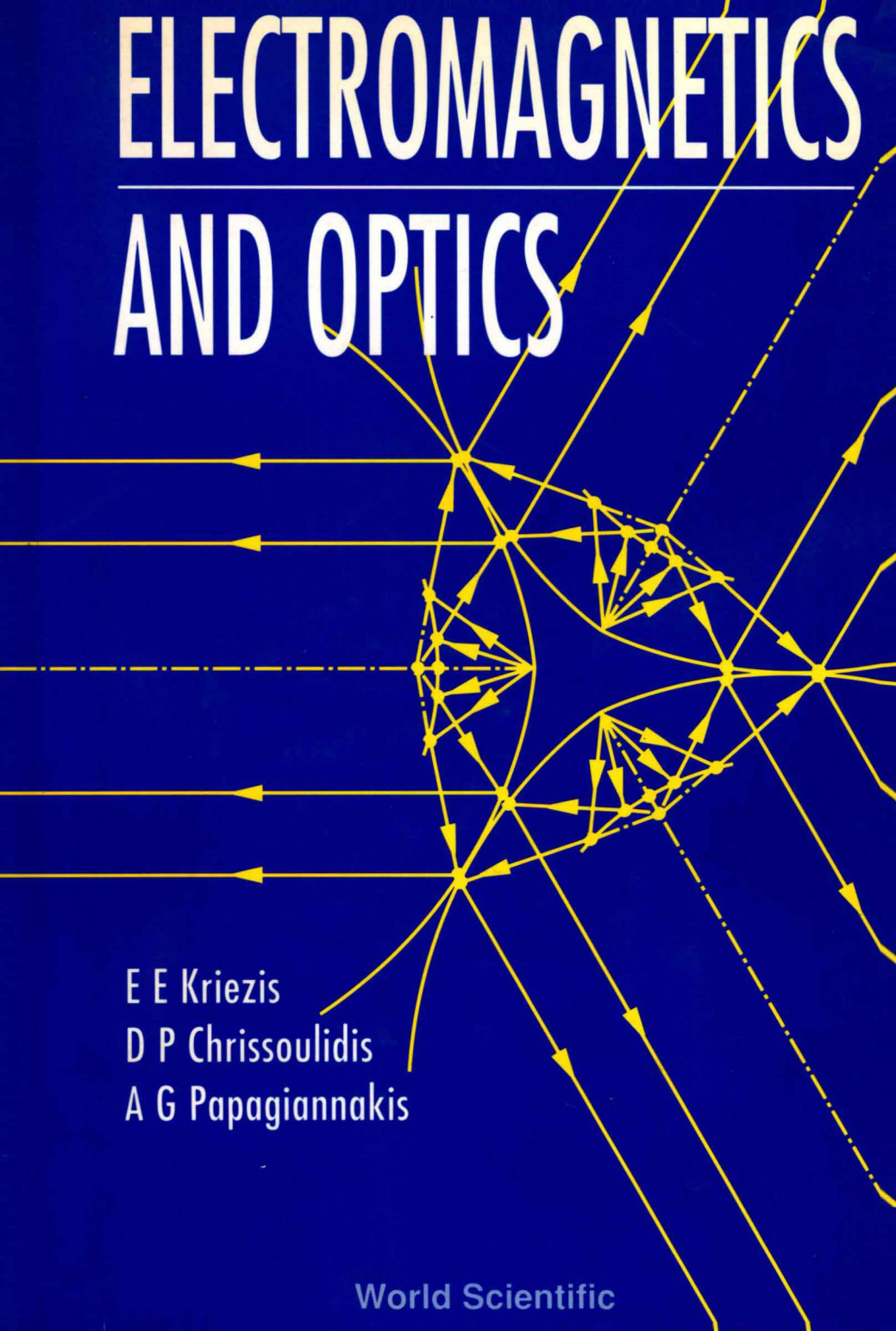


ELECTROMAGNETICS AND OPTICS

An abstract diagram in yellow on a dark blue background. It features several horizontal lines with arrowheads pointing right, representing incident light rays. These rays interact with a series of curved, lens-like shapes. From these interactions, numerous other rays emerge, some as straight lines with arrowheads, others as curved paths, and some as dashed lines. The diagram illustrates complex optical phenomena such as refraction, reflection, and focusing.

