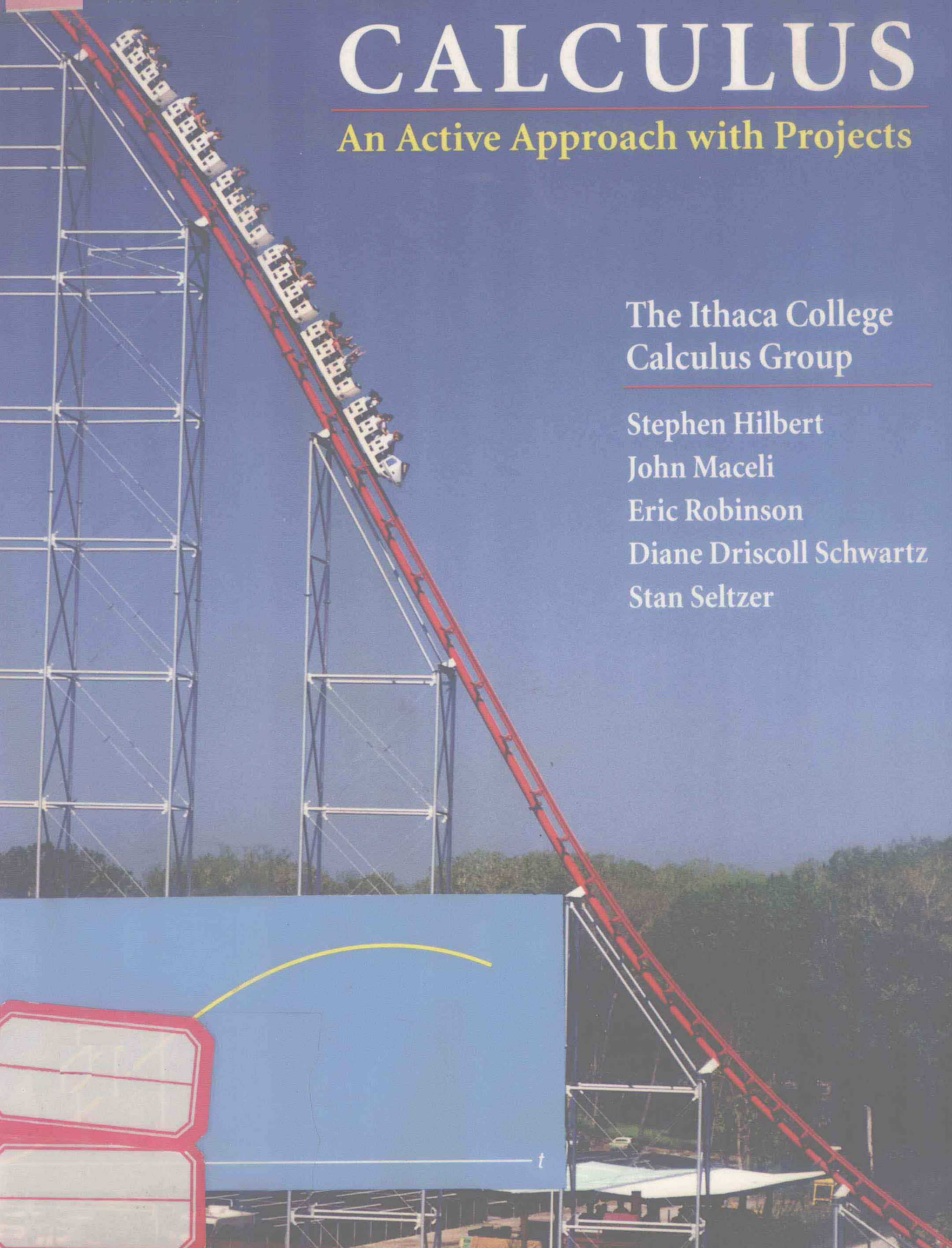
CALCULUS

An Active Approach with Projects



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Preface

Calculus: An Active Approach with Projects is a collection of materials for first-year calculus developed and tested at Ithaca College. It is not a complete textbook, but a complementary volume that can be used successfully in conjunction with any textbook.

We view calculus as a unified subject rather than a linearly ordered sequence of topics and believe that this view should be conveyed to students from the outset of their studies. We have found that students who are actively involved in class are more likely to succeed than those who are passive note-takers. The materials in *Calculus: An Active Approach with Projects* were designed to bring these ideas to life in the classroom.

There are two major sections in the book. The first section contains *activities* that can be done in class or as homework. The second section contains *projects* for the students to work on (usually in teams) outside the classroom.

Activities

The class activities are designed to accustom students to active participation in the course and to introduce some of the material and methods we have identified as important. Activities involve the students in their own learning. Students who have used activities regularly in class tend to make better comments and ask more significant questions about course material than students in more traditional classes. The key to this difference is active participation. This kind of student involvement fosters understanding and retention of course material.

The activities in *Calculus: An Active Approach with Projects* have several purposes. Through the activities, students participate in the development of many of the central ideas of calculus. Activities introduce new calculus topics, often in a guided discovery format. This reduces the amount of formal presentation that the instructor must do. The activities are also an excellent vehicle for promoting cooperative learning in the classroom.

By doing activities, students learn modeling and how to use the top-down approach to solve problems—both are useful for successful completion of projects, and help them solve shorter problems as well. Students who have experienced activities and projects do not regard "word problems" with dismay.

Activities also help students learn how to draw and interpret graphs—a key element in learning ways to represent functions that are not necessarily given as formulas. Finally, the first set of activities provides an overview of most of first-semester calculus. Doing a number of these activities early in the course helps students see the unity of the subject.

Projects

Projects serve to reinforce material already presented, motivate concepts, or introduce topics that might not otherwise be covered. Projects bring out both the relevance and the unity of calculus. Most of the projects are set in "realistic" situations. Most also involve more than one calculus topic, often combining topics in unexpected ways. Students need to broaden their perspective and synthesize ideas in order to complete these projects successfully.

We have students work on the projects in teams of three or four, submitting a single report, although many of the projects have parts that are completed by individual students. To complete a project, each team needs to submit a well written report of its solution. Writing about mathematics may be a new experience for many students, but it is a valuable one. To describe the solution of a significant problem precisely in words requires a deeper understanding than most students gain from just solving many problems that are based on examples found in their notes or textbooks. Most of the projects require about two weeks to complete. In a typical semester course, we have our students do three or four projects.

Many of the projects have "open-ended" parts—that is, parts for which there is not a unique correct answer or approach. These questions encourage the students to brainstorm with their teams and to view mathematics as a subject with creative elements.

A significant benefit of this project-oriented approach is that students learn to solve non-trivial, multi-step problems. Working on the (shorter) activities in a guided classroom environment helps them succeed on projects.

Spiral Approach

The organization and content of the early activities in this book are based on the *spiral approach*. We have found this to be a particularly effective way to teach calculus. Our goal is to present the main ideas of the course early, without getting mired in detail, so that the students will see calculus as a unified subject. The emphasis at this stage is on concepts and relationships, not on technical details.

We use the *calculus of graphs* for this purpose. That is, representing the functions involved almost exclusively in graphical form, and using the familiar ideas of velocity and distance as examples, we examine basic ideas from both differential and integral calculus. Within days, the students have some basic understanding about rates and slopes, concavity, and integration (in the context of obtaining a distance graph when given the corresponding velocity graph). During the rest of the course, students encounter these ideas again and again, each time picking up more of the technical and computational details.

"New" Calculus

Efforts to revise the way calculus is taught have focused on a number of different issues. The materials presented in *Calculus: An Active Approach with Projects* are designed to empower the student to take an active role in her or his own learning. We emphasize the role of calculus as a tool for understanding the world and hence focus on modeling as a central theme. We also emphasize the notion of function and are careful to show that functions can be represented in many different ways: as graphs, as tables of values, as algebraic expressions, as verbal descriptions, as physical relationships, and as theoretical models. These materials are designed to enable any instructor to incorporate these ideas and approaches into their calculus course.

Technology

Some of the activities presuppose the use of either a computer or a graphing calculator. Some of the projects are greatly simplified if some computational device is available to help with the calculations and graphs. The *Instructor's Guide* lists what, if anything, is needed for any particular activity or project. We do not prescribe any particular choice of technology, however. We have always described our materials as *technology independent.* This means that most of the activities and projects require no technology at all, and the few that do require a computer or graphing calculator are presented in such a way that the instructor using the material can choose whatever implementation is available.

The Instructor's Guide

Teaching a calculus course using "new" materials usually requires some modification of one's teaching style and some reorganization of topics. As an aid in this adaptation, we have made available an *Instructor's Guide* to accompany this book. There you will find annotated versions of the activities and projects, including time estimates, background required, teaching suggestions, and topics covered. We have also provided sample curricula, sample test questions, and a set of answers to frequently asked questions about the materials in the book.

Acknowledgments

We wish to thank a number of people whose advice and encouragement have been invaluable to us during this project.

Paul Glenn of Catholic University was an original member of our group and a valuable participant during the initial phase of our project.

Professors William Lucas of Claremont Graduate School and Gil Strang of M.I.T. have contributed invaluable support and advice since we started developing these materials. Professor Tom Tucker of Colgate University has been a member of our advisory board since 1988.

Our colleagues on the faculty of Ithaca College have been generous with time and help. We especially want to acknowledge useful discussion and class testing of material by Jim Conklin, John Rosenthal, and Martin Sternstein of the Department of Mathematics and Computer Science. We also want to thank all our students and the participants in our workshops for helping us to correct flaws and clarify earlier versions of these materials.

Spud Bradley, formerly of the National Science Foundation, and Paul Hamill of Ithaca College provided both help and expertise when we were looking for funding. Finally, we wish to thank the National Science Foundation and Ithaca College for their support of this project.

To the Student

This is a book of activities and projects for calculus. It is designed to help you to understand the basic concepts of calculus and to become a good problem solver.

To use these materials successfully, you should approach them with an open mind and a lot of optimism. The activities are relatively short calculus problems or explorations. Most of them will not look like problems you have seen before. All of them are problems on which almost any first-year calculus student can make significant progress.

Working through the activities should help you understand the basic concepts of calculus and how calculus serves as a tool for understanding the world. The activities will also help you learn ways to approach unfamiliar problems and to make progress in solving them.

The projects are larger problems which will require considerable effort to solve. Many of the projects involve several different calculus ideas, so be prepared to draw on all your mathematical background. Many are also "open-ended" problems. That means that there isn't necessarily just one correct solution. Your ideas for these problems will need to be supported by well thought out explanations that will be convincing to others.

Your instructor will decide just how these materials will be used in your particular course. We hope this book will help make calculus interesting, challenging, and meaningful to you.

Contents

Part I

Activities

Chapter 1

Graphical Calculus

Graphical Calculus and Modeling

The first chapter of this book consists of problems and activities designed to introduce you to many of the important ideas of first-year calculus in a way that encourages you to visualize the objects and actions you are studying. We call this approach the *calculus of graphs.*

We see graphs not only in mathematics, but also in the physical sciences, the social sciences, even the daily newspaper. Graphs are a way of comprehending the world. Graphs give us a way to visualize an ongoing process as a whole. That is, a graph can contain the whole past history of a process, and a prediction of its future progress, in a way that can be comprehended quickly. In short, the usefulness of graphs illustrates the truth of the old saying, "One picture is worth a thousand words."

In many of the activities you will be asked to work with phenomena or functions that have been represented only as graphs, and to sketch graphs to represent real world problems you are studying. In many cases, you will not have a formula that corresponds to the graph—the graph itself contains all the information for the problem. This approach will enable you to see the big picture of calculus, while temporarily postponing many of the technical details.

At the same time, you will get an introduction to *modeling.* Modeling is the process of representing real world problems in mathematical terms so that the methods of mathematics can be used to gain understanding of the original problem. When you sketch a graph based on your observation of some action or from reading a verbal description, you are constructing a mathematical model. The effective use of mathematical models has been responsible for much progress in the physical and social sciences.

The approach to the learning of calculus that we take throughout this book emphasizes both the calculus of graphs to gain understanding and insights, and modeling to help you become a good problem solver.

Chalk toss

Calculus is a study of changes. One form of change is the change in the position of something that is moving. For example, if your instructor throws a piece of chalk into the air and then catches it, the distance of the chalk from the floor changes.

We can record the chalk's position relative to the floor on a graph. The graph will capture the information about how the position changes over time.

1. Watch as your instructor tosses the chalk, and record what you see on a graph.

Exchange papers with the student next to you.

2. Study the graph you just received. In one or two sentences, describe the motion of the chalk that *the graph* describes (even if that does not correspond to what you observed). Be sure to include in your description the answers to the questions:

   "How high did the chalk go?"

   "How long was the chalk in the air?"

3. Briefly discuss both graphs and both verbal descriptions with your neighbor. Decide together which graph you prefer as a solution to the original problem, or what a solution that is better than either one would look like.

4. Watch as your instructor repeats the chalk toss. Based on your previous experience, and your discussion with your neighbor, sketch a new graph.

Watch as your instructor tosses the chalk several more times. After each toss, graph what you see, and compare your new graph with the previous ones.

Your instructor may ask you to discuss your work with another student, and synthesize ideas.

Classroom walk

Your instructor will walk across the front of the classroom. He or she will designate a "starting line" in the front of the room, so there will be a reference point for the trip.

1. You are to graph the displacement, f, of your instructor from the starting line, as a function of time, t.

2. Graph the second trip.

3. Graph the third trip.