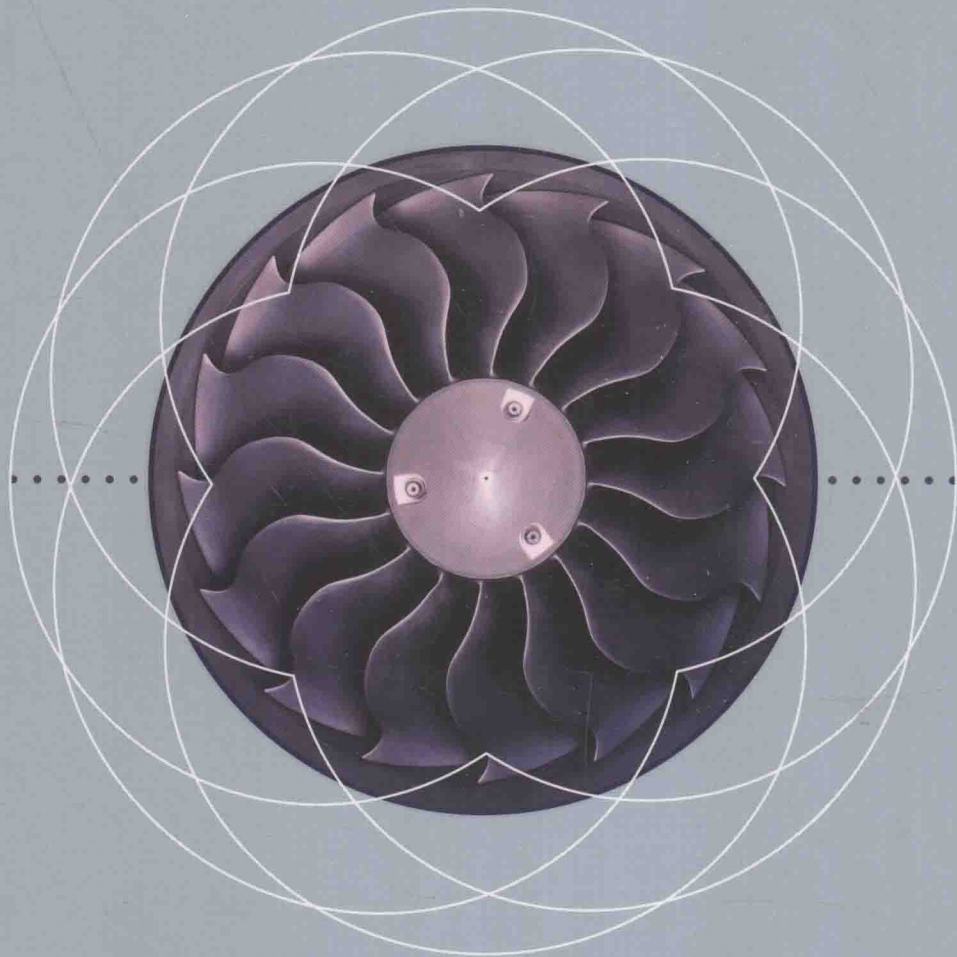


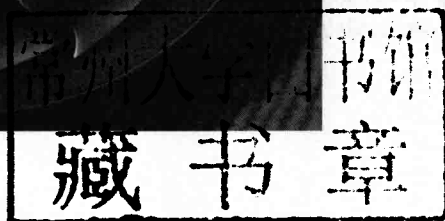
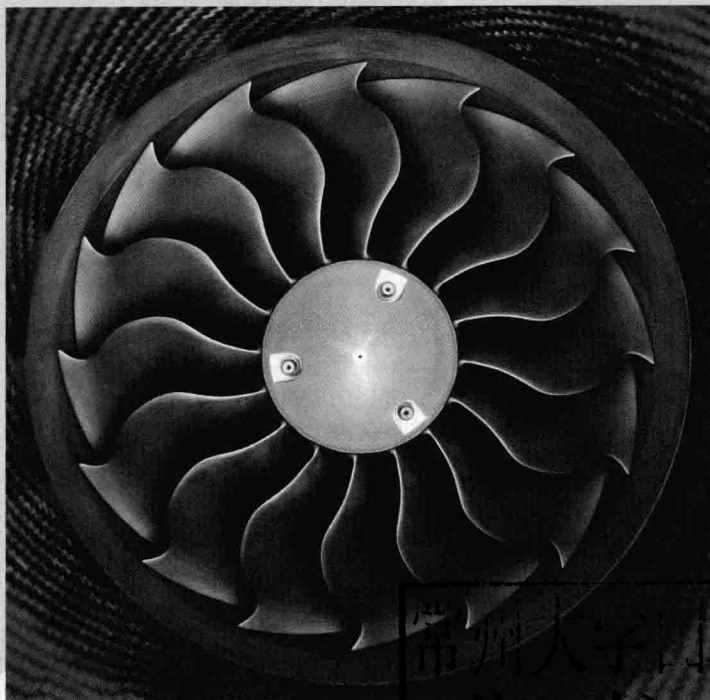
applied Calculus ^{5e}



Waner/Costenoble

Applied Calculus

Fifth Edition



Stefan Waner

Hofstra University

Steven R. Costenoble

Hofstra University



BROOKS/COLE
CENGAGE Learning[®]

Australia • Brazil • Japan • Korea • Mexico • Singapore • Spain • United Kingdom • United States

Applied Calculus, Fifth Edition

Stefan Waner, Steven R. Costenoble

Publisher: Richard Stratton

Senior Acquisitions Editor: Liz Covello

Associate Editor: Jeannine Lawless

Editorial Assistant: Lauren Hamel

Marketing Manager: Ashley Pickering

Marketing Coordinator: Erica O'Connell

Marketing Communications Manager:
Mary Anne Payumo

Content Project Manager: Susan Miscio

Senior Art Director: Jill Ort

Print Buyer: Diane Gibbons

Permissions Editor: Margaret
Chamberlain-GastonProduction Service: MPS Content
Services

Text Designer: Henry Rachlin

Photo Manager: Don Schlotman

Photo Researcher: Pre-Press PMG

Cover Designer: Monica DeSalvo

Cover Image: © Getty Images/
Chip Forelli

Compositor: MPS Content Services

© 2011, 2007 Brooks/Cole, Cengage Learning

ALL RIGHTS RESERVED. No part of this work covered by the copyright herein may be reproduced, transmitted, stored or used in any form or by any means graphic, electronic, or mechanical, including but not limited to photocopying, recording, scanning, digitizing, taping, Web distribution, information networks, or information storage and retrieval systems, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without the prior written permission of the publisher.

For product information and technology assistance, contact us
at **Cengage Learning Customer & Sales Support,**
1-800-354-9706

For permission to use material from this text or product,
submit all requests online at **www.cengage.com/permissions**.
Further permissions questions can be emailed to
permissionrequest@cengage.com.

Library of Congress Control Number: 2009924247

Student Edition:

ISBN-13: 978-1-4390-4923-5

ISBN-10: 1-4390-4923-8

Brooks/Cole20 Davis Drive
Belmont, CA 94002
USA

Cengage Learning products are represented in Canada by Nelson
Education, Ltd.

For your course and learning solutions, visit
www.cengage.com.

Purchase any of our products at your local college store or at our
preferred online store **www.cengagebrain.com**.

PREFACE

Applied Calculus, Fifth Edition, is intended as a one- or two-term course for students majoring in business, the social sciences, or the liberal arts. Like earlier editions, the Fifth Edition of *Applied Calculus* is designed to address the challenge of generating enthusiasm and mathematical sophistication in an audience that is too often underprepared for and uninspired by traditional mathematics courses. We meet this challenge by focusing on real-life applications that students can relate to, by presenting mathematical concepts intuitively and thoroughly, and by employing a writing style that is informal, engaging, and occasionally humorous.

In renewing our commitment to these goals, we have further improved the text by revising the Table of Contents—combining some sections and rearranging others—to promote better flow and organization, enhancing the technology notes to include step-by-step instructions at point of use, and adding notes to the student to provide further clarification and explanation where necessary. Please see the “New to This Edition” section that follows for additional information about these changes.

The Fifth Edition continues to implement support for a wide range of instructional paradigms: from settings using little or no technology to courses taught in computerized classrooms, and from classes in which a single form of technology is used exclusively to those incorporating several technologies. We feature three forms of technology in this text—TI-83/84 Plus graphing calculators, Excel spreadsheets, and powerful online utilities created specifically for this book—in a way that allows them to be integrated or omitted. In particular, our comprehensive support for Excel, both in the text and online, is highly relevant for students who are studying business and economics, where skill with spreadsheets may be vital to their future careers.

Our Approach to Pedagogy

Real World Orientation We are particularly proud of the diversity, breadth, and abundance of examples and exercises included in this edition. A large number are based on real, referenced data from business, economics, the life sciences, and the social sciences. Examples and exercises based on dated information have generally been replaced by more current versions; applications based on unique or historically interesting data have been kept.

Adapting real data for pedagogical use can be tricky; available data can be numerically complex, intimidating for students, or incomplete. We have modified and streamlined many of the real world applications, rendering them as tractable as any “made-up” application. At the same time, we have been careful to strike a pedagogically sound balance between applications based on real data and more traditional “generic” applications. Thus, the density and selection of real data-based applications have been tailored to the pedagogical goals and appropriate difficulty level for each section.

Readability We would like students to read this book. We would also like students to *enjoy* reading this book. Thus, we have written the book in a conversational and student-oriented style, and have made frequent use of question-and-answer dialogues to encourage the development of the student’s mathematical curiosity and intuition. We hope that this text will give the student insight into how a mathematician develops and thinks about mathematical ideas and their applications.

Rigor We feel that mathematical rigor can work easily in conjunction with the kind of applied focus and conceptual approach that are earmarks of this book. We have, especially in the Fifth Edition, worked hard to ensure that we are always mathematically honest without being unnecessarily formal. Sometimes we do this through the question-and-answer dialogues and sometimes through the “Before we go on . . .” discussions that follow examples, but always in a manner designed to provoke the interest of the student.

Five Elements of Mathematical Pedagogy to Address Different Learning Styles

The “Rule of Four” is a common theme in many texts. Implementing this approach, we discuss many of the central concepts **numerically**, **graphically** and **algebraically**, and clearly delineate these distinctions. The fourth element, **verbal communication** of mathematical concepts, is emphasized through our discussions on translating English sentences into mathematical statements, and our Communication and Reasoning exercises at the end of each section. A fifth element, **interactivity**, is integrated through expanded use of question-and-answer dialogues, but is seen most dramatically within the student Web site. Using this resource, students can use interactive tutorials specific to concepts and examples covered in sections and online utilities that automate a variety of tasks, from graphing to regression and matrix algebra. Added recently to the site are more challenging “game” tutorials, with randomized questions, scoring systems, and even “health points.”

Exercise Sets The substantial collection of exercises provides a wealth of material that can be used to challenge students at almost every level of preparation, and includes everything from straightforward drill exercises to interesting and rather challenging applications. The exercise sets have been carefully graded to increase in complexity from basic exercises and exercises that are similar to examples in the text to more interesting and advanced ones, marked in this edition as “more advanced” for easy reference. There are also several much more difficult exercises, designated as “challenging.” The advanced and challenging exercises encourage students to think beyond the straightforward situations and calculations in the earlier exercises. We have also included, in virtually every section, interesting applications based on real data, Communication and Reasoning exercises that help students articulate mathematical concepts, and exercises ideal for the use of technology.

Many of the scenarios used in application examples and exercises are revisited several times throughout the book. Thus, students will find themselves using a variety of techniques, from graphing through the use of derivatives and elasticity, to analyze the same application. Reusing scenarios and important functions provides unifying threads and shows students the complex texture of real-life problems.

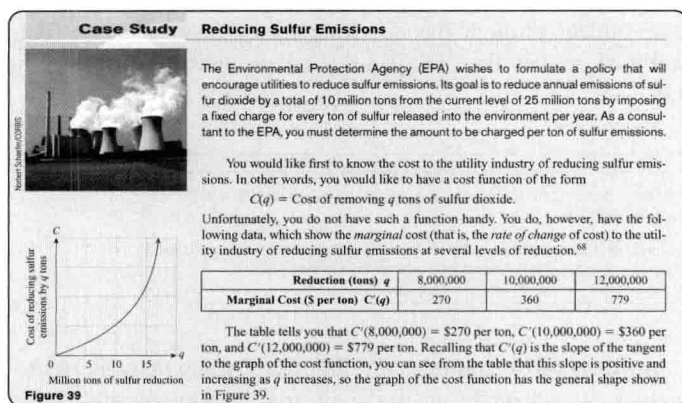
New to This Edition

Content

- Chapter 1 (page 39): Chapter 1 now includes a new section, *Functions and Models*, in which we bring together a variety of applied topics such as revenue, profit, demand, supply, and change over time that occur throughout the book. We have also streamlined the rest of the chapter; functions are now introduced in a single section rather than two, and linear functions and models are similarly discussed in a single section.
- Chapter 4 (page 285): Derivatives of sums and constant multiples are now moved to the beginning of Chapter 4 on techniques of differentiation (where they properly belong).
- Chapter 5 (page 369): Second and higher order derivatives are now discussed in their own section with a great deal of added material on concavity and acceleration.

- Chapter 8 (page 583): We have changed the exposition of constrained maxima and minima to give more emphasis to Lagrange multipliers over substitution methods. Functions of several variables are now discussed in a single section rather than two.
- We now include the discussion of derivatives and integrals of functions involving absolute values in simple closed form, thus expanding the variety of functions available for modeling.
- We have expanded the list of additional optional sections, available to include in custom-published versions of the text, and now offer sections on: Taylor polynomials, the chain rule for multivariate calculus, calculus applied to probability, the extreme value theorem and optimization with boundary constraints for functions of several variables, and determinants and Cramer's rule.

Current Topics in the Applications



- **Case Studies** Each chapter ends with a section entitled “Case Study,” an extended application that uses and illustrates the central ideas of the chapter, focusing on the development of mathematical models appropriate to the topics. The Case Studies have been extensively revised, and in many cases completely replaced by new ones that reflect topics of current interest.
- We have added numerous real data exercises and examples based on topics that are either of intense current interest or of general interest to contemporary students, including *Facebook*, *XBoxes*, *iPhones*, *eBay*, real estate foreclosures and home construction, subprime mortgages, stock market gyrations, travel to Cancun, and the oil industry in the United States and Mexico. (Also see the Index of Companies, in the inside back cover, of the corporations we reference in the applications.)

Exercises

76. Information Highway The amount of information transmitted each month in the early years of the Internet (1988 to 1994) can be modeled by the equation

$$q(t) = \frac{2e^{0.16t}}{3 + 1.5e^{-0.4t}} \quad (0 \leq t \leq 6)$$

where q is the amount of information transmitted each month in billions of data packets and t is the number of years since the start of 1988.³³

- Use technology to estimate $q'(2)$.
- Assume that it costs \$5 to transmit a million packets of data. What is the marginal cost $C'(q)$?
- How fast was the cost increasing at the start of 1990?

Money Stock Exercises 77–80 are based on the following demand function for money (taken from a question on the GRE Economics Test):

$$M_d = 2 \times y^{0.6} \times r^{-0.3} \times p$$

where

- M_d = demand for nominal money balances (money stock)
- y = real income
- r = an index of interest rates
- p = an index of prices

These exercises also use the idea of **percentage rate of growth**:

$$\text{Percentage Rate of Growth of } M = \frac{\text{Rate of Growth of } M}{M} = \frac{dM/dt}{M}$$

- Exercises that are not based entirely on examples in the text are designated as “more advanced” (and indicated by an icon in the exercise set) as a guide for students and instructors.
- We have expanded the exercise sets themselves and carefully reorganized them to gradually increase in level and to include more basic skills exercises that carefully follow the examples.
- Many more of the exercises now have “hints” that either refer to an example in the text where a similar problem is solved, or offer some advice to the student.
- We have added numerous new “communication and reasoning” exercises—many dealing with common student errors and misconceptions—and further expanded the chapter review exercise sections.

87. Formulate a simple procedure for deciding whether to apply first the chain rule, the product rule, or the quotient rule when finding the derivative of a function.

88. Give an example of a function f with the property that calculating $f'(x)$ requires use of the following rules in the given order: (1) the chain rule, (2) the quotient rule, and (3) the chain rule.

89. Give an example of a function f with the property that calculating $f'(x)$ requires use of the chain rule five times in succession.

90. What can you say about composites of linear functions?

* **NOTE** See the section on exponents in the algebra review to brush up on negative and fractional exponents.



using Technology

See the Technology Guide at the end of the chapter for detailed instructions on how to calculate the average rate of change of the function in Example 3 using a TI-83/84 Plus or Excel. Here is an outline:

TI-83/84 Plus

$Y_1 = -8X^2 + 144X + 150$

Home screen: $(Y_1(9.5) -$

$Y_1(8)) / (9.5 - 8)$

[More details on page 280.]

Excel

Enter the headings t , $G(t)$,

Rate of Change in A1–C1

t -values 8, 9.5 in A2–A3

$= -8 * A2^2 + 144 * A2 + 150$

in B2 and copy down to B3

$= (B3 - B2) / (A3 - A2)$ in C2.

[More details on page 282.]

Pedagogy

- **Supplementary Notes** We have added new Notes to the student, located in the side column of the text. These Notes include a wide variety of additional information for the student—further explanation or clarification of a concept, reminders of previously learned material or references for further study, and additional tips for using technology.
- **Technology Notes** have been enhanced to include step-by-step instructions and keystrokes at point of use, to enable better integration of graphing calculators and spreadsheets. As always, these notes can be omitted without loss of continuity.

Hallmark Features

- **Question-and-Answer Dialogue** We frequently use informal question-and-answer dialogues that anticipate the kind of questions that may occur to the student and also guide the student through the development of new concepts. This feature has been streamlined, as has the “Frequently Asked Questions” feature at the end of each section.

Q : Do we always have to calculate the limit of the difference quotient to find a formula for the derivative function?

A : As it turns out, no. In Section 4.1 we will start to look at shortcuts for finding derivatives that allow us to bypass the definition of the derivative in many cases.

- **Before we go on . . . feature** Most examples are followed by supplementary discussions, which may include a check on the answer, a discussion of the feasibility and significance of a solution, or an in-depth look at what the solution means.
- **Quick Examples** Most definition boxes include quick, straightforward examples that a student can use to solidify each new concept.

Functions with Equal Limits

If $f(x) = g(x)$ for all x except possibly $x = a$, then

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x).$$

Quick Example

$$\frac{x^2 - 1}{x - 1} = x + 1 \text{ for all } x \text{ except } x = 1. \quad \text{Write } \frac{x^2 - 1}{x - 1} \text{ as } \frac{(x + 1)(x - 1)}{x - 1} \text{ and cancel the } (x - 1)$$

Therefore,

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} (x + 1) = 1 + 1 = 2.$$

- **Communication and Reasoning Exercises for Writing and Discussion** These are exercises designed to broaden the student’s grasp of the mathematical concepts and develop modeling skills. They include exercises in which the student is asked to provide his or her own examples to illustrate a point or design an application with a given solution. They also include fill-in-the-blank type exercises and exercises that invite discussion and debate. These exercises often have no single correct answer.

TI-83/84 Plus Technology Guide **Section 3.1**

Example 1 (page 193) Use a table to estimate the following limits.

a. $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$ b. $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$

Solution with Technology

On the TI-83/84 Plus, use the table feature to automate these computations as follows:

1. Define $Y_1 = (X^3 - 8) / (X - 2)$ for part (a) or $Y_1 = (e^{2X} - 1) / X$ for part (b).

2. Press 2ND TABLE to list its values for the given values of x . (If the calculator does not allow you to enter values of x , press 2ND TBLSET and set Indpnt to Ask).

Example 4 (page 230) Continuing with Example 3, use technology to compute the average rate of change of

$$G(t) = -8t^2 + 144t + 150 \quad (7.5 \leq t \leq 10.5)$$

over the intervals $[8, 8 + h]$, where $h = 1, 0.1, 0.01, 0.001$, and 0.0001 .

Solution with Technology

1. As in Example 3, enter the function G as Y_1 (using X for t):

$$Y_1 = -8X^2 + 144X + 150$$

2. Now find the average rate of change for $h = 1$ by evaluating, on the home screen,

$$(Y_1(8+1) - Y_1(8)) / 1$$

EXCEL Technology Guide **Section 3.1**

Example 1 (page 193) Use a table to estimate the following limits.

a. $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$ b. $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$

Solution with Technology

1. Set up your spreadsheet to duplicate the table in part (a) as follows:

	A	B	C	D
1	x	f(x)		f(x)
2	1.9	$=(A2^3-8)/(A2-2)$	2.1	
3	1.99		2.01	
4	1.999		2.001	
5	1.9999		2.0001	

2. Next, calculate the average rate of change as shown here:

	A	B	C
1	t	G(t)	
2	8	790	Rate of change over [8, 9.5]
3	9.5	796	$=(B3-B2)/(A3-A2)$

	A	B	C	D
1	t	G(t)		
2	8	790	Rate of change over [8, 9.5]	
3	9.5	796		4

In Example 4, we describe another, more versatile Excel template for computing rates of change. ■

Example 4 (page 230) Continuing with Example 3, use technology to compute the average rate of change of

can easily use this material or not, as they prefer. Groups of exercises for which the use of technology is suggested or required appear throughout the exercise sets.

The Web Site

The authors' Web site, accessible through www.AppliedCalc.com, has been evolving for several years, with growing recognition. Students, raised in an environment in which computers suffuse both work and play, can use their Web browsers to engage with the material in an active way. The following features of the authors' Web site are fully integrated with the text and can be used as a personalized study resource:

- **Interactive Tutorials** Highly interactive tutorials, with guided exercises that parallel the text and a great deal of help and feedback to assist the student.
- **Game Versions of Tutorials** More challenging tutorials with randomized questions that work as games (complete with "health" scores, "health vials," and an assessment of one's performance at the end of the game) are offered alongside the traditional tutorials. These game tutorials, which mirror the traditional "more gentle" tutorials, do not give the student the answers, but instead offer hints in exchange for health points, so that just staying alive (not running out of health) can be quite challenging. Because the questions are randomized and scores are automatically calculated, these tutorials can be used for in-class quizzes, as the authors themselves have done.
- **Detailed Chapter Summaries** Comprehensive summaries with interactive elements review all the basic definitions and problem solving techniques discussed in each chapter. These are a terrific pre-test study tool for students.
- **Downloadable Excel Tutorials** Detailed Excel tutorials are available for almost every section of the book. These interactive tutorials expand on the examples given in the text.
- **Online Utilities** Our collection of easy-to-use online utilities, written in JavaScript, allows students to solve many of the technology-based application exercises directly on the Web page. The utilities available include a function grapher and evaluator that also does derivatives, regression tools, and a numerical integration tool. These utilities require nothing more than a standard Web browser.
- **Chapter True-False Quizzes** Short quizzes based on the key concepts in each chapter assist the student in further mastery of the material.

- **Supplemental Topics** We include complete interactive text and exercise sets for a selection of topics not ordinarily included in printed texts, but often requested by instructors.
- **Spanish** A parallel Spanish version of the entire Web site is also being developed. All of the chapter summaries and many of the tutorials, game tutorials, and utilities are already available in Spanish, with many more resources to come.

Supplemental Material

For Students

Student Solutions Manual by *Waner and Costenoble*

ISBN-10: **1439049904**, ISBN-13: **9781439049907**

The student solutions manual provides worked-out solutions to the odd-numbered exercises in the text as well as complete solutions to all the chapter review tests.

Microsoft Excel Computer Laboratory Manual by *Anne D. Henriksen*

ISBN-10: **0538733209**, ISBN-13: **9780538733205**

This laboratory manual illustrates how Microsoft Excel can be used to solve real-world problems in a variety of scientific, technical, and business disciplines. It provides hands-on experience to demonstrate for students that calculus is a valuable tool for solving practical, real-world problems, while helping students increase their knowledge of Microsoft Excel. The manual is a set of self-contained computer exercises that are meant to be used over the course of a 15-week semester in a separate, 75-minute computer laboratory period. The weekly labs parallel the material in the text.



NetTutor™

Cengage Learning is pleased to provide students with online tutoring and chatting capabilities through NetTutor. NetTutor utilizes a WorldWideWhiteboard application, a Web-based application that allows students and tutors to interact with one another through text and images, and also offers audio and video over Internet Protocol or VOIP, where voice is transmitted through the Internet. The WorldWide Whiteboard software offers the ability to easily create custom groups, which contain course-specific functions, symbol palettes and buttons.

For Instructors

Instructor's Solution Manual by *Waner and Costenoble*

ISBN-10: **1439049831**, ISBN-13: **9781439049839**

The instructor's solutions manual provides worked-out solutions to all of the exercises in the text.

PowerLecture CD with ExamView

ISBN-10: **1439049890**, ISBN-13: **9781439049891**

This comprehensive CD-ROM contains the Instructor's Solutions Manual, PowerPoint lecture notes, and ExamView computerized testing to create, deliver, and customize tests. The PowerLecture CD also includes a multimedia library containing all of the art from the book in MS PowerPoint as well as individual JPEG files.

Test Bank

The test bank—available on the companion Web site—contains numerous multiple choice and free response questions for those instructors who prefer a more traditional method of test preparation.



Enhanced WebAssign

Instant feedback and ease of use are just two reasons why WebAssign is the most widely used homework system in higher education. WebAssign allows you to assign, collect, grade, and record homework assignments via the Web. Now this proven homework system has been enhanced to include links to textbook sections, video examples, and problem-specific tutorials. Enhanced WebAssign is more than a homework system—it is a complete learning system for math students.

Acknowledgments

This project would not have been possible without the contributions and suggestions of numerous colleagues, students, and friends. We are particularly grateful to our colleagues at Hofstra and elsewhere who used and gave us useful feedback on previous editions. We are also grateful to everyone at Cengage Learning for their encouragement and guidance throughout the project. Specifically, we would like to thank Carolyn Crockett and Liz Covello for their unflagging enthusiasm and Jeannine Lawless for whipping the book into shape.

We would also like to thank Jerrold Grossman for his meticulous critique of the manuscript, Lynn Lustberg and the production team for their patience in dealing with our often shifting demands, and the numerous reviewers and accuracy checkers who provided many helpful suggestions that have shaped the development of this book.

Doug Burkholder, *Lenoir-Rhyne University*

Leslie Cohn, *The Citadel*

Melanie Fulton, *High Point University*

Jerrold W. Grossman, *Oakland University*

Celeste Hernandez, *Richland College*

Dean Leoni, *Edmonds Community College*

Michael J. Kallagher, *Washington State University*

Karla Karstans, *University of Vermont*

Vincent Koehler, *University of Vermont*

Karl Kruppstadt, *University of Minnesota, Duluth*

Robert H. Lewis, *Fordham University*

David Miller, *William Paterson University*

Phillip Miller, *Indiana University Southeast*

Jack Y. Narayan, *SUNY Oswego*

Sergei Ovchinnikov, *San Francisco State University*

Lauri Papay, *Santa Clara University*

Jean Nicholas Pestieau, *Suffolk County Community College*

Leela Rakesh, *Central Michigan University*

Nelson de la Rosa, *Miami Dade College, Kendall*

Arthur Rosenthal, *Salem State College*

Carol H. Serotta, *Cabrini College*

Mary Ann Teel, *University of North Texas*

Tzvetalin S. Vassilev, *North Carolina Central University*

Marie A. Vitulli, *University of Oregon*

Richard West, *Francis Marion University*

Donna Wilson, *University of Texas at El Paso*

Stefan Waner

Steven R. Costenoble

BRIEF CONTENTS

CHAPTER 0	Precalculus Review	1
CHAPTER 1	Functions and Linear Models	39
CHAPTER 2	Nonlinear Functions and Models	123
CHAPTER 3	Introduction to the Derivative	191
CHAPTER 4	Techniques of Differentiation with Applications	285
CHAPTER 5	Further Applications of the Derivative	369
CHAPTER 6	The Integral	451
CHAPTER 7	Further Integration Techniques and Applications of the Integral	517
CHAPTER 8	Functions of Several Variables	583
CHAPTER 9	Trigonometric Models	649

CONTENTS

CHAPTER 0

Precalculus Review

1

Introduction

- 0.1 Real Numbers 2
- 0.2 Exponents and Radicals 7
- 0.3 Multiplying and Factoring Algebraic Expressions 17
- 0.4 Rational Expressions 22
- 0.5 Solving Polynomial Equations 24
- 0.6 Solving Miscellaneous Equations 30
- 0.7 The Coordinate Plane 34

CHAPTER 1

Functions and Linear Models

39

Introduction

- 1.1 Functions from the Numerical, Algebraic, and Graphical Viewpoints 40
- 1.2 Functions and Models 56
- 1.3 Linear Functions and Models 75
- 1.4 Linear Regression 94

KEY CONCEPTS 104

REVIEW EXERCISES 104

CASE STUDY Modeling Spending on Internet Advertising 106

■ **TECHNOLOGY GUIDES** 110

CHAPTER 2

Nonlinear Functions and Models

123

Introduction

- 2.1 Quadratic Functions and Models 124
- 2.2 Exponential Functions and Models 136
- 2.3 Logarithmic Functions and Models 151
- 2.4 Logistic Functions and Models 165

KEY CONCEPTS 176

REVIEW EXERCISES 176

CASE STUDY Checking up on Malthus 179

■ **TECHNOLOGY GUIDES** 185

CHAPTER 3 Introduction to the Derivative 191

Introduction

- 3.1 Limits: Numerical and Graphical Approaches 192
- 3.2 Limits and Continuity 204
- 3.3 Limits and Continuity: Algebraic Approach 210
- 3.4 Average Rate of Change 224
- 3.5 Derivatives: Numerical and Graphical Viewpoints 239
- 3.6 The Derivative: Algebraic Viewpoint 263

KEY CONCEPTS 273

REVIEW EXERCISES 273

CASE STUDY Reducing Sulfur Emissions 277

■ **TECHNOLOGY GUIDES** 280

CHAPTER 4 Techniques of Differentiation with Applications 285

Introduction

- 4.1 Derivatives of Powers, Sums, and Constant Multiples 286
- 4.2 A First Application: Marginal Analysis 302
- 4.3 The Product and Quotient Rules 313
- 4.4 The Chain Rule 325
- 4.5 Derivatives of Logarithmic and Exponential Functions 337
- 4.6 Implicit Differentiation 352

KEY CONCEPTS 359

REVIEW EXERCISES 359

CASE STUDY Projecting Market Growth 362

CHAPTER 5 Further Applications of the Derivative 369

Introduction

- 5.1 Maxima and Minima 370
- 5.2 Applications of Maxima and Minima 384
- 5.3 Higher Order Derivatives: Acceleration and Concavity 398
- 5.4 Analyzing Graphs 414
- 5.5 Related Rates 422
- 5.6 Elasticity 432

KEY CONCEPTS 440

REVIEW EXERCISES 440

CASE STUDY Production Lot Size Management 444

■ **TECHNOLOGY GUIDES** 448

CHAPTER 6 The Integral**451**

Introduction

6.1 The Indefinite Integral 452**6.2** Substitution 467**6.3** The Definite Integral: Numerical and Graphical Approaches 478**6.4** The Definite Integral: Algebraic Approach and the Fundamental Theorem of Calculus 496

KEY CONCEPTS 506

REVIEW EXERCISES 506

CASE STUDY Spending on Housing Construction 509■ **TECHNOLOGY GUIDES** 514**CHAPTER 7 Further Integration Techniques and Applications of the Integral****517**

Introduction

7.1 Integration by Parts 518**7.2** Area Between Two Curves and Applications 527**7.3** Averages and Moving Averages 536**7.4** Applications to Business and Economics: Consumers' and Producers' Surplus and Continuous Income Streams 545**7.5** Improper Integrals and Applications 555**7.6** Differential Equations and Applications 565

KEY CONCEPTS 573

REVIEW EXERCISES 573

CASE STUDY Estimating Tax Revenues 575■ **TECHNOLOGY GUIDES** 581**CHAPTER 8 Functions of Several Variables****583**

Introduction

8.1 Functions of Several Variables from the Numerical, Algebraic, and Graphical Viewpoints 584**8.2** Partial Derivatives 602**8.3** Maxima and Minima 610**8.4** Constrained Maxima and Minima and Applications 619**8.5** Double Integrals and Applications 628

KEY CONCEPTS 637

REVIEW EXERCISES 637

CASE STUDY Modeling College Population 640■ **TECHNOLOGY GUIDES** 646

CHAPTER 9 Trigonometric Models**649**

Introduction

- 9.1** Trigonometric Functions, Models, and Regression 650
- 9.2** Derivatives of Trigonometric Functions and Applications 664
- 9.3** Integrals of Trigonometric Functions and Applications 672

KEY CONCEPTS 680

REVIEW EXERCISES 680

CASE STUDY Predicting Airline Empty Seat Volume 681■ **TECHNOLOGY GUIDES** 687

ANSWERS TO SELECTED EXERCISES A1

INDEX I1

O

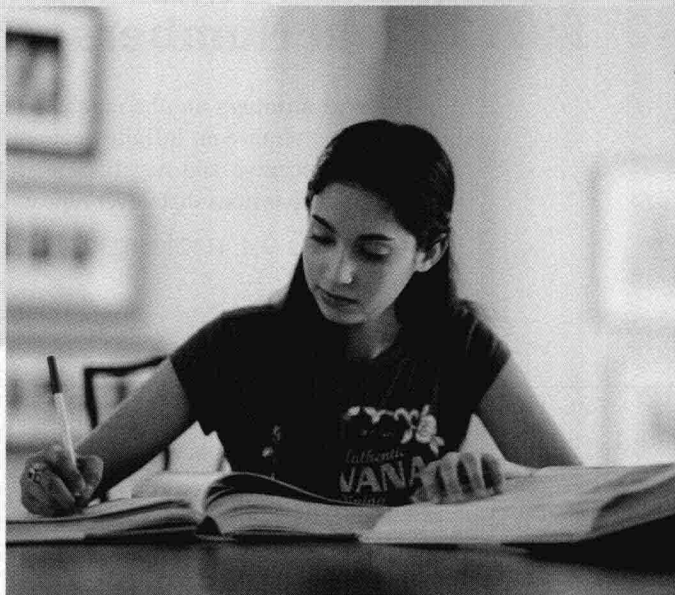
Precalculus Review

- 0.1** Real Numbers
- 0.2** Exponents and Radicals
- 0.3** Multiplying and Factoring Algebraic Expressions
- 0.4** Rational Expressions
- 0.5** Solving Polynomial Equations
- 0.6** Solving Miscellaneous Equations
- 0.7** The Coordinate Plane

Web Site

www.AppliedCalc.org

- At the Web site you will find section-by-section interactive tutorials for further study and practice.



DreamPictures/Taxi/Getty Images

Introduction

In this appendix we review some topics from algebra that you need to know to get the most out of this book. This appendix can be used either as a refresher course or as a reference.

There is one crucial fact you must always keep in mind: The letters used in algebraic expressions stand for numbers. All the rules of algebra are just facts about the arithmetic of numbers. If you are not sure whether some algebraic manipulation you are about to do is legitimate, try it first with numbers. If it doesn't work with numbers, it doesn't work.

0.1 Real Numbers

The **real numbers** are the numbers that can be written in decimal notation, including those that require an infinite decimal expansion. The set of real numbers includes all integers, positive and negative; all fractions; and the irrational numbers, those with decimal expansions that never repeat. Examples of irrational numbers are

$$\sqrt{2} = 1.414213562373 \dots$$

and

$$\pi = 3.141592653589 \dots$$

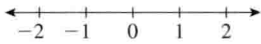


Figure 1

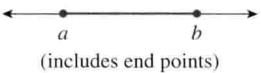
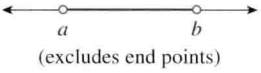


It is very useful to picture the real numbers as points on a line. As shown in Figure 1, larger numbers appear to the right, in the sense that if $a < b$ then the point corresponding to b is to the right of the one corresponding to a .

Intervals

Some subsets of the set of real numbers, called **intervals**, show up quite often and so we have a compact notation for them.

Interval Notation

Here is a list of types of intervals along with examples.

	<i>Interval</i>	<i>Description</i>	<i>Picture</i>	<i>Example</i>
Closed	$[a, b]$	Set of numbers x with $a \leq x \leq b$	 (includes end points)	$[0, 10]$
Open	(a, b)	Set of numbers x with $a < x < b$	 (excludes end points)	$(-1, 5)$
Half-Open	$(a, b]$	Set of numbers x with $a < x \leq b$		$(-3, 1]$
	$[a, b)$	Set of numbers x with $a \leq x < b$		$[0, 5)$