E E Kriezis
D P Chrissoulidis
A G Papagiannakis

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ELECTROMAGNETICS --- AND OPTICS

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ELECTROMAGNETICS AND OPTICS

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ELECTROMAGNETICS

AND OPTICS

To our wives:

Zaharoula

Ariadne

Christine

Preface

*Εἴτε φῶς εἴτ' ἀήρ ἐστὶ τὸ μεταξὺ τοῦ
ορωμένου καὶ τοῦ ὀμματός, ἡ διὰ τούτου
κίνησις ἐστὶν ἡ ποιοῦσα τὸ ορᾶν.¹
Αριστοτέλης*

Modern communication systems encompass information-carrying and information processing devices. The need for ever higher capacity has gradually linked progress in communications with light signals traveling through low-loss fibres and optical devices. Multichannel fibre links of enormous capacity are currently in use in developed parts of the world. Moreover, the scientists' dream of optical supercomputers that will be able to perform numbercrunching at the speed of light, has renewed interest in optics as well as in electromagnetics, since light is a high-frequency electromagnetic radiation.

Although both electromagnetics and optics are based on Maxwell's equations, their common origin is only superficially realised by students of applied physics or electrical engineering. Deeper physical insight might be achieved by treating electromagnetics and optics in parallel and from common ground, thus enlightening the natural link between them.

By presenting principles, theory and phenomena of electromagnetics and optics, this book contributes to the task of bridging the two disciplines. Implicit in our effort is the general communication problem.

Wavefunctions and rays used by electromagnetics and optics, respectively, to represent the information-carrying signal are associated with solutions of

¹ Whether light or air is the medium between the visible object and the eye, the motion through this medium is what produces vision.

Aristotle, De Anima, 438^b3-5

the wave and ray equations. Waveguides and resonators used in optical applications are analysed from the standpoint of both field theory and ray theory, the dual approach aimed at revealing to the reader the interrelations between electromagnetics and optics.

The book covers five subjects in fourteen chapters. The subjects covered are light/matter interactions, guidance, resonance, processing and applications. An outline of the contents of each chapter follows.

Starting from Maxwell's equations, the wave equation is formulated in Chapter 1. Solutions are given for isotropic and anisotropic media. Reflection/refraction of a uniform plane wave at a dielectric boundary, the Brewster angle and the critical angle are treated next. Chapter 1 is concluded with a discussion concerning the concept of coherent radiation and the uncertainty principle, chiefly for use in subsequent chapters.

The asymptotic solution of the wave equation in the high-frequency limit leads to geometrical optics, which is examined in Chapter 2. The Huygens principle and the concept of optical path length are used to formulate the eikonal and ray equations. The laws of geometrical optics are derived next by using Fermat's principle. Variational principles and quantum considerations in optics are finally discussed.

The laws of geometrical optics lead to ray theory, which is presented in Chapter 3. Several forms of light/matter interactions, such as reflection, refraction, diffraction and scattering in the optical band, are treated. Ray tracing by the ABCD transition matrix as well as rays in layered and inhomogeneous media are examined. A hybrid wave/ray approach is used on some occasions.

Guidance of light by dielectric waveguides is treated in Chapters 4 and 5. Step-index, graded-index and weakly guiding structures are analysed, appropriate wavefunctions are determined and rays are traced. Brief discussions of coupling and dispersion are also included.

Chapter 6 deals with resonance. Oscillating waveforms in microwave resonators are determined in rectangular, cylindrical and spherical cavities bounded by metallic walls. Interest is next focussed on optical resonators. Repetitive ray paths are determined through the ABCD matrix method. The Fabry-Perot etalon is treated next. Finally, an introduction to paraxial resonator theory is presented with reference to geometrical and perturbational stability of optical cavities.

The Gaussian beam is introduced in Chapter 7. Starting from the scalar wave equation for the beam, the corresponding waveforms are determined. Trans-

formation and propagation of the Gaussian beam are also examined.

Problems related with the laser are examined in Chapter 8. The basic equation of the laser is determined and the broadening mechanism is discussed. Oscillation of the laser and the gain saturation are also treated.

Chapter 9 deals with Fourier optics. Considering light as a scalar disturbance, the features of light propagation and the associated filtering process are thoroughly investigated. The transmittance function of a discontinuity surface is introduced next; basic reflection/refraction phenomena with regard to the thin lens are treated, thus establishing a link between Fourier and Gaussian optics.

