for the overall scope of the subject matter. Furthermore, many students may use the book later as a reference and consequently may end up learning many of the omitted topics on their own.

In the next section of this preface a number of outlines are provided for one-semester courses at either the sophomore level or the junior/senior level and with either a matrix-oriented emphasis or a slightly more theoretical emphasis. To further aid the instructor in the choice of topics, three sections have been designated as optional and are marked with a dagger in the table of contents. These sections are not prerequisites for any of the following sections in the book. They may be skipped without any loss of continuity.

Ideally the entire book could be covered in a two-quarter or two-semester sequence. Although two semesters of linear algebra has been recommended by the LACSG, it is still not practical at many universities and colleges. At present there is no universal agreement on a core syllabus for a second course. Indeed, if all of the topics that instructors would like to see in a second course were included in a single volume, it would be a weighty (and expensive) book. An effort has been made in this text to cover all of the basic linear algebra topics that are necessary for modern applications. Furthermore, two additional chapters for a second course are available for downloading from the Internet. See the special Prentice Hall Web page discussed earlier.

SUGGESTED COURSE OUTLINES

I. Two-Semester Sequence

In a two semester sequence it is possible to cover all 39 sections of the book. Additional flexibility is possible by omitting any of the three optional sections in Chapters 2, 5, and 6. One could also include an extra lecture demonstrating how to use the MATLAB software.

II. One-Semester Sophomore-Level Course

A. A Basic Sophomore-Level Course

Chapter 1	Section 1–5		7 lectures
Chapter 2	Section 1–2		2 lectures
Chapter 3	Section 1–6		9 lectures
Chapter 4	Section 1–3		4 lectures
Chapter 5	Section 1–6		9 lectures
Chapter 6	Section 1–3		4 lectures
		Total	35 lectures

B. The LACSG Matrix Oriented Course

The core course recommended by the Linear Algebra Curriculum Study involves only the Euclidean vector spaces. Consequently, for this course you should omit Section 1 of Chapter 3 (on general vector spaces) and all references and exercises involving function spaces in Chapters 3 to 6. All of the topics in the LACSG core syllabus are included in the text. It is not necessary to introduce any supplementary materials. The LACSG recommended 28 lectures to cover the core material. This is possible if the class is taught in lecture format with an additional recitation section meeting once a week. If the course is taught with recitations, the author feels that the following schedule of 35 lectures is perhaps more reasonable.

Chapter 1	Sections 1–5		7 lectures
Chapter 2	Sections 1–2		2 lectures
Chapter 3	Sections 2–6		7 lectures
Chapter 4	Sections 1–3		2 lectures
Chapter 5	Sections 1–6		9 lectures
Chapter 6	Sections 1,3–5		8 lectures
		Total	35 lectures

III. One-Semester Junior/Senior-Level Courses

The coverage in an upper division course is dependent on the background of the students. Below are two possible courses with 35 lectures each.

A. Course 1

Chapter 1	Sections 1–5	6 lectures
Chapter 2	Sections 1–2	2 lectures
Chapter 3	Sections 1–6	7 lectures
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1–7	10 lectures
	Section 8 if time allows	
Chapter 7	Section 4	1 lecture

B. Course 2

Review of Topics in Chapters 1–3		5 lectures
Chapter 4	Sections 1–3	2 lectures
Chapter 5	Sections 1–6	10 lectures
Chapter 6	Sections 1–7	11 lectures
	Section 8 if time allows	
Chapter 7	Sections 4–7	7 lectures
	If time allows, Sections 1–3	

SUPPLEMENTARY MATERIALS

- **ATLAST Computer Exercises for Linear Algebra.** The ATLAST book described earlier in this preface is available at no extra charge when ordered as a bundle with this textbook. The ISBN for ordering the two-book bundle is 0-13-096706-8. The ATLAST M-files may be downloaded for free from the ATLAST Web site. The *ATLAST Mathematica Notebooks* may also be downloaded for free from the ATLAST Web site.
- **Visualizing Linear Algebra using Maple,** by Sandra Keith, is available as a companion volume to this book. The manual is offered as a bundle with this book at a special discount price. The ISBN for ordering the two-book bundle is 0-13-074174-4.
- **Understanding Linear Algebra Using MATLAB,** by Irwin and Margaret Kleinfeld, is available as a companion volume to this book. This manual is also available as part of a bundle at a special discount price. This ISBN for ordering the two-book bundle is 0-13-060945-5.
- **Instructor's Solution Manual.** A solutions manual is available to all instructors teaching from this book. The manual contains complete solutions to all the nonroutine

exercises in the book. The manual also contains answers to many of the elementary exercises that were not already listed in the answer key section of the book.

- **Student Guide to Linear Algebra with Applications.** The manual is available to students as a study to accompany this textbook. The manual summarizes important theorems, definitions, and concepts presented in the textbook. It provides solutions to some of the exercises and hints and suggestions on many other exercises.
- **Web Supplements.** The Prentice Hall Web site for this book has an impressive collection of supplementary materials. It is expected to be functional by October 15, 2001. The URL for the Web site is:

http://www.prenhall.com/leon

ACKNOWLEDGMENTS

The author would like to express his gratitude to the long list of reviewers that have contributed so much to all six editions of this book. Thanks also to the many users who have send in comments and suggestions. Thanks also to James Bunch, University of California, San Diego, for his many helpful suggestions.

A special thanks is due the reviewers of the sixth edition:

Steven Kaliszewski, Arizona State University
Anil Nerode, Cornell University
Lanita Presson, University of Alabama in Huntsville
Gilberto Schleiniger, University of Delaware
Carl Swenson, Seattle University

Many of the revisions and new exercises in this latest edition are a direct result of their comments and suggestions.

Thanks to Michael Downes for his help with the LaTeX cls and macro files and thanks to Judith Leon for all of her help and encouragement. Special thanks to Mathematics Editor, George Lobell, for his help in planning the new edition and to Production Manager Bob Walters for his work on coordinating production of the book. Thanks to the entire editorial, production, and sales staff at Prentice Hall for all their efforts.

Finally, the author would like to acknowledge the contributions of Gene Golub and Jim Wilkinson. Most of the first edition of the book was written in 1977–1978 while the author was a Visiting Scholar at Stanford University. During that period the author attended courses and lectures on numerical linear algebra given by Gene Golub and J. H. Wilkinson. Those lectures have greatly influenced this book.

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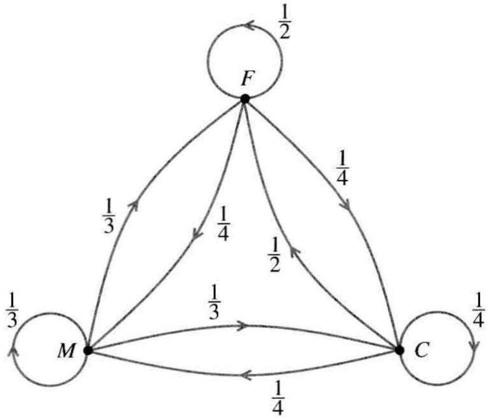
CONTENTS

*Web: The Supplemental Chapters 8 and 9 can be downloaded from the web page: (www.prenhall.com/Leon)
†Optional sections.



CHAPTER 1

MATRICES AND SYSTEMS OF EQUATIONS

Probably the most important problem in mathematics is that of solving a system of linear equations. Well over 75 percent of all mathematical problems encountered in scientific or industrial applications involve solving a linear system at some stage. By using the methods of modern mathematics, it is often possible to take a sophisticated problem and reduce it to a single system of linear equations. Linear systems arise in applications to such areas as business, economics, sociology, ecology, demography, genetics, electronics, engineering, and physics. Therefore, it seems appropriate to begin this book with a section on linear systems.

1 SYSTEMS OF LINEAR EQUATIONS

A *linear equation in n unknowns* is an equation of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, a_2, \ldots, a_n$ and b are real numbers and $x_1, x_2, \ldots, x_n$ are variables. A *linear system* of m equations in n unknowns is then a system of the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned} \tag{1}$$

where the a_{ij}'s and the b_i's are all real numbers. We will refer to systems of the form (1) as $m \times n$ linear systems. The following are examples of linear systems:

(a) $\begin{aligned} x_1 + 2x_2 &= 5 \\ 2x_1 + 3x_2 &= 8 \end{aligned}$ (b) $\begin{aligned} x_1 - x_2 + x_3 &= 2 \\ 2x_1 + x_2 - x_3 &= 4 \end{aligned}$ (c) $\begin{aligned} x_1 + x_2 &= 2 \\ x_1 - x_2 &= 1 \\ x_1 \quad &= 4 \end{aligned}$

System (a) is a 2×2 system, (b) is a 2×3 system, and (c) is a 3×2 system.

By a solution to an $m \times n$ system, we mean an ordered n-tuple of numbers $(x_1, x_2, \ldots, x_n)$ that satisfies all the equations of the system. For example, the ordered pair $(1, 2)$ is a solution to system (a), since

$$1 \cdot (1) + 2 \cdot (2) = 5$$
$$2 \cdot (1) + 3 \cdot (2) = 8$$

The ordered triple $(2, 0, 0)$ is a solution to system (b), since

$$1 \cdot (2) - 1 \cdot (0) + 1 \cdot (0) = 2$$
$$2 \cdot (2) + 1 \cdot (0) - 1 \cdot (0) = 4$$

Actually, system (b) has many solutions. If α is any real number, it is easily seen that the ordered triple $(2, \alpha, \alpha)$ is a solution. However, system (c) has no solution. It follows from the third equation that the first coordinate of any solution would have to be 4. Using $x_1 = 4$ in the first two equations, we see that the second coordinate must satisfy

$$4 + x_2 = 2$$
$$4 - x_2 = 1$$

Since there is no real number that satisfies both of these equations, the system has no solution. If a linear system has no solution, we say that the system is *inconsistent*. If the system has at least one solution, we say that it is *consistent*. Thus system (c) is inconsistent, while systems (a) and (b) are both consistent.

The set of all solutions to a linear system is called the *solution set* of the system. If a system is inconsistent, its solution set is empty. A consistent system will have a nonempty solution set. To solve a consistent system, we must find its solution set.

2×2 Systems

Let us examine geometrically a system of the form

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

Each equation can be represented graphically as a line in the plane. The ordered pair (x_1, x_2) will be a solution to the system if and only if it lies on both lines. For example, consider the three systems

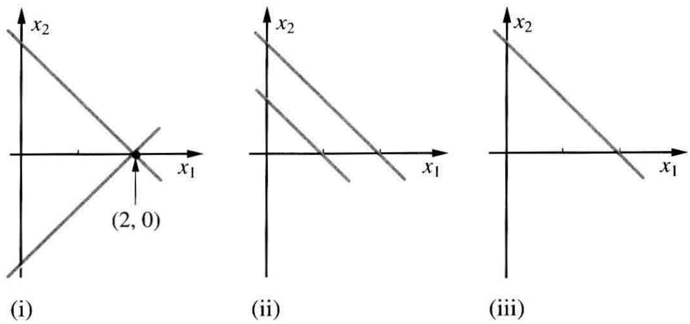


FIGURE 1.1.1

(i) $\begin{aligned} x_1 + x_2 &= 2 \\ x_1 - x_2 &= 2 \end{aligned}$ (ii) $\begin{aligned} x_1 + x_2 &= 2 \\ x_1 + x_2 &= 1 \end{aligned}$ (iii) $\begin{aligned} x_1 + x_2 &= 2 \\ -x_1 - x_2 &= -2 \end{aligned}$

The two lines in system (i) intersect at the point $(2, 0)$. Thus $\{(2, 0)\}$ is the solution set to (i). In system (ii) the two lines are parallel. Therefore, system (ii) is inconsistent and hence its solution set is empty. The two equations in system (iii) both represent the same line. Any point on that line will be a solution to the system (see Figure 1.1.1).

In general, there are three possibilities: the lines intersect at a point, they are parallel, or both equations represent the same line. The solution set then contains either one, zero, or infinitely many points.

The situation is similar for $m \times n$ systems. An $m \times n$ system may or may not be consistent. If it is consistent, it must either have exactly one solution or infinitely many solutions. These are the only possibilities. We will see why this is so in Section 2 when we study the row echelon form. Of more immediate concern is the problem of finding all solutions to a given system. To tackle this problem, we introduce the notion of *equivalent systems*.

Equivalent Systems

Consider the two systems

(a) $\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ x_2 &= 3 \\ 2x_3 &= 4 \end{aligned}$ (b) $\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ -3x_1 - x_2 + x_3 &= 5 \\ 3x_1 + 2x_2 + x_3 &= 2 \end{aligned}$

System (a) is easy to solve because it is clear from the last two equations that $x_2 = 3$ and $x_3 = 2$. Using these values in the first equation, we get

$$\begin{aligned} 3x_1 + 2 \cdot 3 - 2 &= -2 \\ x_1 &= -2 \end{aligned}$$

Thus the solution to the system is $(-2, 3, 2)$. System (b) seems to be more difficult to solve. Actually, system (b) has the same solution as system (a). To see this, add the first two equations of the system: