

ADVANCED SCIENCE AND TECHNOLOGY SERIES

Optical Waveguide Analysis

MASANORI KOSHIBA

ADVANCED SCIENCE AND TECHNOLOGY SERIES

OPTICAL WAVEGUIDE ANALYSIS

by **Masanori Koshiha**

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Introduction

The rapid developments in fields such as fiber optics communication engineering and integrated optical electronics have expanded the interest and increased expectations about guided-wave optics, in which optical waveguides play a central role. Optical waveguides for optical fibers and optical integrated circuits utilize a wave phenomenon that traps the light locally and guides it in any direction, although their propagation lengths differ greatly. In order to develop new optical communication systems or optical devices, we need to fully understand the principles of optical guiding, while obtaining accurate quantitative propagation characteristics of waveguides and utilizing them effectively in actual design.

This book offers an easy-to-understand introduction to the analysis of optical waveguides, which is indispensable in optical technology research and development, while describing the physical principles of the optical waveguide phenomenon. Focusing on a simple explanation of the principles of optical waveguides and the minimum requirements of optical waveguide analysis, the book should be useful to both undergraduate and graduate students in various related fields, as well as engineers and researchers who have just begun their study of optical waveguide analysis technology.

Maxwell's equations are used to calculate the propagation characteristics of optical waveguides. It is, however, rather rare to obtain a precise analytic solution, and therefore, a precise analysis of optical waveguides is generally considered to be difficult. For this reason, various methods of optical waveguide analysis have been developed. These methods may be broadly classified into two categories: analytical approximation solutions, and numerical solution using computers. In order to analyze and design optical waveguides, it is necessary to study these analysis methods in depth, and to be able to use these methods as tools. Some of the representative analysis methods are introduced in this book.

The book consists of ten chapters. Chapter 1 describes the types of optical waveguides and gives an overview of the analysis techniques. Chapter 2 reviews the fundamentals of light and wave motion, and describes the wave theory, which is the standard theory for analysis of optical waveguides, and the ray theory, which provides a simpler approach to the physical side of optical waveguide phenomenon. Chapter 3 presents the analysis of three-layer optical dielectric waveguides, which are the most simple and basic optical waveguides, and the uniform-core optical fiber. These belong to a small group of waveguides for which precise solutions can be obtained, and also represent all fundamental aspects of the optical waveguide phenomenon. It is important, therefore, to develop a full understanding of these subjects. Chapter 4 introduces the ray-approximation method and the Wentzel-Kramers-Brillouin (WKB) method, which are known as typical analytical approximation solutions for graded 2-D optical waveguides, in which the refractive index changes gradually in a certain direction. Chapter 5 introduces typical analytical approximation solutions for 3-D optical waveguides, namely, the Marcanti method, the effective index method and the equivalent network method. These methods of analysis are not only widely used because of their ease of handling, but also contain the essence of the optical waveguide phenomenon, providing clues for understanding the principles of waveguiding in optical waveguides for which a precise solution is difficult. Therefore, it is important to read this chapter carefully. After reading through this chapter, the reader should have sufficient knowledge of analysis to be able to calculate propagation characteristics of optical waveguides. There are, however, cases in which the analytical accuracy of an analytical approximation solution is not sufficient, or cases in which application of an analytical approximation solution is no longer possible, depending on the structure of the optical waveguide. In these cases, we are forced to depend on numerical solutions. There are numerous types of numerical solutions for optical waveguide analysis, not all of which can be discussed in the limited space of this book. Only one type of numerical solution is introduced here: the finite element (FE) method, which is known as the general-purpose numerical solution for optical waveguides and is gradually being used by more researchers. In Chapter 6 and subsequent chapters, examples of actual analyses are given to demonstrate that the finite element method can be used effectively in the analysis of various optical waveguides, including 2-D optical waveguides, 3-D optical waveguides, axisymmetric optical fiber, non-axisymmetric optical fiber, and nonlinear optical waveguides. In addition, the book lists as many papers as possible on finite element analysis of optical waveguide modes. The author hopes that they will be useful in understanding the research trends in this field.

Since the purpose of this book is to promote an understanding of the fundamentals of optical waveguide analysis, the waveguides selected for analysis are those with material constants, such as structure and refractive index, which do not change in the direction of propagation. However, in order to construct an optical integrated circuit, it is also very important to consider the issues of interconnection between waveguides and coupling of parallel waveguides. The author hopes to have another opportunity to discuss these problems in the future, but for the present, the textbooks on these subject listed in the Bibliography should be helpful to the reader.

In case the explanations or assumptions presented in this book are unclear or inappropriate due to the author's inadequacy, the reader's patience and criticism are appreciated. It is the author's wish that this book will contribute to the dissemination and development of the optical waveguide analysis technology in the area of waveguide optics, including optical fiber communication engineering and integrated optical electronics.

In proceeding with the study that provided the foundation of this book, the author received instruction and assistance from a number of people through academic conferences, research societies, and symposia, and would like to take this opportunity to express his heartfelt gratitude. Special thanks are due to Professor Takanori Okoshi of the University of Tokyo Research Center for Advanced Scientific Technology for his extensive instructions, encouragement, and valuable advice, and for giving the author the opportunity to publish this work. The author would also like to express his gratitude to Instructor Kazuya Hayata, who cooperated with the author in the research and development of optical waveguide analysis technology in the author's research lab; the alumni and students of the research lab; and last but not least, the Editorial Staff of Asakura Shoten, who have been very supportive during the writing of this book.

October, 1990

Masanori Koshiba

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1.

Types of Optical Waveguides and Analytical Techniques

This chapter introduces the types of optical waveguides, which are classified into 2-D and 3-D optical waveguides for optical integrated circuits and axisymmetrical and non-axisymmetrical optical fibers for optical communication. The methods of optical waveguide analysis, classified into analytical approximation solutions and numerical solutions, are also outlined in this chapter.

1.1 Outline of Optical Waveguides

Optical waveguides, which trap light locally and guide it in a specific direction, can be roughly classified into optical waveguides for optical integrated circuits and optical fibers.

In the microwave band, electromagnetic fields are generally distributed within a finite area surrounded by conductors, as in waveguides and coaxial lines. On the other hand, it is difficult to construct a waveguide mainly from metal in the visible wave band, since the metal will behave as a substance having a complex permittivity with a large absolute value. For this reason, waveguides are usually constructed by combining appropriate dielectrics, as shown in Fig. 1.1, thus distributing an electromagnetic field over an infinite area. By loading a thin film with a higher refractive index than either the substrate or the upper cladding on the substrate surface, the light can be trapped inside this film. Furthermore, as

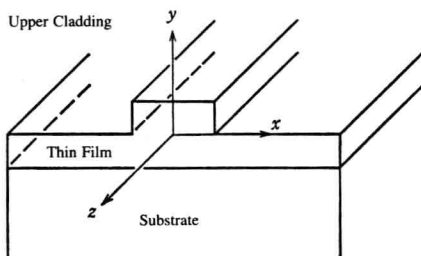


Fig. 1.1 Optical Waveguide

the thickness of the film increases, the effective refractive index (or effective index) sensed by the light increases. Therefore, in case of the waveguide shown in Fig. 1.1, the light is pulled toward the central area inside the thin film, resulting in the propagation of light in the z direction, with the power of the light concentrated in a certain part of the cross section of the waveguide.

Waveguides in which the refractive index changes in stages are called step-index (SI) optical waveguides, while those with a gradual refractive index change are called graded-index (GI) optical waveguides.

An optical waveguide that is uniform in the direction of propagation, as shown in Fig. 1.1, is the most basic type of waveguide, but this alone is not sufficient for construction of an optical integrated circuit. In reality, an appropriate combination of various forms of optical waveguides as shown in Fig. 1.2 is placed on the substrate to construct an optical circuit with desired features. Corner-bent waveguides, S-shaped waveguides, and bent waveguides are used to change the direction of the lightwaves. Tapered waveguides are used to change the width of waveguides; branching waveguides and crossed waveguides are used for splitting, combining, and interference; and optical-waveguide directional couplers and two-mode waveguide couplers are used for coupling. Waveguide gratings, with a periodic structure in the direction of propagation, plays many important roles in the optical integrated circuit, such as wavelength filter, mode converter, reflector, resonator, demultiplexer, etc. Waveguide gratings are also used widely as a laser element, such as a distributed Bragg reflector (DBR) laser or a distributed feedback (DFB) laser.

Although waveguides come in various forms and with a variety of functions, as shown above, the fact remains that the optical waveguide that is uniform in the direction of propagation, as shown in Fig. 1.1, is the basic form of waveguide. The information regarding the propagation characteristics of the optical waveguide shown in Fig. 1.1 is the most basic and important information required when designing the types of waveguides shown in Fig. 1.2. Therefore, because of space limitations, the discussion in this book will be restricted to optical waveguides in which material constants such as structure and refractive index do not change in the direction of propagation.

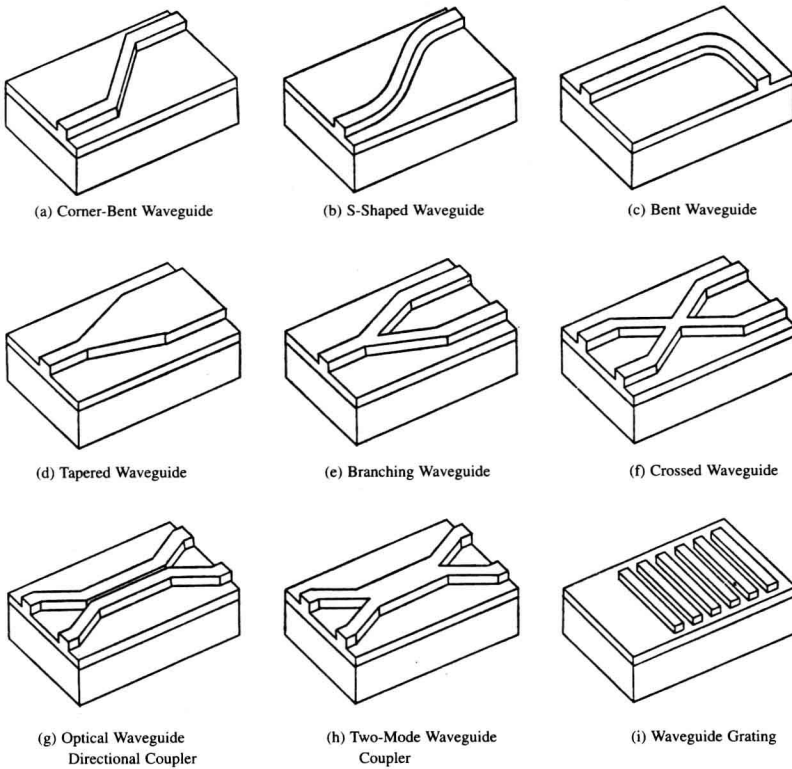


Fig. 1.2 Examples of Optical Waveguide Usage

1.2 Waveguides for Optical Integrated Circuits

(1) 2-D Optical Waveguides

Waveguides that trap the light only in the direction of thickness are called 2-D optical waveguides or slab optical waveguides.

a. Stepped 2-D Optical Waveguides

Fig. 1.3 illustrates the simplest, most basic waveguide structure. Here, n_f , n_s and n_c represent the refractive indexes of the thin film, substrate, and upper cladding, respectively. When the upper cladding is air, as in most cases, $n_c=1$. This

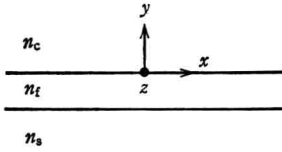


Fig. 1.3 Three-Layer Dielectric Waveguide

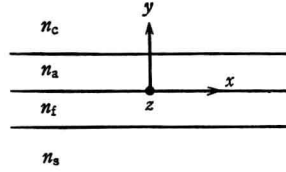


Fig. 1.4 Four-Layer Dielectric Waveguide

type of optical waveguide is called a three-layer (trilayer) dielectric waveguide or an asymmetric slab waveguide. The relationship among the refractive indexes is $n_c < n_s < n_t$, and the light is trapped inside the thin film. Therefore, the thin film is sometimes called the waveguide layer. When $n_s = n_c$, the waveguide is called a symmetric slab waveguide. Four-layer dielectric waveguides (shown in Fig. 1.4) which are used frequently, have an additional layer of thin film to control the propagation characteristics of light. For these n_a is the refractive index of the added layer, which can be either above or below the waveguide layer.

b. Graded 2-D Optical Waveguides

Optical waveguides manufactured by thermomigration or ion exchange are called graded optical waveguides because the refractive index changes gradually. Fig. 1.5 illustrates the simplest structure of a graded 2-D optical waveguide.

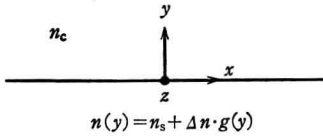


Fig. 1.5 Graded 2-D optical waveguide.

The refractive index distribution, $n(y)$, can be obtained as follows:

$$n(y) = n_s + \Delta n \cdot g(y) \quad (1.1)$$

Here n_s is the refractive index of the substrate, Δn is the maximum amount of change in the refractive index, and $g(y)$ describes the distribution function of the refractive index in the y direction. The value of the distribution function is usually between 1 and 0. Exponential functions, Gaussian functions, error-compensating functions, linear functions, second-order functions, etc., are used as the distribution function, and are expressed, respectively, as follows:

$$g(y) = \exp(y/d_y) \quad y \leq 0 \quad (1.2)$$

$$g(y) = \exp(-y^2/d_y^2) \quad y \leq 0 \quad (1.3)$$

$$g(y) = \operatorname{erfc}(-y/d_y) \quad y \leq 0 \quad (1.4)$$

$$g(y) = \begin{cases} 1 + y/d_y & -d_y \leq y \leq 0 \\ 0 & y \leq -d_y \end{cases} \quad (1.5)$$

$$g(y) = \begin{cases} 1 - y^2/d_y^2 & -d_y \leq y \leq 0 \\ 0 & y \leq -d_y \end{cases} \quad (1.6)$$

Here d_y is the diffusion depth in the y direction. The error-compensating function, erfc , is defined as

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-v^2) dv$$

Fig. 1.6 shows the changes in the refractive index distribution function, $g(y)$, in the direction of the depth of substrate. Generally, the refractive index distribution of the graded optical waveguides that are actually manufactured takes the form of a Gaussian function or an error-compensating function.

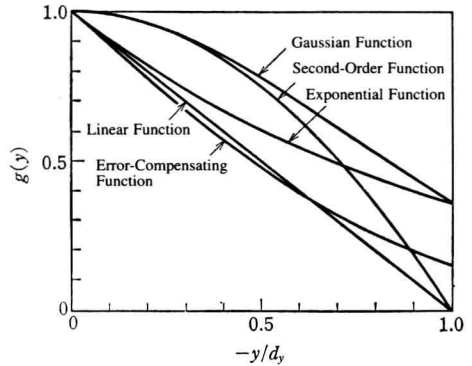


Fig. 1.6 Refractive Index Distribution Function, $g(y)$

(2) 3-D Optical Waveguides

A 2-D optical waveguide can trap light in the direction of the thickness (y direction), but allows light to spread in the horizontal direction (x direction). In order to facilitate the construction of optical integrated circuits, various types of 3-D optical waveguides, or optical channel waveguides, which trap the light in both x and y directions, have been devised.

a. Stepped 3-D Optical Waveguides

Fig. 1.7 illustrates the common types of 3-D optical waveguides: embedded waveguide, rectangular dielectric waveguide, dielectric strip waveguide (or raised waveguide), rib waveguide, strip-loaded waveguide, ridge waveguide, metal-clad waveguide, and trapezoidal rib waveguide. The metal-clad waveguide enables trapping of light in the horizontal direction by utilizing the decrease of the effective refractive index caused by the loading of metal film. The trapezoidal rib waveguide is shown here as an example of a trapezoidal waveguide. Some waveguides inevitably end up with a trapezoidal cross section, due to the manufacturing process.