The results of Fourier optics are also compared with the results of the scalar theory of diffraction, which is developed in Chapter 10. Kirchhoff's diffraction theory is presented there and the Fresnel/Fraunhofer approximations are analysed. Several representative applications are included in Chapter 10.

The scalar diffraction theory is used in Chapter 11 to explain the process of holographic wavefront reconstruction. Properties of the reconstructed images and beams are investigated. A qualitative description of various kinds of holograms concludes chapter 11.

Elements of optical radiation are presented in Chapter 12. Radiometric and photometric quantities are defined and discussed. Thermal radiation and the field of light are also examined there.

Chapter 13 deals with optical techniques and components applied to remote sensing. Photographic imaging by aerial cameras and the operation of radiometers are discussed in the first part of the chapter, which refers to passive remote sensing. Optical data processing is discussed next, chiefly as a prerequisite for the ensuing description of the optical processor used by synthetic aperture radars.

Finally, chapter 14 examines the fundamental principles of the non-linear optics. Classical atomic models are described and the coupled wave equations are treated. A generalisation of the laws of optics is thus obtained.

Considering that the contemporary problems (dielectric waveguides, holography, lasers etc.) of the electrical engineer require a more advanced background, we feel that the proposed book will help senior-level students and engineers not only to understand the various physical problems but also to cope with applications.

We are deeply indebted to all those who have assisted in the realisation of this book. Many thanks are due to Prof. J.A. Tsoukalas for his valuable

suggestions and discussions. We are also grateful to our colleagues Dr. T. Tsiboukis, Dr. S. Panas, Dr. C. Antonopoulos, Dr. P. Hagouel, Dr. G. Sergiadis and Dr. M. Zervas for the valuable discussions and their comments. In addition, thanks are due to Dipl. Eng. E. Kriezis for his meticulous checks of several parts of the manuscript. We further thank the unknown reviewers engaged by the publishers. Finally, we gratefully acknowledge the work of Mrs. J. Hassekidou-Maragou and Miss V. Altintzi in preparing the drawings of the book.

Thessaloniki, 1991

E.E. Kriezis
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Contents

Preface Contents

vii
xi

1	REVIEW OF THE EQUATIONS OF ELECTROMAGNETIC FIELD	1
1.1	Poynting's Vector - Intensity of Light, 1	
1.2	Wave Equation in Isotropic Media, 3	
1.3	Wave Propagation in Anisotropic Media, 8	
1.4	Reflection and Refraction at a Dielectric Boundary, 19	
1.5	Brewster's Angle - Critical Angle, 24	
1.6	Coherent Radiation, 27	
1.7	Uncertainty Principle, 29	
2	GEOMETRICAL OPTICS	33
2.1	Introduction, 33	
2.2	Eikonal Equation and Ray Equation, 35	
2.3	Applications of Ray Equation, 42	
	2.3.1 Rays in a Spherically Inhomogeneous Medium, 42	
	2.3.2 Rays in a Cylindrically Inhomogeneous Medium, 45	
2.4	Fermat's Principle, 48	
2.5	Boundary Conditions, 51	
2.6	Variational Principles in Optics, 54	
2.7	Quantum Considerations in Optics, 59	
2.8	Exercises, 64	
3	RAY THEORY	67
3.1	Introduction, 67	
3.2	Mirror Reflection, 68	
3.3	Prism/Lens Refraction, 77	
3.4	Ray Tracing by the ABCD Matrix, 85	
3.5	Rays in Non-Uniform Media, 93	
3.6	Rays in Layered Structures, 100	
3.7	Reflection/Refraction Gratings, 107	
3.8	Simple Scattering Processes, 113	
3.9	Exercises, 117	

4	PLANAR DIELECTRIC WAVEGUIDES	119
4.1	Introduction, 119	
4.2	Dielectric Film, 123	
4.2.1	TM-Modes, 124	
4.2.2	TE-Modes, 127	
4.2.3	Weakly Guiding Film, 130	
4.3	Dielectric Slab, 132	
4.3.1	Total Internal Reflection, 132	
4.3.2	TE-Modes, 137	
4.3.3	TM-Modes, 141	
4.3.4	Weakly Guiding Slab, 142	
4.4	Coupled Slab Waveguides, 146	
4.5	Rectangular Dielectric Waveguide, 151	
4.6	Graded-Index Slab, 155	
4.7	Rays in Dielectric Slabs, 158	
4.8	Exercises, 161	
5	FIBRES	163
5.1	Introduction, 163	
5.2	Step-Index Fibre, 166	
5.3	Graded-Index Fibre, 178	
5.4	Rays in Fibres, 183	
5.5	Pulse Distortion in Single-Mode Fibres, 192	
5.6	Mode Conversion/Coupling, 196	
5.7	Exercises, 201	
6	RESONATORS	203
6.1	Introduction, 203	
6.2	Oscillating Waveforms in Microwave Resonators, 203	
6.2.1	The Rectangular Cavity, 205	
6.2.2	The Cylindrical Cavity, 208	
6.2.3	The Spherical Cavity, 212	
6.2.4	Resonant Mode Density, 217	
6.3	Repetitive Ray Paths in Optical Resonators, 219	
6.3.1	Mode Spectrum of Two-Mirror Optical Cavity, 223	
6.3.2	ABCD Matrix of Stable Optical Resonator, 224	
6.4	The Fabry-Perot Etalon, 227	
6.5	Paraxial Resonator Theory, 232	
6.5.1	Geometrical Stability, 232	
6.5.2	Perturbational Stability, 234	
6.5.3	Stable and Unstable Optical Resonators, 236	
6.5.4	Mode Volume, 239	
6.6	Exercises, 240	

7	GAUSSIAN BEAM	241
7.1	Introduction, 241	
7.2	Scalar Wave Equation, 241	
7.3	Gaussian Beam, 245	
7.4	Higher-Order Beam Modes, 254	
7.5	Transformation of Gaussian Beam and q-Parameter, 255	
7.6	ABCD Matrix for the Gaussian Beam, 259	
7.7	Propagation of the Gaussian Beam, 264	
7.8	Exercises, 270	
8	LASER	271
8.1	Introduction, 271	
8.2	Spontaneous and Stimulated Emission. Absorption, 272	
8.3	The Concept of Laser. Basic Equation of Laser, 276	
8.4	Lineshape Broadening, 279	
	8.4.1 Doppler Broadening, 280	
	8.4.2 Homogeneous Radiative Lifetime Broadening, 282	
	8.4.3 Collision Broadening, 285	
8.5	Laser Modes, 286	
8.6	The Electron Oscillator Model of the Atomic Transition, 289	
8.7	The Laser Oscillation Condition, 291	
8.8	Gain Saturation, 296	
8.9	Exercises, 306	
9	FOURIER OPTICS	309
9.1	Introduction, 309	
9.2	Light: Vector or Scalar Disturbance?, 309	
9.3	Propagation of the Light Disturbance, 310	
	9.3.1 Spatial Frequencies of a Wave Disturbance, 311	
	9.3.2 Propagation of the Angular Spectrum, 313	
	9.3.3 The Propagation as a Spatial Filter, 316	
	9.3.4 Let's go Afar, 319	
	9.3.5 Let's go Even Farther, 326	
9.4	Transmittance function of a Discontinuity Surface, 327	
9.5	Refraction, 329	
9.6	Reflection, 333	
9.7	The Thin Lens, 336	
9.8	Exercises, 337	
10	SCALAR THEORY OF DIFFRACTION	341
10.1	Introduction, 341	
10.2	Formulation of the Problem - Kirchhoff's Boundary Conditions, 342	

- 10.3 Fresnel - Kirchhoff Diffraction, 347
- 10.4 The Rayleigh - Sommerfeld Diffraction Equation, 349
- 10.5 Fresnel Diffraction, 352
- 10.6 Fraunhofer Diffraction, 357
- 10.7 Comparison of Fourier Optics and STD, 359
- 10.8 Diffraction of a Plane Wave, 363
- 10.9 Fraunhofer Diffraction by a Rectangular Aperture, 365
- 10.10 Fraunhofer Diffraction by a Circular Aperture, 366
- 10.11 Fraunhofer Diffraction by Grating of Harmonic Amplitude Variation, 368
- 10.12 Fraunhofer Diffraction by Grating of Harmonic Phase Variation, 370
- 10.13 Fresnel Diffraction by a Rectangular Aperture, 373
- 10.14 Diffraction by an Infinite Slit, 376
- 10.15 Fresnel Zones - Vibration Curve, 380
- 10.16 Fresnel Zone Plate, 383
- 10.17 Exercises, 386

11 OPTICAL HOLOGRAPHY

389

- 11.1 Introduction, 389
- 11.2 A Qualitative Description of Holography, 390
- 11.3 About Coherence, 391
- 11.4 Interference of two Wave Disturbances, 394
- 11.5 The Recording Medium, 396
 - 11.5.1 The Photographic Film, 397
 - 11.5.2 The Hurter-Driffield Curve, 397
 - 11.5.3 The Transmittance vs. Exposure Curve, 398
- 11.6 Hologram Construction, 399
- 11.7 Wavefront Reconstruction, 402
 - 11.7.1 The Transmitted Disturbance, 402
 - 11.7.2 The Choice of the Reference Waves, 404
 - 11.7.3 The Third Diffraction Term, 405
 - 11.7.4 The Fourth Diffraction Term, 406
- 11.8 Properties of the Reconstructed Waves, 407
- 11.9 Hologram Types, 418
 - 11.9.1 Fresnel Hologram, 418
 - 11.9.2 Fourier Transform Hologram, 418
 - 11.9.3 Lensless Fourier Transform Hologram, 419
 - 11.9.4 Image Hologram, 420
 - 11.9.5 Computer Holograms, 420
 - 11.9.6 Other Types of Holograms, 421
- 11.10 Volume Holograms, 421
- 11.11 Applications of Holography, 426

- 11.11.1 Three-Dimensional Characteristics, 426
- 11.11.2 Spherical Reference Waves, 427
- 11.11.3 High Density, 427
- 11.11.4 High Speed, 427
- 11.11.5 Coherence and Interference, 427
- 11.11.6 Phase Inversion, 429
- 11.11.7 Accuracy, 429
- 11.12 Exercises, 429

12 OPTICAL RADIATION

431

- 12.1 Introduction, 431
- 12.2 Radiometric Quantities, 432
 - 12.2.1 Spectral Distribution of Radiation, 439
- 12.3 Photometric Quantities, 442
- 12.4 Thermal Radiation, 446
 - 12.4.1 The Wien Displacement Law, 447
 - 12.4.2 The Planck Radiation Law, 449
- 12.5 The Field of Light, 454

13 OPTICAL TECHNIQUES IN REMOTE SENSING

459

- 13.1 Introduction, 459
- 13.2 Photographic Imaging, 461
- 13.3 Electro-Optical Sensors, 471
- 13.4 Optical Data Processing, 476
- 13.5 Application to SAR signals, 487

14 NON-LINEAR OPTICS

497

- 14.1 Introduction, 497
- 14.2 The General Model of the Anharmonic Oscillator, 498
- 14.3 Maxwell's Equations, 503
- 14.4 Second Harmonic Generation, 505
- 14.5 Self-Focusing, 508
- 14.6 Stimulated Raman Scattering, 513
- 14.7 Brillouin Scattering, 516

APPENDIXES

A THE FOURIER TRANSFORM

523

- A.1 The One-Dimensional Fourier Transform, 523
- A.2 The Two-Dimensional Fourier Transform, 526
- A.3 The Three-Dimensional Fourier Transform, 529
- A.4 The Hankel and Fourier-Bessel Transforms, 530

A.5	The Angular Spectrum of a Disturbance, 534	
A.6	Fourier Transform Properties, 536	
B	THE SCALAR GREEN'S FUNCTION	539
B.1	Green's Function for the Scalar Helmholtz Equation, 539	
B.1.1	Posing the Problem, 539	
B.1.2	Definition of Green's Function, 540	
B.1.3	A Fundamental Property of Green's Function, 543	
B.1.4	Computation of Green's Function, 544	
B.2	The Radiation Condition and the Free Space Green's Function, 544	
B.3	Green's Function in Two Dimensions, 548	
C	ELECTROMAGNETIC FIELD AND QUANTUM CONSIDERATION	551
	References	557
	Subject Index	561

Chapter 1

Review of the Equations of Electromagnetic Field

1.1 Poynting's Vector - Intensity of Light

It is well known that light is an electromagnetic phenomenon which conforms to *Maxwell's equations*. The possibility of linking light to electromagnetism has been considered by several predecessors of Maxwell. Maxwell's equations, in differential form, are as follows:

$$\nabla \times \vec{\mathcal{E}} = - \frac{\partial \vec{\mathcal{B}}}{\partial t} \quad (1.1)$$

$$\nabla \times \vec{\mathcal{H}} = \frac{\partial \vec{\mathcal{D}}}{\partial t} + \vec{\mathcal{J}} \quad (1.2)$$

$$\nabla \cdot \vec{\mathcal{B}} = 0 \quad (1.3)$$

$$\nabla \cdot \vec{\mathcal{D}} = \varrho \quad (1.4)$$

The *polarisation* $\vec{\mathcal{P}}$ and *magnetisation* $\vec{\mathcal{M}}$ are introduced through the well-known constitutive relations:

$$\vec{\mathcal{D}} = \epsilon_0 \vec{\mathcal{E}} + \vec{\mathcal{P}} \quad (1.5)$$

$$\vec{\mathcal{B}} = \mu_0 (\vec{\mathcal{H}} + \vec{\mathcal{M}}) \quad (1.6)$$

The *surface power density* is represented by the *Poynting vector*:

$$\vec{\mathcal{J}} = \vec{\mathcal{E}} \times \vec{\mathcal{H}} \quad (1.7)$$

If harmonic time variation is assumed, the *electric field intensity* $\vec{\mathcal{E}}$ and the *magnetic field intensity* $\vec{\mathcal{H}}$ are expressed as follows:

$$\vec{\mathcal{E}} = \text{Re}\{\vec{E}e^{j\omega t}\} = \frac{1}{2}(\vec{E}e^{j\omega t} + \vec{E}^*e^{-j\omega t}) \quad (1.8)$$

$$\vec{\mathcal{H}} = \text{Re}\{\vec{H}e^{j\omega t}\} = \frac{1}{2}(\vec{H}e^{j\omega t} + \vec{H}^*e^{-j\omega t}) \quad (1.9)$$

with F^* as the complex conjugate of F . By using the expressions of Eqs.(1.8 & 1.9) for $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$, Eq.(1.7) is written as follows:

$$\vec{\mathcal{J}} = \frac{1}{4}(\vec{E} \times \vec{H}^* + \vec{E}^* \times \vec{H}) + \frac{1}{4}(\vec{E} \times \vec{H}e^{2j\omega t} + \vec{E}^* \times \vec{H}^*e^{-2j\omega t}) \quad (1.10)$$

If the relations $\text{Re}\{\vec{E} \times \vec{H}^*\} = (\vec{E} \times \vec{H}^* + (\vec{E} \times \vec{H}^*)^*)/2$, $(\vec{E} \times \vec{H}^*)^* = \vec{E}^* \times \vec{H}$ and $(\vec{E} \times \vec{H}e^{2j\omega t})^* = \vec{E}^* \times \vec{H}^*e^{-2j\omega t}$ are used, Eq.(1.10) can be shortened as follows:

$$\vec{\mathcal{J}} = \frac{1}{2}\text{Re}\{\vec{E} \times \vec{H}^*\} + \frac{1}{2}\text{Re}\{\vec{E} \times \vec{H}e^{2j\omega t}\} \quad (1.11)$$

It should be noted that only the second term in the right-hand side of Eq.(1.11) is time-dependent, its frequency being double that of $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$. The vector

$$\vec{S}_c = \frac{1}{2}\vec{E} \times \vec{H}^* \quad (1.12)$$

is defined as the *complex Poynting vector*. Maxwell's equations, as adapted to harmonic time variation, will next be used to determine \vec{S}_c . Eqs.(1.1 & 1.2) are written as follows:

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \quad (1.13)$$

$$\nabla \times \vec{H} = j\omega\vec{D} + \vec{J} \quad (1.14)$$

with $\vec{J} = \text{Re}\{\vec{J}e^{j\omega t}\}$. The following relation can easily be proved from Eqs.(1.13 & 1.14):

$$-\vec{H}^* \cdot \nabla \times \vec{E} + \vec{E} \cdot \nabla \times \vec{H}^* = \vec{E} \cdot \vec{J}^* + j\omega(\vec{B} \cdot \vec{H}^* - \vec{E} \cdot \vec{D}^*) \quad (1.15)$$

The vector identity $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B}$ indicates that Eq.(1.15) may be transformed into the following:

$$-\nabla \cdot (\vec{E} \times \vec{H}^*) = \vec{E} \cdot \vec{J}^* + j\omega(\vec{B} \cdot \vec{H}^* - \vec{E} \cdot \vec{D}^*) \quad (1.16)$$

By using Eqs.(1.5, 1.6 & 1.12) and integrating Eq.(1.16) throughout a given volume V , yields: