Nonlinear Phenomen

in Fluids, Solids and Other Complex Systems

Nonlinear Phenomena
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Edited by

Patricio Cordero
Bruno Nachtergaele
Departamento de Física
Facultad de Ciencias Físicas y Matemáticas
Universidad de Chile
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PREFACE

NONLIN 2, – The second Latin American Workshop on Nonlinear Phenomena – was held in Santiago from 6 to 14 September 1990. Several physicists from the region and from the first world met during eight days to discuss aspects on nonlinear phenomena in physics. During the morning sessions series of outstanding review lectures were given by leading physicists in the field. In the afternoon these were followed by a great number of research reports and we believe that the combined publication of the lecture notes and a selection of research reports will make this volume a valuable tool for future work on the topics.

It is not common to have a Latin American physics meeting with so many relevant physicists from both the first world and the southern region, and not very often that the best Latin American physicists get together. The active presence of participants from Argentina, Brazil, Chile, France, Italy, Mexico, the United States of America, Uruguay and Venezuela was crucial to make the meeting such a success. Most sessions were followed by vivid discussions. We are convinced that meetings with these characteristics are essential for the further development of scientific activity in third world countries.

The Workshop focused its attention on a few aspects of nonlinear phenomena, as they appear in fluids, solids, cellular automata, neural networks and other complex and/or disordered systems. For the readers convenience the material has been divided into four chapters, but this division should not be taken too strictly. Many papers combine different aspects of nonlinear phenomena, or refer to a variety of applications.

The invited lecturers were: P. Bak, R.B. Griffiths, P.C. Hohenberg, L.P. Kadanoff, A.J. Libchaber, J.L. Morán-López and M.A. Virasoro. Our attempt to publish the lecture notes of all of them has not been completely successful.

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The organizers are very much indebted to the following persons whose assistance was essential in one or more aspects of the organization of the conference: Jacobo Rapaport, Igor Saavedra, Nelson Zamorano, Naum Joel, Francisco Brieva, and also to the students Dino Risso and Marianne Takamiya. We are also grateful to Carmen Belmar and Maltilde Gálvez.

The Editors

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Part 1:
FLUIDS

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THERMAL TURBULENCE

A. LIBCHABER, E. MOSES, X.Z. WU, and G. ZOCCHI

University of Chicago, Physics Department and The Research Institutes
5640 So. Ellis Avenue, Chicago, IL 60637 USA

1. INTRODUCTION

In his referred paper on the problem of turbulence [1] Landau wrote "Although the turbulent motion has been extensively studied from different points of view, the very essence of the phenomenon is still lacking clearness. The problem may appear in a new light if the process of initiation of turbulence is examined thoroughly." As experimentalists, we undertook a program to study the unsteadiness of the laminar motion, starting from the onset of free convection.

Following Threlfall [2], we used helium gas at low temperature as the fluid, for two main reasons. First a large span of Rayleigh numbers can be obtained by changing the gas density, going up to the liquid-gas critical point. Second, at helium temperatures, the kinematic viscosity can reach very small values, of the order h/m. [3]. Thus, large Rayleigh (Ra) and Reynolds (Re) numbers can be reached in a closed system of reasonable size, with solid thermal stability and well defined boundary conditions. Precisely, in our last cell size, 40 cm height, we can get close to $Ra = 10^{16}$ and Re of the order of 10^8. Not able to visualize the flow in He gas we concentrate on statistical measurements, heat flux, histograms of temperature fluctuations, large scale velocity, power spectra.

An optical study of the turbulent flow was also developed by Zocchi et al [4]. In essence, we have shown that a possible way to attack the problem is to concentrate on the dynamics of the coherent structures in the flow: how they are formed, how they propagate, and what they carry. the situation is not too complicated because apparently there aren't too many different structures in the cell; here we distinguish three kinds: heat carrying plumes, vorticity carrying swirls, and waves propagating along the boundary layers. We give a qualitative description of the interplay between these objects in the "life cycle" of the cell, and we show that there are regions in the flow where the coherent structures are dominant and regions where they cannot penetrate.

2. THE MATHEMATICAL PROBLEM

Let us consider the following physical system: a fluid is contained in a closed box, which we take, for definiteness, to be a cube (this is the geometry of the present experiment). The sides of the box are thermally insulated from the outside environment (for example by means of a vacuum jacket); the top is maintained at a constant, uniform temperature (let's call it 0), and the bottom at a higher temperature Δ. The box is in a uniform gravitational field.

This is a non equilibrium system: there is a temperature difference between the bottom and top, and correspondingly a heat flux through the system. To state the problem in a simple mathematical form we consider the equations for the velocity and temperature fields of the fluid in the Boussinesq approximation, which consists in neglecting the variations of the fluid's properties with temperature everywhere except in the buoyancy term (this effect has obviously to be retained since it provides the forcing in the momentum equation). The equations of motion (which express the conservation of momentum, energy and mass) then read:

$$
\begin{aligned}
(\partial_t + \mathbf{u}\nabla)\mathbf{u} &= -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + g\alpha\theta\hat{\mathbf{z}} \\
(\partial_t + \mathbf{u}\nabla)\theta &= \kappa\nabla^2\theta \qquad (1) \\
\nabla\mathbf{u} &= 0
\end{aligned}
$$

Here $\mathbf{u}(\mathbf{x},t)$, $p(\mathbf{x},t)$, $\theta(\mathbf{x},t)$ are the velocity, pressure and temperature fields, ρ is the density, ν the kinematic viscosity, α the expansion coefficient and κ the thermal diffusivity of the fluid, and g is the acceleration of gravity. $\mathbf{z}$ is the unit vector in the upward direction. The forcing term $g\,\alpha\,\theta\,\mathbf{z}$ is well defined when one specifies the boundary conditions for the temperature (only temperature differences matter, not the absolute temperature).

The boundary conditions are that the velocity is zero everywhere at the walls of the box, while the temperature is zero at the top, Δ at the bottom, and the normal derivative of the temperature is zero at the sidewalls (since there is no heat flux out of the sidewalls). If we choose a Cartesian coordinate system parallel to the sides of the box, with the origin at the center of the box and the z-axis pointing upwards, and calling L the length of the side of our cubic box, these boundary conditions are:

$$\begin{aligned}
&\mathbf{u}=0 \ , \ \theta=0 \quad \text{at} \quad z=L/2 \ , \ -L/2 \le x,y \le L/2 \\
&\mathbf{u}=0 \ , \ \theta=\Delta \quad \text{at} \quad z=-L/2 \ , \ -L/2 \le x,y \le L/2 \\
&\mathbf{u}=0 \ , \ \partial_x\theta=0 \text{ at} \quad x=\pm L/2 \ , \ -L/2 \le y,z \le L/2 \\
&\mathbf{u}=0 \ , \ \partial_y\theta=0 \text{ at} \quad y=\pm L/2 \ , \ -L/2 \le x,z \le L/2
\end{aligned} \tag{2}$$

To specify the problem completely we also need an initial condition, but we will not worry about this here. We note again that the equations (1) are only approximate: apart from the "Boussinesq approximation", other effects have been neglected; for example, the heat equation should contain a term describing the generation of heat from viscous dissipation, etc.

As always in fluid mechanics, in order to understand what the control parameters really are one puts the equations and boundary conditions in dimensionless form. To do that we take L , L^2/κ and Δ as the units of length, time, and temperature respectively. The choices of L and Δ are obliged in order to put the boundary conditions (2) in non-dimensional form, whereas the choice of L^2/κ as the time scale contains an arbitrariness of a power of the Prandtl number $\sigma = \nu/\kappa$, in other words $(L^2/\kappa)\,\sigma^\gamma$ (for any γ) would be an equally suitable choice. This arbitrariness with respect to the Prandtl number is present, as we will see in chapter V , in all simple scaling arguments on convection.

Transforming the equations (1) (i.e. with the change of variables $x \to x/L$, $t \to t/(L^2/\kappa)$, $u \to (L/\kappa)\,u$, $\theta \to \theta/\Delta$ and $p/\rho \to (L^2/\kappa^2)\,p/\rho$) and giving to the new variables the same name as the old ones one finds:

$$\begin{aligned}
(\partial_t + \mathbf{u}\nabla)\mathbf{u} &= -\frac{1}{\rho}\nabla p + \sigma\nabla^2\mathbf{u} + \sigma Ra\,\theta\hat{\mathbf{z}} \\
(\partial_t + \mathbf{u}\nabla)\theta &= \nabla^2\theta \\
\nabla\mathbf{u} &= 0
\end{aligned} \tag{3}$$

where $\sigma = \nu/\kappa$ is the Prandtl number and $Ra = \dfrac{g\alpha\Delta L^3}{\nu\kappa}$ is the Rayleigh number.The boundary conditions become $\theta = 0$ at $z = 1/2$, $\theta = 1$ at $z = -1/2$, etc. So the problem, in this formulation, depends only on two parameters, Ra and σ . There is one known solution of these equations and boundary conditions, and it is the purely diffusive one, that is the velocity field is zero and the temperature profile is linear with height. This solution is stable for $Ra < R_c$, and unstable for $Ra > R_c$, where R_c is a critical Rayleigh number independent of σ , which can be determined theoretically by a linear stability analysis [5] , or experimentally by measuring the heat flux , and is found to be about 2000. For $Ra > R_c$ convection starts, and the flow becomes more and more complex as the Rayleigh number is increased, going from a time independent to a simple periodic, then chaotic, and finally turbulent behaviour. The range of Rayleigh numbers of the present experiment (~ 10^7 - 10^9) is well past the chaotic regime, and the flow is turbulent.

Let us now look at the dimensional arguments made above a little differently, because it is thus possible, just by stating the problem and without even starting to solve it, to appreciate what the questions and difficulties are. In most experiments (including the present one) the imposed boundary conditions are actually different from (2) , in that the heat flux through the system is held constant, and not the temperature drop Δ . That is, one usually heats the bottom plate with a constant power P, and regulates the temperature of the top plate at a given value (let's call it 0). So instead of the boundary condition $\theta = \Delta$ at $z = -L/2$ we now have $L^2 \rho c_p \kappa \, \partial_z \theta = P$ at $z = -L/2$; c_p is the specific heat, so $\rho c_p \kappa$ is the thermal conductivity, $\rho c_p \kappa \, \partial_z \theta$ is the heat flux at the plate, and L^2 is the area of the plate. To put this new problem in dimensionless form we again take L and L^2/κ as the units of length and time, but looking at the new boundary condition it is clear that we have now to take $P/(\rho c_p \kappa L)$ as the unit of temperature. The resulting equations are exactly the same as (3) , only with the number $\mathcal{L} = (g\alpha P L^2)/(\rho c_p \kappa^2 \nu)$ instead of the Rayleigh number. The boundary conditions are also all the same except that instead of $\theta = 1$ at $z = -1/2$ we now have $\partial_z \theta = 1$ at $z = -1/2$, in other words we go from "Dirichlet" boundary conditions to "Neumann" boundary conditions at the lower plate.

We would now like to venture some speculations regarding this state of affairs. If we do an experiment imposing the heat flux, we nonetheless expect the lower plate to finally settle down at some temperature, i.e. the system will choose a particular temperature drop Δ . This is observed experimentally: there is a one to one relation between the Rayleigh number and the heat flux. From a mathematical point of view though, if the system is determined with the condition $\partial_z \theta = 1$ at the plate, we expect it to be overdetermined (i.e. a solution may not exist) with the extra condition θ = some number, uniformly valid on the plate. We know of course what the solution of the puzzle is: the system respects the condition θ = const. not instantaneously and globally on the plate, but only on average. That is, there are fluctuations. Conversely, if one imposes the temperature drop Δ then there will be fluctuations of the heat flux, and the condition $\partial_z \theta)_{z=-1/2}$ = some number will hold only on average.

3. SCALING

Let us pick up again the problem of convection in our box, as defined by the equations and boundary conditions (1) and (2) of chapter II. First, we want to estimate the size δ of the boundary layer. As always, we start with dimensional analysis. As we saw in chapter II , there is a priori only one lengthscale in the problem, L , and the control parameters are Ra and σ . Thus the boundary layer size must have the form :

$$\delta \sim L\, f(Ra, \sigma) \tag{4}$$

where f is an unknown function (we suppose here to be in a regime with no hysteresis or dependence on initial conditions, so that f is an honest single - valued function). This is just dimensional analysis, and it contains no hypothesis other than saying that the equations (1) and (2) correctly describe the physical system. The scaling assumption consists in assigning a particular form to the function f ; since Ra is for us always very large, it is natural to write the Rayleigh number dependence as a power law (the rationale being that we are writing, for the quantity considered, the first term of an expansion in the small parameter 1/Ra) ; the Prandtl number is for us however of order unity, so we leave this dependence indicated by an unknown function g :

$$\delta \sim L\ Ra^{\alpha}\, g(\sigma) \tag{5}$$

Experimentally it has been shown [2], [3] that quantities like the Nusselt number Nu (and hence δ , which is related to it) have a simple power law behaviour over a large range of Ra , so the assumption (5) is justified. For the rest of the chapter we will ignore the Prandtl number dependence, since experimental data on this aspect are scarce (in our turbulent regime), and not much can be done theoretically by simple scaling. The origin of the difficulty, as pointed out in chapter II , is that with two transport coefficients , ν and κ , one can form an infinite number of different time scales, differing by powers of σ , so in any formula involving time scales there is, as far as scaling is concerned, an indeterminacy of a power of σ . Thus we set, in (5) , $\sigma = 1$:

$$\delta \sim L\ Ra^{\alpha} \tag{6}$$