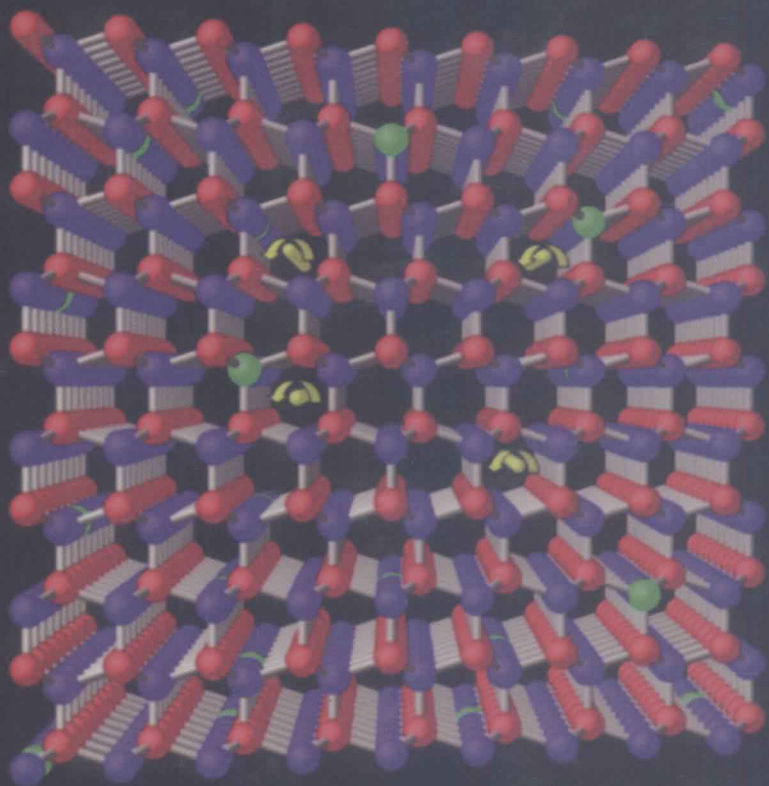


INTRODUCTION TO **Spintronics**



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Dedication

S. B. dedicates this book to Bimalendu, Bela, Anuradha and Saumil Bandyopadhyay. M. C. thanks his wife Maureen for her constant support and dedicates this book to the memory of his father and father-in-law.

Preface

This is a textbook intended to introduce a student of engineering, materials science, and/or applied physics, to the field of *Spintronics*. While the term “spintronics” may have different connotations for different people, in this textbook it deals primarily with the science and technology of using the spin degree of freedom of a charge carrier to store, encode, access, process and/or transmit information in some way. That role had been traditionally delegated to the “charge” of an electron, not its “spin”. Interest in spintronics is motivated by a long standing belief that replacing charge with spin in signal processing devices may yield some advantages in terms of increased processing speed, lower power consumption and/or increased device density. While this may not be always true, there are some scenarios where it *may* become true in the near future. In this textbook, we place particular emphasis on identifying situations where “spin” may have an advantage over “charge”, and where it may not (see, in particular, Chapters 12-14).

The recent advent of quantum computing has added a new dimension. The spin polarization of a single electron can exist in a coherent superposition of two orthogonal spin polarizations (i.e., mutually anti-parallel spin orientations) for a relatively long time without losing the phase coherence. The charge degree of freedom, on the other hand, loses phase coherence much faster. Therefore, spin has become the preferred vehicle to host a quantum bit (or “qubit”) which is a coherent superposition of two orthogonal states representing classical logic bits 0 and 1. The potential application of spin to scalable quantum logic processors has a short history, but has provided a tremendous boost to spintronics.

This textbook is expected to equip the reader with sufficient knowledge and understanding to conduct research in the field of spintronic devices, particularly semiconductor based spintronic devices. We assume that readers have first year graduate level knowledge of device engineering, solid state physics and quantum mechanics. At Virginia Commonwealth University, parts of this material were used to teach a one-semester course in spintronics to graduate students who had this background. The experiment was successful and the feedback was quite encouraging. This course is now planned to be offered in a distance learning format to graduate students in six Virginia universities, as part of an endeavor funded by a Partnership for Innovation (PFI) grant from the US National Science Foundation.

At the University of Cincinnati, parts of this material (particularly Chapters 3, 4, 5 and 14) were used to teach a course on quantum computing. That

experiment too was immensely successful and provided much of the impetus for writing this textbook.

This book is organized into fifteen chapters. The first chapter provides a historical perspective to those who have had little or no exposure to this field. It traces the early history of spin, the anomalous Zeeman effect, and ends with the accidental discovery of “spin” by Stern and Gerlach in 1922.

Chapter 2 introduces the quantum mechanics machinery needed to understand spin physics, as well as analyze spin transport and general spin dynamics in solid state structures. It also introduces the concept of Pauli spin matrices, the Pauli Equation, and finally its relativistic refinement - the Dirac Equation. Since, in this textbook, we will never encounter any situation where relativistic corrections become important, we will not have any occasion to use the Dirac Equation. The Pauli Equation will be sufficient for all scenarios. Nonetheless, it is important to gain an appreciation for the Dirac Equation, since the quantum mechanical nuances associated with spin cannot be fully absorbed without an understanding of Dirac’s seminal work.

Chapter 3 introduces the Bloch sphere concept since it is a very useful tool to visualize the dynamics of a spin-1/2 particle (e.g., an electron), or qubit encoded in the spin of an electron, under the action of external magnetic fields. Applications of the Bloch sphere concept are elucidated with a number of examples. A spinor is viewed as a radial vector in the Bloch sphere, and this serves as a nice visualization tool for students interested in quantum computing and other applications of spintronics. All coherent motions of the spinor (where spin does not relax) are essentially excursions on the surface of the Bloch sphere.

Chapter 4 deals with an important application of the Bloch sphere concept, namely the derivation of Rabi oscillation and the Rabi formula for coherent spin rotation or spin flip. These have important applications in many spin-related technologies such as electron spin resonance spectroscopy, nuclear magnetic resonance, and ultimately solid state versions of quantum computing. This chapter is somewhat mathematical and ‘seasons’ the student to deal with the algebra (and recipes) necessary for calculating quantities that are important in spintronics. If needed, this chapter can be skipped at first and revisited later.

Chapter 5 introduces the concept of the ‘density matrix’, pure and mixed states, Bloch equations (that describe the temporal relaxation of spin), the Bloch ball concept, and the notion of the longitudinal (T_1) and transverse (T_2) relaxation times. Several numerical examples are also presented to strengthen key concepts. Since here we allow incoherent dynamics of the spin, the motion of the spinor is no longer constrained to the Bloch sphere. The ‘Bloch sphere’ actually refers only to the surface of the sphere and excludes the interior. If spin relaxes so that the norm of the sphere’s radius is no longer conserved, then we have to allow excursions into the interior of the Bloch sphere. Therefore, we extend the Bloch sphere concept to the ‘Bloch ball’ concept. This chapter contains advanced concepts and may also be skipped at first reading.

Chapter 6 introduces the rather important topic of spin-orbit interaction which is at the heart of many spintronic devices, since it offers a “handle” to manipulate spins. We focus on the two spin-orbit interactions that are predominant in the conduction band of most semiconductors: the Rashba interaction arising from structural inversion asymmetry, and the Dresselhaus interaction arising from bulk (crystallographic) inversion asymmetry. These two interactions form the basis of spintronic field effect transistors where the current flowing between two of the transistor’s terminals is modulated by influencing the spin-orbit interaction in the device via a potential applied to the third terminal. In this chapter, we also discuss an important physical phenomenon that is caused by spin-orbit interaction in a solid: the intrinsic spin Hall effect.

In Chapter 7, we derive the electron dispersion relations (energy versus wavevector) of electrons in quasi two- and one-dimensional structures (quantum wells and wires) in the presence of Rashba and Dresselhaus spin-orbit interactions, as well as an external magnetic field. We also derive the spin eigenstates, which allows us to deduce the spin polarization of carriers in any band. All this is accomplished by solving the Pauli Equation. This is an example of how the Pauli Equation is applied to solve a real life problem. The dispersion relations are important to understand many physical phenomena, a particularly intriguing example of which is the ‘Spin Galvanic Effect’, where spin polarization in the carrier population gives rise to an electric current (charge current) without a battery! We place special emphasis on how the dispersion relations are modified by an external magnetic field. This is important since it ultimately helps the student to appreciate how an external magnetic field can affect the performance of spin-based devices. Ultrasensitive magnetic field sensors, based on the modification of the energy dispersion relations by an external magnetic field, are a direct application of this theory. An example of this is discussed in Chapter 7.

Chapter 8 discusses spin relaxation of conduction electrons in metals and semiconductors. We focus on four primary spin relaxation mechanisms: the D’yakonov-Perel, the Elliott-Yafet, the Bir-Aronov-Pikus and hyperfine interactions with nuclear spins, since these are dominant in the conduction band of semiconductors and therefore are most important in device contexts. Because spin relaxation limits the performance of most, if not all, spin-based devices, it is a vital issue. Ultimately, the aim of all device engineers and physicists is to reduce the rate of spin relaxation in spin devices, in order to make them more robust and useful. Spin relaxation also has peculiarities that are completely unexpected and without parallel in solid state physics. We present one example where spin can relax in *time* but not in *space*.

Chapter 9 introduces the more advanced concepts of exchange and spin-spin interaction. These form the basis of ferromagnetism and also the basis of single spin computing schemes that are dealt with in Chapters 13 and 14.

Chapter 10 is an introduction to spin transport in solid state structures in the presence of spin relaxation. We focus on two basic models: the drift-

diffusion model of spin transport, and the semi-classical model that goes beyond the drift-diffusion model. The “spin” drift-diffusion model is very similar to the “charge” drift-diffusion charge model applied to bipolar transport; the “up-spin” and “down-spin” carriers assume roles analogous to electrons and holes. However, it has limitations. One limitation that we emphasize with specific examples is that it fails to describe essential features of spin transport, even qualitatively, if electrons are traveling “upstream”, against the force exerted on them by an electric field. In this Chapter, we present many examples of how spin relaxes in time and space in quasi one-dimensional structures in the presence of D’yakonov-Perel’ spin relaxation mechanism, since it is usually the dominant mechanism for spin relaxation in semiconductor structures. These examples are based on the semi-classical model and therefore applicable to both low field transport and high field (hot electron) transport. The semiclassical model is based on combining the Liouville equation for the time evolution of the spin density matrix with the Boltzmann Transport Equation for time evolution of the carrier momentum in the presence of scattering and external electric fields.

In Chapter 11, we discuss *passive* spintronic devices such as spin valves and devices based on the giant magnetoresistance effect. Most commercial spintronic products that are currently available (magnetic read heads for reading data in computer hard disks or entertainment systems such as Apple iPods, and magnetic random access memory) utilize these passive devices. Therefore, an adequate understanding of these devices is vital for engineers. We also discuss the important notions of spin injection efficiency, spin extraction and the recently discussed spin blockade. This is a long chapter with many topics and it is intended to introduce the reader to important concepts encountered in the modern spintronics literature.

Chapter 12 introduces *active* spintronic devices, specifically what we call “hybrid spintronic devices” such as Spin Field Effect Transistors and Spin Bipolar Transistors. We explain the physical basis of how these devices operate and what their shortcomings are. We make a simple estimate of their performance figures in order to project a realistic picture of whether they are or not competitive with traditional electronic devices that are currently extant. Regardless of their actual device potential, these devices are standard bearers that aroused early interest in the field among engineers and applied physicists. They will remain forever enshrined in the ‘Spintronics Hall of Fame’.

Chapter 13 introduces “monolithic spintronic devices” with Single Spin Logic paradigm as the example. This chapter goes beyond isolated devices and actually discusses complete systems, such as “all-spin-logic-gates” for combinational and sequential circuits (e.g., the arithmetic logic unit of a processor). We elucidate many pertinent issues such as power dissipation, and show that this is where “spin” may have a fundamental advantage over “charge”. This is an area that has remained neglected, but deserves particular attention since, unlike quantum computing, single spin classical logic does not require phase

coherence of spin. Consequently, it is robust and much easier to implement than quantum computers.

Chapter 14 is an introduction to the field of spin-based reversible logic gates (that can, in principle, compute without dissipating energy) and spintronic embodiments of quantum computers. This is a rapidly advancing field, extremely popular among many spintronic researchers, and discoveries are made at a fast pace. This chapter is written mostly for engineers and applied physicists (not computer scientists or theoretical physicists), and should provide them with the preliminary knowledge required to delve farther into this field.

Chapter 15 is a stand-alone chapter that can be treated as an *Appendix*. At first sight, it will appear unrelated to spintronics, which it is, but it has been included for a reason. There are many instances in this book when a student will have to recollect or re-familiarize herself/himself with some key results of quantum mechanics. Rather than making a trip to the nearest library, it would be more convenient to have a ‘quantum mechanics primer’ handy where these key results have been re-derived. This chapter is included for completeness and comprehensiveness. The reader can refer to it if and when necessary.

This work is a preliminary attempt at writing a textbook covering some important topics in spintronics. By its very nature, it must be incomplete and omit many topics that are both important and interesting. We have focused mostly on electron spin, and with the sole exception of discussing hyperfine nuclear interactions, we have ignored nuclear spin altogether. Hence, we do not discuss such well-known phenomena as the Overhauser effect, which is more relevant to nuclear spin. Another area that we have intentionally not covered in any detail is the extremely important area of *organic spintronics*. Organic semiconductors (mostly hydrocarbons) have weak spin-orbit interactions, so that spin relaxes slowly in these materials compared to inorganic semiconductors. Hence, they have a major advantage over inorganics when it comes to applications where spin relaxation must be suppressed, such as in spin based classical or quantum computing. Some reviews have appeared in the literature covering organic spintronics. Since this field is so young and progressing so fast, we considered it premature to dwell on this topic at this time. Future editions of this textbook, if there are any, will surely address this area.

This textbook also heavily emphasizes transport phenomena as opposed to optical phenomena dealing with the interaction of polarized photons with spin-polarized electrons and holes. Delving into “opto-spintronics” would have easily added a couple hundred pages to the 500-odd pages in this textbook. Our own expertise is more in transport phenomena, which has led us to focus more on transport. However, there are many excellent books (although not “textbooks”) available that deal with opto-spintronics, and the interested reader can easily find an assortment of literature in that area.

Finally, some acknowledgements are due. Many of our associates have contributed indirectly to this book. They are our students, laboratory interns and

post-doctoral research associates, past and present. They include Prof. Sandipan Pramanik of University of Alberta, Canada, who was a graduate student and then a post-doctoral researcher at Virginia Commonwealth University (VCU) at the time this book was composed, Mr. Bhargava Kanchibotla, a graduate student at VCU who provided the T_2 data in cadmium sulfide nanostructures from his experiments (figure 8.4), Mr. Sivakumar Ramanathan and Mr. Sridhar Patibandla, two graduate students at VCU working with S. B. who took the first spintronics graduate course offered at VCU by S. B. and provided valuable feedback, Mr. Harsh Agarwal, a summer undergraduate intern visiting VCU from the Banaras Hindu University Institute of Technology, Varanasi, India, who computed and generated some of the plots in Chapter 13, and Mr. Amit Trivedi, another summer undergraduate intern from the Indian Institute of Technology, Kanpur, who performed some of the calculations in Chapter 14.

At the University of Cincinnati, graduate student Mr. Junjun Wang and intern Ms. Lindsay Ficke contributed immensely to the generation of data and plots. We also thank graduate student Mr. Nishant Vepachedu for proof-reading the manuscript. We remain grateful to all of them.

Finally, in spite of all our best efforts, a few typographical errors may have escaped detection. We will remain grateful to any reader who points them out to us. Our e-mails are sbandy@vcu.edu and mcahay@ececs.uc.edu.

Welcome to the world of spintronics!

TABLE 0.1
Table of Universal Constants

Free electron mass (m_0)	9.1×10^{-31} Kg.
Dielectric constant of free space (ϵ_0)	8.854×10^{-12} Farads/meter
Electronic charge (e)	1.61×10^{-19} Coulombs
Reduced Planck constant (\hbar)	1.05×10^{-34} Joules-sec
Bohr radius of ground state in H atom (a_0)	$0.529 \text{ \AA} = 5.29 \times 10^{-11}$ meters
Bohr magneton (μ_B)	9.27×10^{-24} Joules/Tesla

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