

A Survey of Mathematics with Applications

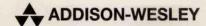
FIFTH EDITION

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Monroe Community College

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Monroe Community College



An imprint of Addison Wesley Longman, Inc.

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Library of Congress Cataloging-in-Publication Data

```
Angel, Allen R., 1942–
A survey of mathematics with applications / Allen Angel, Stuart Porter. — 5th ed.
p. cm.
Includes index.
ISBN 0-201-84600-4. — ISBN 0-201-85761-8
1. Mathematics. I. Porter, Stuart R., 1932–
II. Title.
QA39.2.A54 1996
510—dc20
96-43388
CIP
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To my wife, Kathy, and my sons, Robert and Steven A.R.A.

To my family: Joyce, Lisa, Tod, Teri, Brian, Adam, Andrew, Matthew, Emily, Molly S.R.P.

Math: It's All Around Us!

We present A Survey of Mathematics with Applications, fifth edition, with that vision in mind. Our primary goal in writing this book was to give students a text that they can read, understand, and enjoy while learning how mathematics affects the world around them. Numerous applied examples motivate topics. A variety of interesting applied exercises demonstrate the real-life nature of mathematics and its importance in the students' lives.

The text is intended for students who require a broad-based general overview of mathematics, especially those majoring in the liberal arts, elementary education, the social sciences, business, nursing, and allied health fields. It is particularly suitable for those courses that satisfy the minimum competency requirement in mathematics for graduation or transfer.

New and Expanded Features

In this edition we made several important improvements in presentation.

- A modified design and increased page size allow for easier reading and referencing of material.
- New Group Projects appear at the end of each chapter to facilitate cooperative/ group learning.
- Approximately 40% of the exercises are new.
- · The number of writing exercises was increased significantly.
- There is increased focus on data and graphical analysis in both the textual material and exercise sets.
- More problem Solving/Group Activity exercises were added to the section exercises.
- The number of Research Activities was increased.
- The number of examples was increased throughout the text to promote student understanding.

Content Revision

In addition we revised and expanded certain topics to introduce new material and to increase understanding.

- Chapter 1, Critical Thinking Skills, was expanded. Additional examples and explanations are intended to promote student problem solving.
- · Chapter 3, Logic, was reorganized for greater clarity.
- Chapter 6, Algebra, Graphs, and Functions, was expanded to include coverage of variation.
- Metric System coverage was expanded to an entire chapter, Chapter 8, and a new section on Dimensional Analysis was added.
- Chapter 9, Geometry, was expanded to include more coverage of Topology.
- Chapter 12, Probability, was expanded to include a new section on the binomial probability formula.
- Chapter 13, Statistics, has been expanded to include a new section 13.8, on Correlation and Regression.

Continuing Features

Several features appear throughout the book, adding interest and provoking thought.

- Problem Solving Beginning in Chapter 1, students are introduced to problem solving and critical thinking. The theme of problem solving is then continued throughout the text, and special problem-solving exercises are presented in the exercise sets.
- Critical Thinking Skills In addition to a focus on Problem Solving, the book also features sections on Inductive Reasoning and the important skills of Estimation and Dimensional Analysis.
- Profiles in Mathematics Brief historical sketches and vignettes present the stories of people who have advanced the discipline of mathematics.
- Chapter Openers Interesting and motivating photo essays introduce each chapter and illustrate the real-world nature of the chapter topics.
- **Did You Know** . . . These colorful, engaging, and lively boxed features highlight the connections of mathematics to history, to the arts and sciences, to technology, and to a broad variety of disciplines and student majors.

Instructor's Supplements

Instructor's Solutions Manual This manual contains detailed, worked-out solutions to all the exercises in the text, and answers to the Group Projects.

OmniTest³

OmniTest³ is available in DOS-based and Macintosh formats. This new version of this easy-to-use software was developed for Addison-Wesley by ips Publishing, a leader in computerized testing and assessment.

- The Macintosh format makes full use of the Macintosh graphical user interface.
- DOS user interface is easy to learn and operate. Its Windows look-alike structure lets the user easily choose and control the items as well as the format for each test.
- Make-up exams, customized homework assignments, and multiple test forms can be quickly and easily created.
- OmniTest³ is algorithm driven—meaning that the program can automatically handle insertion of new numbers into the same equation—creating hundreds of variations of that equation.
 - a) The numbers are constrained to keep answers reasonable, so that a virtually endless supply of parallel versions of the same test can be created.
 - **b)** With this new version of OmniTest the values shown in the model problem may be "locked in."
- OmniTest³ is keyed section by section to the text, allowing selection of questions that test individual objectives from that section.
- Instructor-generated questions may be entered by way of OmniTest³'s sophisticated editor—complete with mathematical notation.

Printed Test Bank The Test Bank includes three alternative tests per chapter. These items may be used as actual tests or as references for creating tests with or without the computer.

Test Generator/Editor for Mathematics with Quiz Master (CLAST

Version) Is a computerized test generator that lets instructors select test questions by CLAST objective or make use of ready made tests. The software is algorithm driven so that regenerated number values maintain problem types, and provide a large number of test items in both multiple-choice and open-response formats for one or more test forms. The IBM/Windows editor lets instructors modify existing questions or create their own including graphics and accurate mathematics symbols. The Macintosh version allows instructors to add their own questions. Tests created with the Test Generator can be used with Quizmaster, which records student scores as they take tests on a single computer network and prints reports for students, classes, or courses.

Videotapes Correlated to each important topic are available to departments. Contact your Addison-Wesley Sales Consultant.

Student's Supplements

Student's Solutions Manual This manual contains detailed worked-out solutions to all the odd-numbered section exercises and to all Review and Chapter Test exercises. Students will find this manual very helpful.

Guide to CLAST Mathematical Competency (State of Florida) This guide provides all the necessary material to help students prepare for the computational portion of the CLAST test. It includes worked-out examples and practice for CLAST skills, as well as a practice test. Optional topics in trigonometry are provided for those who wish to brush up in this area as well.

Interactive Mathematics Tutorial Software with Management System Is an innovative package that is CLAST objective based, self paced, and algorithm driven to provide unlimited opportunity for review and practice. Tutorial lessons provide examples, progress-check questions, and access to an on-line glossary. Practice problems include hints for first incorrect responses, solutions, and online tools to aid in computation and understanding. The optional management system records student scores on disk, and lets instructors print diagnostic reports for individual students or classes. **Student Versions**, which include record keeping and practice tests, may be purchased by students for home use. (IBM DOS/Windows and Macintosh).

Acknowledgments

We appreciate the suggestions offered by many students and faculty members for improving our text. In particular, we thank those who reviewed this edition of the text: Helen G. Bass, Southern Connecticut State University; Una Bray, Skidmore College; Linda F. Crabtree, Metropolitan Community College; Richard DeCesare, Southern Connecticut State University; Theresa A. Geiger, St. Petersburg Junior College; Leo Andrew Lusk, West Virginia University; Frances O. McDonald, Delgado Community College; Will Miles, Tri-County Technical College; Joanne Peeples, El Paso Community College; and Donna Weir, Union County College. We would also like to thank Larry Clar of Monroe Community College for reading the galleys and checking answers.

Our wives, Kathy and Joyce, and our children, Robert and Steven, and Tod, Teri, Lisa, and Brian deserve our special thanks. Without their support and great sacrifice, this book could not have become a reality.

It is our pleasure to acknowledge the assistance given us by the staff at Addison Wesley Longman, Inc. In particular, we appreciate the advice and encouragement of our acquisitions editor, William Poole and our project manager, Christine O'Brien. We would also like to thank our production editors, Jenny Bagdigian and Sandra Rigney, and our art editor, Susan London-Payne.

We also acknowledge the contribution of our supplement authors: Christine Dunn and Gary Egan of Monroe Community College, for the *Instructor's Solutions Manual* and *Student's Solutions Manual*; Evelyn Woodward for the *CLAST Supplement*; and ips Publishing, Inc., Vancouver, Washington, for OmniTest³.

We would also like to thank all the reviewers and students who have provided valuable suggestions for past editions of this book. The success of this book is a result of the cumulative constructive suggestions we have received over the years, and we want our reviewers to know we appreciate their help. The following is a list of reviewers for the first through fourth editions, and their affiliation at the time they provided their review.

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James Wooland

Florida State University

Mathematics is an exciting, living study. It has applications that shape the world around you and influence your everyday life. We hope that as you read through this book you will realize just how important mathematics is and gain an appreciation of both its usefulness and its beauty. We also hope to teach you some practical mathematics that you can use in your everyday life and that will prepare you for further courses in mathematics.

Our primary purpose in writing this text was to provide material that you could read, understand, and enjoy. To this end we have used straightforward language and tried to relate mathematical concepts to everyday experiences. We have also provided many detailed examples for you to follow.

The concepts, definitions, and formulas that deserve special attention have been either boxed or set in boldface type. The exercises are graded so that the more difficult problems appear at the end of the exercise set. The problems with exercise numbers set in color are writing exercises. At the end of most exercise sets are Problem Solving/Group Activity exercises that contain challenging or exploratory exercises. At the end of each chapter are Group Projects which reinforce the material learned or provide related material.

Each chapter has a summary, review exercises, and a chapter test. When studying for a test, be sure to read the chapter summary, work the review exercises, and take the chapter test. The answers to the odd-numbered exercises, all review exercises, and all chapter test exercises appear in the Answer section in the back of the text. However, you should use the answers only to check your work.

It is difficult to learn mathematics without becoming involved. To be successful, we suggest you read the text carefully and work each exercise in each assignment in detail. Check with your instructor to determine which supplements are available for your use.

We welcome your suggestions and your comments. Our address is Monroe Community College, Rochester, NY 14623. Good luck in your adventure in mathematics!

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Chapter 1 Critical Thinking Skills

ife constantly presents new problems. The great inventors, scientists, scholars, politicians, and artists make their contributions to civilization by confronting and solving problems. To learn the techniques for solving problems requires practice and patience. But once the basic principles are understood, they can be applied to each new challenge.

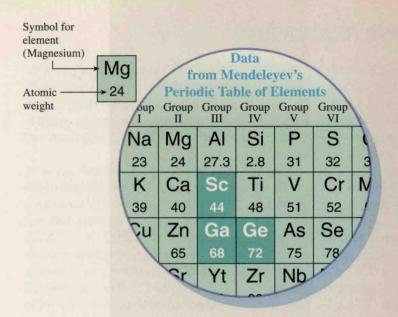
The goal of this chapter is to help you master the skills of reasoning, estimating, and problem solving. These skills will aid you in solving the problems in the remainder of this book, as well as problems that you will encounter in everyday life.

Problem solving requires critical thinking: considering the information given, deciding what question must be answered, making and acting on a plan to find the answer, and



Charles Lindbergh, who made the first solo flight across the Atlantic in 1927, had to solve many problems, such as the route over the ocean and the financing of the trip. Most important, the aircraft design had to be modified to carry enough fuel for the 3600-mile trip. To keep the load light, Lindbergh flew without a copilot, parachute, or radio and carried only five sandwiches and a quart of water on his 33-hour, 32-minute trip.

checking the answer. Mathematics provides the tools needed to think critically and solve problems. Every day, you make decisions that require you to use critical thinking skills. Sometimes, the problem may be one that can be solved by computation. For example, in a drugstore you may be choosing between two bottles of shampoo. One is 6 ounces and costs \$2.95, the other is 8 ounces and costs \$3.50. Which is the better price? The answer to this problem lies in finding the unit cost: If you compute the cost for 1 ounce of each



Seeing patterns often helps in solving problems. In 1869, chemists had isolated 63 of 109 of the chemical elements known today, but had not found any apparent underlying order. In that year, Russian chemist Dmitri Mendeleyev saw a pattern. He proposed a table of the elements organized by increasing atomic weight and grouped according to similar properties. To make this method work, he had to predict the existence of three then-unknown elements: scandium, gallium, and germanium.

shampoo, you will find that the 8-ounce bottle costs the least per ounce.

Sometimes solving a problem may require you to make a reasonable estimate, to look for clues, or to experiment with several possible solutions before choosing the best solution. Often the most important part of solving a problem is just understanding what question must be answered.



ne way of determining the answer to the question "how many" is to estimate, using a small sampling of a larger group. This technique can be used to guess how many jelly beans are in a jar, or how many people are in attendance at a political rally.