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GAS-SOLID FLOWS

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FOREWORD

Over the past two decades major advances have been made both in numerical and experimental fluid mechanics. Some of these advances include the development of large numerical codes for solving complex flow regimes and the development of laser velocimetry which has greatly increased the experimental capabilities for fluid measurement. Gas-solid flow research, in particular, has been a beneficiary. For example, the interaction between suspended particles and a supporting turbulent flow can be simulated and non-intrusive measurements can record the velocity of particles transported in a complex flow field. Presently, researchers are continuing to make use of these newly developed techniques and the techniques are commonly used to solve more practical engineering problems in industry.

In light of this progress and in an effort to broaden the range of interests within the Multiphase Flow Committee, the organizing committee is hoping to establish this symposium as a biennial event. The objective of the symposium is to provide a forum for both researchers and engineers to discuss recent advances or applications involving gas-solid flows. The sessions are expected to address topics such as:

- analytical or numerical models for gas-solid flows
- recent advances in LDV techniques (particle sizing, concentration measurements, etc,)
- flow visualization of gas-solid flows
- fluid-particle turbulent interaction
- flow of non-spherical particles
- engineering applications (fluidized beds, erosion, combustion, separation devices, etc.)

although any topic related to gas-solid flows will be of interest.

Finally, the editor wishes to thank the authors, reviewers and session chairmen for their contribution to the symposium. He also extends special gratitude to R. A. Bajura for his continued advice during the organization of the symposium and to the invited speakers, D. E. Stock and S. L. Lee, for their willing cooperation and assistance in organizing the symposium.

John T. Jurewicz
Symposium Chairman

CONTENTS

PARTICLE DISPERSION IN TURBULENT GAS FLOW

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Abstract

Particle dispersion in a lightly loaded gas flow is governed by the scale of the gas phase turbulence, the physical size and mass of the particles, and the crossing trajectories effect. This paper reviews the influence of these three parameters on particle motion in homogeneous turbulence. Experimental and numerical work on particle dispersion in grid generated turbulence, pipe flow and jet flow is reviewed. finally, areas where current theories cannot explain experimental results and areas where experimental results are lacking are pointed out.

RECENT DEVELOPMENT OF PARTICLE DEPOSITION IN A TURBULENT SUSPENSION FLOW

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ABSTRACT

Highlights from laser-Doppler anemometer measurements of turbulent flows of a two-phase suspension are first described and peculiar features of flow behavior at the particle level pointed out. A new analytical approach based on the particle's dynamical response to turbulent fluctuations is then outlined to explain those measured peculiar features of flow behavior.

INTRODUCTION

The deposition of solid particles or droplets from a turbulent suspension flow to channel walls has been the central subject of a number of theoretical treatments. Most of these treatments are based on the conventional three-layer flow structure in the vicinity of the wall, the viscous sublayer, the buffet zone and the turbulent core, which has been established from studies of single-phase fully developed turbulent channel flows. In the turbulent core and much of the buffet zone, particles are assumed to be laterally transported by turbulent diffusion to the edge of the viscous sublayer. From there, they are assumed to coast across the sublayer to form deposition on the wall.

Unfortunately, these conventional treatments all contain an adjustable empirical constant which is not universal for all flow systems. For instance, Wildi [1] compared the prediction from one of such treatment with the result of wall deposition measurement of a mist flow of droplets over a size range. With the empirical constant adjusted to produce reasonable prediction for the larger droplets, the treatment generates an awkward under prediction of four orders of magnitude for the smaller droplets. This apparent drawback points to the question of the correctness of the very physics assumed in these theoretical treatments and thus calls for the local in-situ dynamic measurement of the two-phase suspension turbulent flow itself.

MEASUREMENT OF DYNAMIC BEHAVIOR OF SMALL PARTICLES IN NEAR-WALL REGION BY LASER-DOPPLER ANEMOMETRY

Particle sizing in a flow of suspension is a fundamental subject in a variety of industrial processes of practical importance. The conventional methods consist of collecting the particulates on a certain collector and then analyzing them with a size analyzer counter. The major drawback common to all these methods is that the measurement gives the distribution of number flux density of particulates in the flow rather than the distribution of number density of particulates in the flow. Much more desirable techniques for this kind of measurement are obviously those that make no use of intrusive mechanical probes, thereby avoiding the creation of disturbances in the flow and that at the same time are capable of making in-situ velocity measurements of both phases and size measurement of the particulates in the suspension with high resolution. Two of such techniques are the laser-Doppler anemometry (LDA) schemes developed specially for two-phase suspension flows: one for the smaller particulate size range from a few microns to about 240μm [2] and another for the large particulate size range of over about 240μm [3]. Since most practical applications concern themselves with particulates of up to a couple hundred microns in size, the scheme for the small size range is more useful for dynamic sizing.

The conventional laser-Doppler anemometry (LDA) is a technique which utilizes scattered light from very small tracer particles in a fluid to measure the velocity of that fluid. In principle, the laser anemometer is linear, needs no calibration, and measures the local velocity independently of the fluid properties. A relatively small optical measuring volume (240μm, for instance) and inherently fast response give it the ability to follow rapidly changing velocities in the fluid without the introduction of disturbances of other physical measuring probes [4]. This technique has been developed into a powerful research tool in studying single-phase flows in the last two decades, and its extension to two-phase suspension flows, primarily due to the inherent difficulties concerning particle sizing [5,6], has met success only in recent years [7].

The new generation of LDA for two-phase suspension measurement derives the particle sizing information from the very Doppler signal from which the information on the particle velocity is derived, thus ensuring the maintenance of the same level of spatial and temporal resolutions [8]. In addition to the frequency which is related to the particle velocity, the amplitude and path time of the Doppler signal are also analyzed. By a suitable electronic discrimination scheme, the central core of the measuring volume, Fig. 1, can be isolated in which the particle's size becomes readily deducible.

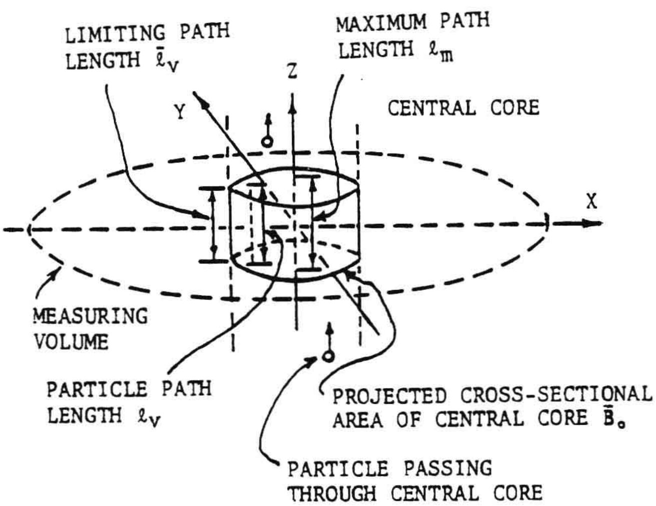

Fig. 1. Sketch of electronically isolated central core of optical measuring volume (from ref. [8]).

Typically, for each run of the experiment a total of from 20,000 to 100,000 particle Doppler signals together with Doppler signals from small (submicron size) contaminates, which serve as tracer particles for the carrier fluid in between every two successive particle Doppler signals, are collected to formulate the statistics of velocity and particle size of the suspension.

The optical system for this scheme is shown in the sketch of Fig. 2 in which the two velocity optical channels are separated by the polarities of the respective beams. Electronic circuits, mostly custom-built, are used to process the signals coming from the two photomultiplier tubes and the final outputs, in digital form, are fed to a PDP-11/34 minicomputer through the custom-designed computer interfaces and the PDP-11/34 interface. The data are stored in the memory of a hard disc and analyzed by software. The data acquisition system has been so automated that it is possible to collect as many data points as required under software control.

An experiment was conducted of the turbulent flow of a dilute water droplet-air suspension with a droplet size range of up to 100μm inside a vertical 10 mm by 25 mm rectangular channel [9]. Results obtained include the droplet size and number density distributions and the droplet size and two-dimensional velocity distributions for a total of 20,000 droplets

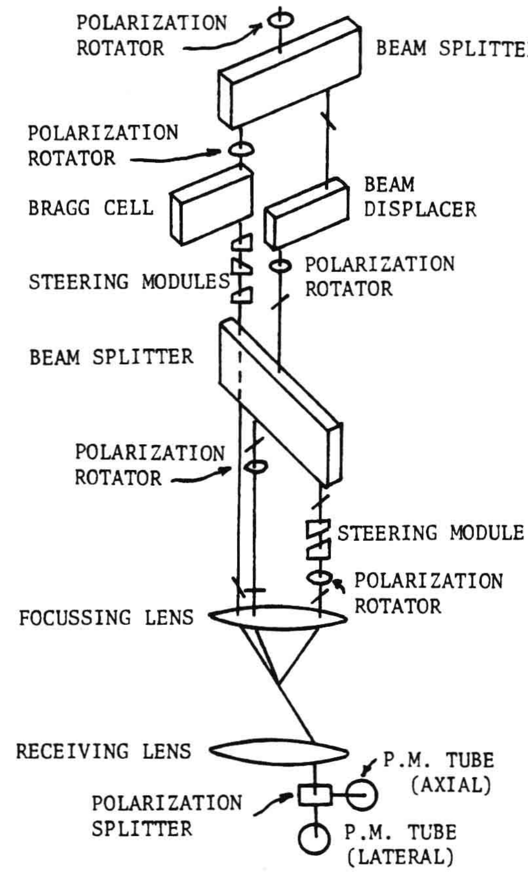

Fig. 2. Sketch of optical arrangement (from ref. [8]).

individually measured at each of a number of measuring locations across the channel. Figure 3 shows plots of droplet number density against distance from the wall for droplets of eight size ranges each of which is designated by its median value from 10 to 95 μm. The mean transverse velocities are indicated by the horizontal arrows the length of which represents the relative magnitudes. It is clear that droplets smaller than 50 μm are generally found to tend to move towards the wall in the near-wall region with the most vigorous transverse migration exhibited by droplets of about 30 μm. The conventional theoretical treatments are all totally incapable of providing an explanation to this newly discovered phenomenon.

MEASUREMENT OF DYNAMIC BEHAVIOR OF LARGE PARTICLES IN NEAR-WALL REGION BY LASER-DOPPLER ANEMOMETRY.

By a scheme developed by Durst and Zaré [10] for the determination of the velocity of a large spherical particle from the beat frequency of the moving fringe pattern formed by the two reflected beams from the particle, Lee and Durst [11] measured the turbulent upward flow of a glass particle-air suspension with one uniform size particles in a pipe of inner radius of 2.09 cm. The local time-mean axial velocities of the particles and the air at the various radial locations for particles of 100 μm, 200 μm, 400 μm and 800 μm are plotted in Figures 4a, 4b,

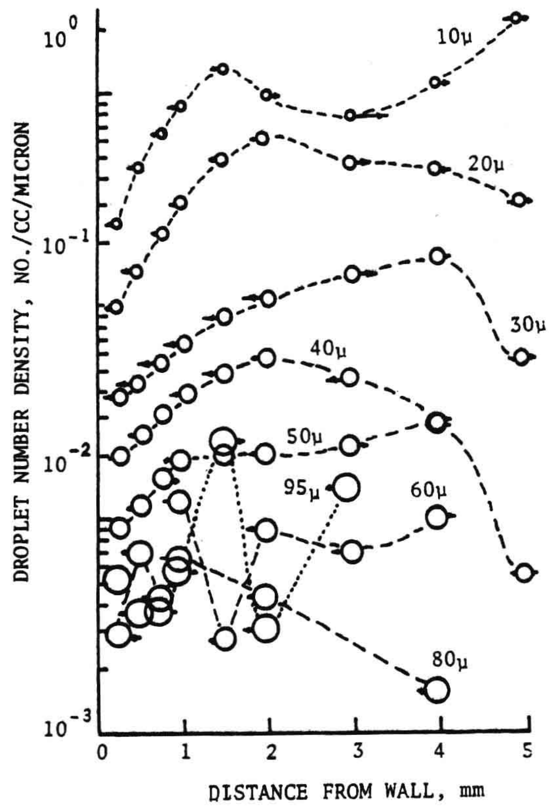

Fig. 3. Sample sketch of droplet behavior in a suspension channel flow (from ref. [9]).

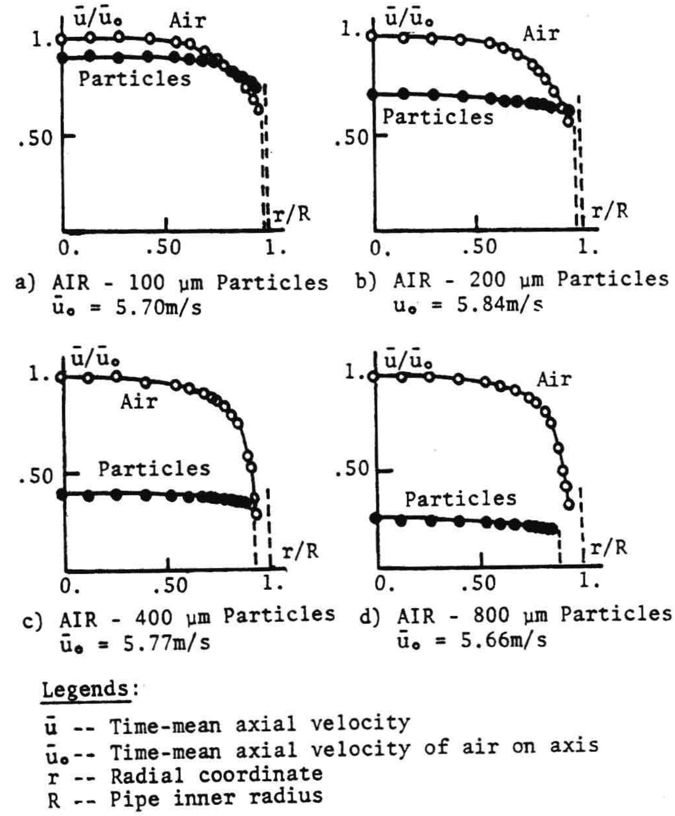

Fig. 4. LDA-measurements in two-phase flows: mean velocity profiles of air and glass spheres in upward pipe flow (from ref.[11])

4c and 4d, respectively. Generally, the particles were found to progressively lag behind the air due to the effect of gravity, the exception being the situation in the near-wall region for the 100 μm and 200 μm particles in which the particles were found to lead the air. The thickness of this region was about 20% and 10% of the pipe radius for the 100 μm and 200 μm, respectively. On the other hand, for the two larger particles of 400 μm and 800 μm, a sizable particle-free near-wall region was found, the thickness of which depended on the particles' size. These newly discovered peculiar phenomena in the near-wall region again clearly defy any prediction of conventional theoretical treatments.

A NEW THEORY OF DEPOSITION BASED ON A PARTICLE'S DYNAMICAL RESPONSE CHARACTERISTICS TO FLUID TURBULENCE

In analyzing a particle's behavior in turbulent flow, Rouhianinen and Stachiewicz [12] made use of the idea of frequency response of the particle developed by Hjelmfelt and Mockros [13]. An important result is the ratio of the amplitude of oscillation of the particle to that of the fluid as a function of the oscillation frequency of the fluid. In treating the problem of wall deposition, Lee and Durst [11,14] introduced the model of the particle's response characterized by a cut-off frequency below which the particle responds fully to the fluid oscillation and above which the particle is insensitive to the fluid oscillation. This cut-off frequency is a function of the particle's size, the ratio of density of the particle to that of the fluid and the fluid kinematic viscosity. The particle's motion is determined by turbulent diffusion for fluid oscillation frequencies smaller than the cut-off frequency and the mean, or quasi-laminar, motion of the fluid for fluid oscillation frequencies greater than the cut-off frequency. In an application to the two-phase turbulent pipe flow, a most energetic frequency of the fluid was evaluated as a function of radial location from existing results for the turbulent pipe flow of a single-phase fluid. Matching of the cut-off frequency from a particle's frequency response and the most energetic frequency from the turbulent fluid motion then determines the cut-off radius for the particle within the pipe. Within the cut-off radius lies the turbulent diffusion core and outside the cut-off radius the annular quasi-laminar region for the particle. The cut-off radius increases with the decrease of particle size for the same flow. Figure 5 shows a sketch of this new flow field classification.

AN EXPLANATION TO MEASURED MIGRATORY BEHAVIOR OF SMALL PARTICLES

We now return to the aforementioned, newly discovered phenomenon of most vigorous migration of particles of about 30 μm in the near-wall region of a suspension duct flow as shown in the sketch of Fig. 3. According to the new theory, up to the cut-off radius, the particle's transverse motion is determined by turbulent diffusion in the turbulent core. At the cut-off radius, the particle is ejected into the quasi-

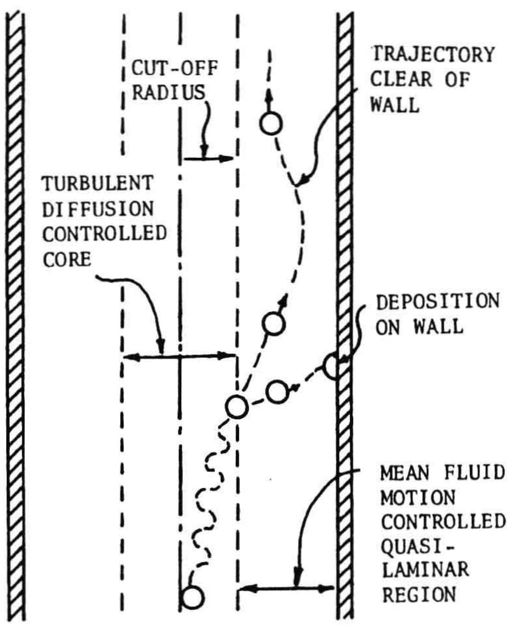

Fig. 5. New Flow Field Classification

laminar region to coast towards the wall. Since the cut-off radius increases with the decrease of particle size for the same flow, the smallest particle will have the thinest quasi-laminar region to cross. However, because the ratio of resistance to inertia forces is inversely proportional to particle size, the smallest particle will experience the severest resistance and therefore slow down most rapidly, as shown in the top sketch of Fig. 6.

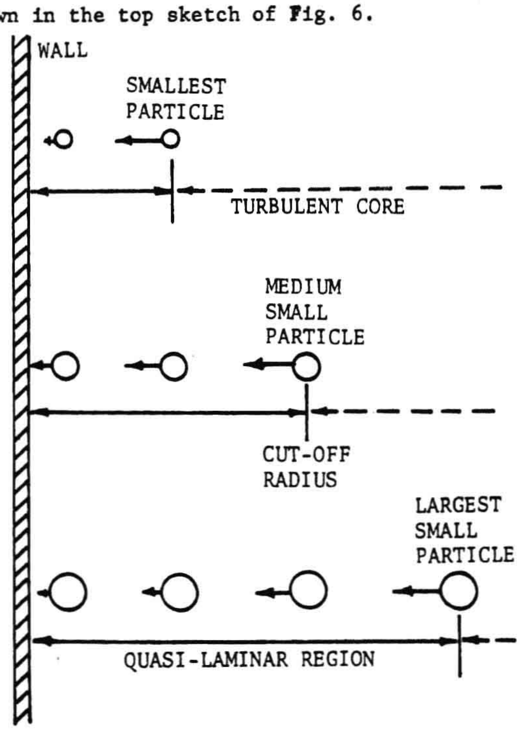

Fig. 6. Migratory behavior of small particles

On the other hand, the largest small particle will experience the lightest resistance over the thickest quasi-laminar region and therefore slow down considerably on reaching the wall as shown in the bottom sketch of Fig. 6. Somewhere between these two extremes, there exists an optimum-size particle which will possess the highest migratory velocity near the wall as shown in the middle sketch of Fig. 6. This may very well be the case of particles of about 30 μm in the aforementioned experiment.

AN EXPLANATION TO MEASURED MIGRATORY BEHAVIOR OF LARGE PARTICLES

Next, we return to the aforementioned, newly discovered phenomena of the creation of a particle-free zone and the existance of a slip-velocity inversion region near the wall in the turbulent upward flow of a glass particle-air suspension in a pipe as shown in the sketches of Fig. 4. According to the new theory, for the flow under consideration, the cut-off radius vanishes at a particle size of 21 μm and remains zero for larger sizes. In other words, as far as these particles (100 μm, 200 μm, 400 μm and 800 μm) are concerned, the whole flow across the pipe has become quasi-laminar and their motions are controlled by the mean fluid motion.

In the theories of laminar boundary-layer flows of a two-phase suspension by Otterman and Lee [15,16], Lee and Chan [17], and DiGiovanni and Lee [18], the particles at the edge of the boundary layer have negligible transverse velocity and the fluid is leading the particles in longitudinal velocity. The use of the transverse shear-slip lift force first derived by Saffman [19] helps the creation of a low particle concentration zone at the wall which has been verified by the experiments of Lee and Einav [20].

Use will be made of this shear-slip lift force, whose direction depends on the direction of the shear, in the analysis of the quasi-laminar flow of a suspension in a vertical pipe. The particle's migratory behavior is linked to the directional reversal of this lift force on two sides of the transverse matching position of the longitudinal velocities of the fluid and the particle in the near-wall region as shown in the sketches of Fig. 7. For a given main flow in which the particle is lagging behind the fluid in the longitudinal direction, the particle coming into the near-wall region initially experiences a combined resistance of drag and lift forces. If the initial transverse particle velocity in the interior of the flow is high enough for it to pass through the matching location, the lift force will reverse its direction and help propel the particle towards the wall. Therefore, this explains the existence of a slip-velocity inversion region near the wall for the 100 μm and 200 μm particles as shown in Fig.'s 4a and 4b, respectively.

On the other hand, if the initial transverse particle velocity in the interior is not high enough for it to reach the matching location, the particle will be kept away from the wall in a way similar to the finding of two-phase suspension boundary-layer studies mentioned above. Therefore, this explains the creation of a particle-free zone near the wall for the 400 μm and 800 μm particles as shown in Fig.'s 4c and 4d respectively.

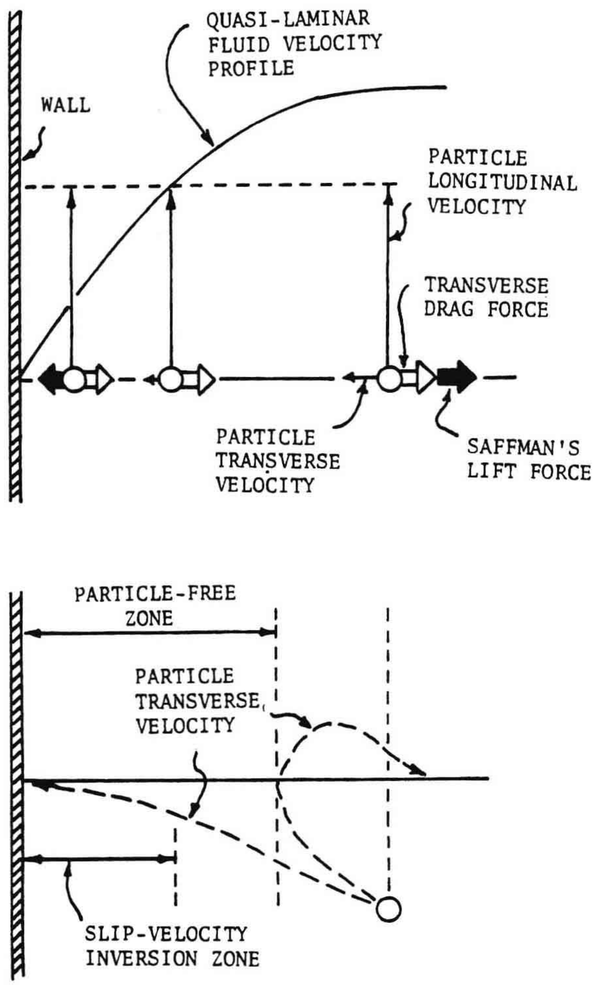

Fig. 7. Migratory behavior of large particles

CONCLUSION

Development of laser-Doppler anemometry in dynamic particle sizing has brought about new discoveries of migratory behavior of particles in the near-wall region which defy the conventional theoretical treatments on particle deposition. These new discoveries in turn have stimulated the successful development of a new dynamical response theory to provide the needed explanations.

ACKNOWLEDGEMENT

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ON THE RELATIONSHIP BETWEEN BROWNIAN MOTION AND THE STATISTICAL MOTION OF DISCRETE PARTICLES IN A TURBULENT FLOW

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ABSTRACT

It is shown that Brownian Motion is a special case of particle motion in a turbulent fluid for which

(a) the particle Reynolds no. is very much less than unity,

(b) the timescale of fluid motion is much shorter than the particle relaxation time,

(c) the systematic drag and fluctuating driving force are thermodynamically related through the fluctuation-dissipation theorem. This implies equipartition of energy of particle and fluid motion which is invalid in hydrodynamic turbulence.

By using an expression for the particle partial pressure the particle diffusion coefficient in a turbulent fluid is derived in a similar manner to the way Einstein derived the Brownian diffusion coefficient. In addition the Fokker Planck equation for Brownian motion is generalised to encompass particle motion in a turbulent flow. One of the terms in this equation are shown to be important in the 2 phase flow equations.

INTRODUCTION

The statistical motion of small particles suspended in a turbulent flow is a problem of great practical importance both industrially and environmentally and has received considerable attention in the past. Our own particular interest in the phenomenon is motivated by a need to know the plate out of in core activated particulate from the gas coolant of an Advanced Gas Cooled Reactor (AGR) both for safety assessment and normal operating conditions. Particle sizes may range from submicron up to 1000 μm in diameter. Our particular concern is with those particles greater than a micron which by virtue of their inertial response to changes in flow, cannot be assumed to follow the random fluctuations in the turbulent coolant. Values for the particle diffusion coefficient are necessarily of great importance in determining the particulate plateout onto a reactor component exposed to the flow.

There is an obvious similarity between this type of motion and that of Brownian (B.M.). In the one case (B.M.) the random driving forces originate in the random collisions of particles with fluid molecules whilst in the other case this agency is provided by fluid induced drag forces derived from the random interaction of particles with turbulent eddies. Recently this analogy has been explored by extending the application of the fluctuation-dissipation theorem of equilibrium thermodynamics to turbulent motion (Gitterman and Steinberg, 1980).

The apparently interesting result of this examination is that in homogeneous stationary turbulence the long time particle diffusion coefficient as with that for B.M. decreased with increasing particle diameter. This was in contradiction to previous theoretical work on the subject, notably Tchen (1947), Hinze (1959), Reeks (1977) and Pismen and Nir (1978). Here it was shown in general that in the absence of any external force, e.g. gravity, the particle diffusion coefficient approached an assymptotic limit with increasing particle diameter. More precisely this limiting diffusion coefficient $\varepsilon_{ij}(\infty)$ was given by

$$\varepsilon_{ij}(\infty) = \int_0^\infty R_{E_{ij}}(s)\, ds \qquad (1)$$

where $R_{E_{ij}}$ is the Eulerian velocity autocorrelation of the turbulence in a frame moving with the mean velocity of the flow.

More importlantly, the Gitterman and Steinberg result is also at variance with recent accurate measurements of the particle diffusion coefficient made by Wells and Stock (1983). They were mostly concerned with the measurement of the effect of crossing trajectories (Yudine, 1959). Measurements were made of the diffusion coefficients of 5 μm (low inertia) and 57 μm (high inertia) spherical particles. They found that for large diffusion times in the absence of drift the particle diffusion coefficients for both sizes were approximately equal (in fact the diffusion coefficient of the 57 μm size was slightly greater than that of the 5 μm). They also found good agreement with measurements based on the particle Eulerian velocity autocorrelation, all of which is consistent with equation (1). The beauty of their experiment was that they were able to separate out inertial effects from crossing trajectory effects, a feature not possible in the experiment (Soo, 1967) quoted by Gitterman and Steinberg as confirmation of their formula. In fact almost all of the particle size dependence of particle diffusion coefficient observed in that experiment was most probably attributable to crossing trajectories.

This paper was initiated in order to resolve this dichotomy of theoretical approach. However, in the process of examining the fundamental statistical differences between B.M. and particle turbulent motion (T.M.) it has taken on aspects more important than our original intent.

For example we shall approach the problem of particle motion in a turbulent fluid by considering the concept of partial pressure. By exploiting the Clausius Virial theorem commonly used in thermodynamics we shall obtain an expression for the pressure exerted by an ensemble of particles and then use this expression to obtain the particle diffusion coefficient. In this respect we shall adopt an argument used by Einstein in his original treatise on B.M. (Einstein, 1905). Here, the reader may recall, the diffusion coefficient was derived by considering the equilibrium of Brownian particles in a gravitational potential. Einstein looked upon the equilibrium as arising from either

(a) a balance between the pressure gradient and the gravitational force per unit volume, or

(b) a balance between the current due to gravitational settling and the gradient diffusion current.

The particle diffusion coefficient we obtain does not have the same form as for B.M. Indeed, it has the same form and lack of inertial dependence as that predicted by earlier work previously quoted. We shall show that this difference exemplifies the invalidity of the fluctuation dissipation theorem in hydrodynamic turbulence.

Our attention is then drawn to a consideration of the transport equation for particles in a turbulent fluid. We shall consider the simplest way the Fokker Planck equation for B.M. might be generalised to encompass motion in which the particle relaxation time is comparable or less than the time-scale of fluid motion. We do this by considering in addition a diffusion current in phase space proportional to the spatial gradient of the phase space concentration. This extra term turns out to be completely consistent with the existence of the partial pressure term previously derived and indeed we shall use it to evaluate the constant of proportionality (phase space diffusion coefficient).

THE FLUCTUATION DISSIPATION THEOREM

The origin of this theorem is based upon the fact that in a thermodynamic system like B.M. the impact of the molecules gives rise to not only a fluctuating random force but in addition a systematic frictional force. This means that the frictional force and the fluctuating force must be related in some way because they are of the same origin. The fluctuation-dissipation theorem is an expression of this fact. What is important to realise is that it also implies some other basic property of the system. In a classical thermodynamic system like B.M. it embodies the notion of thermodynamic equilibrium i.e. the number of accessible states, $\Omega(E)$, available to a system with energy E is of the form

$$\Omega(E) \sim e^{E/kT} \qquad (2)$$

where T is the absolute temperature and k is Boltzmann's constant. A manifestation of this concept is that there exists an equipartition of energy betwen the particle and the molecules of the surrounding fluid. The systematic velocity imparted to the particle is regarded as only slightly perturbing the equilibrium of the molecules in the immediate vicinity and that these rapidly establish a new equilibrium with the surrounding fluid. If the particle velocity $\underline{V}$ changes in time it must do so in a manner such that a new quasi-steady equilibrium is established at each value of time. Quite naturally the average frictional force must relate back to the energy ΔE imparted to the fluid as if that energy were associated with a new system in quasi-steady equilibrium with energy $(E+\Delta E)$, i.e.

$$\Omega(E+\Delta E) \sim e^{(E+\Delta E)/kT} \qquad (3)$$

It can be shown (Reif, 1965) that as a result of (3), the average frictional force $\langle F \rangle$ is

$$\langle F \rangle = \frac{1}{kT} \langle F \Delta E \rangle \qquad (4)$$

with

$$\Delta E = - \int_0^t v(s)F(s)ds \qquad (5)$$

We have finally

$$\langle F(t) \rangle = - \frac{1}{kT} \int_o^t v(s)\langle F(o)F(t-s)\rangle ds \qquad (6)$$

This is more specifically a statement of the 2nd fluctuation dissipation theorem (Kubo, 1966). The first dissipation theorem is the one more commonly quoted and relevant to Brownian motion. Here one recognises that the time scale of v(s) is much greater than F(s). Hence we may write equation (6) as

$$\langle F \rangle = - \frac{v}{kT} \int_o^\infty \langle F(o)F(t)\rangle \, dt$$

and the friction coefficient ζ is thus

$$\zeta = \frac{1}{kT} \int_o^\infty \langle F(o)F(t)\rangle dt \qquad (7)$$

The 2nd fluctuation dissipation theorem is clearly a more general statement of the relationship between the average systematic force and its fluctuating counterpart for classical thermodynamic systems. Kubo (1966) gave an even more general statement of the theorem when he extended it to quantised systems. But the general features of the proof are the same.

The new density matrix of the system perturbed by an external force is calculated to first order in the unperturbed equilibrium density matrix.

It is true that one can begin with a generalised form of the Langevin equation of motion of the form

$$\frac{dv}{dt} + \int_o^t \gamma(t-s)v(s)ds = \frac{f(t)}{m} \qquad (8)$$

where f(t) is the fluctuating force, and m the mass of the particle, and derive a purely formal relationship between $\langle f(o)f(t)\rangle$ and $\gamma(t)$, i.e.

$$\gamma(t) = \frac{\langle f(o)f(t)\rangle}{\langle v^2\rangle m^2} \qquad (9)$$

But in no way can this be regarded as a causal relationship in the same way that is implied by e.g. equation (7). It is merely a formal property of the equation of motion. It only becomes a causal relationship of meaningful importance when we recognise some extra property of the system i.e.

$$m\langle v^2\rangle = kT \qquad (10)$$

This is the essential flaw of the analysis of Gitterman and Steinberg. They write the equation of motion for a particle in a turbulent fluid as a generalised Langevin equation.

$$\frac{dv}{dt} + \int_o^t \gamma(t-s)v(s)ds = \beta u(t) \qquad (11)$$

Here u(t) is the local velocity of the fluid and β is a constant of the motion whose meaning is somewhat obscured by the fact that it is not the normal Stokes relaxation time. Quite formally in accord with equation (9)

$$\gamma(t) = \beta^2 \frac{\langle u(o)u(t)\rangle}{\langle v^2\rangle} \qquad (12)$$

and

$$\varepsilon(\infty) = \frac{(\langle v^2\rangle)^2}{\beta^2 \int_o \langle u(o)u(t)\rangle dt} \qquad (13)$$

where $\varepsilon(\infty)$ is the long time particle diffusion coefficient (although they do not make this clear, equation (13) is only correct for $\gamma(o)\tau_f^2 \ll 1$ where τ_f is the fluid integral scale). As before equation (13) is interesting but quite useless in predicting the behaviour of ε in relation to that of the fluid since we require some extra knowledge of $\langle v^2\rangle$. As with B.M. Gitterman and Steinberg base $\langle v^2\rangle$ upon the equipartition of energy. Although equipartition of energy is true for B.M. it is inconsistent with the form of the energy spectrum for hydrodynamic turbulence.

It is quite impossible to follow the same route for hydrodynamic turbulence as for B.M. in pursuit of a fluctuation dissipation theorem. We cannot make such simple statements equivalent to

$$\Omega(E) \sim e^{E/kT}$$

and indeed if one were to exist it would relate to the totality of the turbulent field i.e. all spatial and temporal moments of the turbulence velocity field, and be quite unusable in a practical sense.

Let us now consider equation (11). There seems no merit in using this equation as a basis for motion rather than the simpler local equation commonly used i.e.

$$\frac{dv}{dt} = \beta(u-v) \qquad (14)$$

Here

$$\beta = \zeta/m$$

Indeed this equation has a more sound theoretical basis. We are implying here that β is determind by the average motion of the molecules within an eddy. That we can do so implies that the timescale of molecular motion is much shorter than the period over which both u and v may vary. u and v are in fact quasi-steady. The shear stresses are evaluated to first order in the fluid velocity gradients, regarding these quantities as perturbations on the equilibrium phase space density for a flow of constant spatial velocity u. It is true that in general the stresses ought to be non local, but the linear approximation would appear quite adequate for flows normally encountered in nature. To reject this hypothesis would be to question the validity of the Navier Stokes equation for hydrodynamic turbulence. There seems adequate proof that it is a correct representation of the process even though its solution is unknown (see e.g. Leslie, 1973, Batchelor, 1971). The intractibility of the turbulence problem is associated with the non linear nature of the convection, rather than an incorrect representation of the viscous stress. Straightforward application of equation (14) would give equation (1) for the long time particle diffusion coefficient. The fact that it is different from the diffusion coefficient for B.M.

$$\varepsilon(\infty) = \frac{kT}{\zeta} \qquad (15)$$

need not be of any real concern. It is simply that in B.M. ζ is related to the correlation function of the random driving force through the 1st fluctuation-dissipation theorem whereas in T.M. it is not. More transparently in either case

$$\varepsilon = \frac{1}{\zeta^2} \int_o^\infty \langle f(o)f(s)\rangle ds \qquad (16)$$

where

$$\int \langle f(o)f(s)\rangle ds = \zeta^2 \int \langle u(o)u(s)\rangle ds \qquad \text{T.M.} \qquad (17)$$

$$= \zeta kT \qquad \text{B.M.} \qquad (18)$$

Substitution of equations (17) and (18) gives equations (1) and (15) respectively.

DERIVATION OF THE PARTICLE DIFFUSION COEFFICIENT FROM THE PARTIAL PRESSURE OF PARTICLES IN A TURBULENT FLUID

It is instructive to recall the way Einstein in 1905 first derived the well-known expression (15) for the diffusion coefficient of a Brownian particle. Using thermodynamic arguments he was able to show that the pressure p exerted by the particles was identical to form to that of a dilute solution of solute molecules, namely

$$p = kT\rho \qquad (19)$$

where ρ is the dilute number density of the particles suspended in the fluid.

He then considered isothermal equilibrium of these particles under the action of a constant force K in the x-direction. The equilibrium concentration was then given by a balance between the net applied force per unit volume and the pressure gradient, i.e.

$$K\rho - \frac{\partial p}{\partial x} = 0 \qquad (20)$$

or using (19)

$$K\rho - kT\,\frac{\partial \rho}{\partial x} = 0 \qquad (21)$$

Alternatively we could regard this equilibrium as a balance between a constant current provided by K of the form $\frac{K\rho}{\zeta}$ and a diffusion current $-\,\varepsilon\,\frac{\partial \rho}{\partial x}$ where ε is the long time particle diffusion coefficient i.e.

$$\frac{K\rho}{\zeta} - \varepsilon\,\frac{\partial \rho}{\partial x} = 0 \qquad (22)$$

Comparing (21) with (20) we obtain equation (15).

It suggests that we might obtain an expression for the particle diffusion in T.M. by applying similar arguments. We require, however an equivalent expression for the particle partial pressure. To do this we shall use the Clausius virial theorem of thermodynamics that is normally used to obtain the correction terms for the pressure of a non ideal gas due to the intermolecular forces.

Let the particles be suspended in an isotropic and stationary turbulent fluid of volume V, and let their number density be ρ and their individual mass m. We suppose that the equation of motion of a particle is of the form given in (14) with the addition of a force $\underline{X}$. Thus, for one component we have

$$m\ddot{x}_i = X_i - \zeta\dot{x}_i + \zeta\, u_i(t) \qquad (23)$$

where $\underline{x}$ is the position of the particle at time t. Multiply (23) by $\frac{1}{2}\, x_i$ and summing over i. We have

$$\frac{1}{4}\, m\,\frac{d^2}{dt^2}\, x_i x_i + \frac{1}{4}\,\frac{d}{dt}\,\zeta x_i x_i = \frac{1}{2}\, m\dot{x}_i\dot{x}_i + \frac{1}{2}\, X_i x_i + \frac{1}{2}\,\zeta x_i u_i \qquad (24)$$

Summing over all particles within the volume and assuming equilibrium means

$$\frac{3}{2}\, V\rho m\langle v^2\rangle = \frac{1}{2}\,\zeta V\rho\langle x_i(t)u_i(t)\rangle - \frac{1}{2}\sum_{\text{all particles}}\langle x_i X_i\rangle \qquad (25)$$

where $\langle\rangle$ means an ensemble average and $\langle v^2\rangle = \langle v_1{}^2\rangle = \langle v_2{}^2\rangle = \langle v_3{}^2\rangle$. The virial $-\frac{1}{2}\,\Sigma\,\langle x_i X_i\rangle$ is made up of all the forces between the particles (in this case zero) and the stresses across the physical or geometrical boundary. The stress/unit area in this case is the pressure, and we may insist that the pressures on any boundary or across any internal surface are always equal in the absence of surface tension and external fields of force. It is clear therefore that after converting a surface integral to a volume integral (Fowler, 1966)

$$-\frac{1}{2}\,\Sigma\,\langle x_i X_i\rangle = \frac{3}{2}\, p\, V \qquad (26)$$

and that substituting in (25) we have

$$\frac{p}{\rho} = m\langle v^2\rangle + \frac{1}{3}\,\zeta\,\langle x_i(t)u_i(t)\rangle \qquad (27)$$

We may simplify the R.H.S. still further by noting that from the equation of motion

$$\langle v^2\rangle = \beta\int_0^\infty e^{-\beta s}\,\langle u(o)u(s)\rangle\, ds \qquad (28)$$

and

$$\langle X_i(t)u_i(t)\rangle = 3\int_0^\infty (1-e^{-\beta s})\langle u(o)u(s)\rangle ds \quad \text{for } t\to\infty \qquad (29)$$

where $\beta = \frac{\zeta}{m}$, and the indices are redundant. Thus equation (27) reads

$$\frac{p}{\rho} = \zeta\int_0^\infty \langle u(o)u(s)\rangle ds \qquad (30)$$

In this formula we must interpret u(s) as the fluid velocity at time s measured along a particle trajectory.

With the aid of equations (27) and (29) it is interesting to draw the analogy of B.M. and T.M. with an ideal and non ideal gas. For an ideal gas we have

$$\frac{p}{\rho} = kT$$

as with B.M. For a non ideal gas the R.H.S. is modified by the addition of contributions to the virial from intermolecular forces. In T.M. these contributions are from the resistive motion due to the fluid. We note from (29) this contribution vanishes when $\beta^{-1} \gg$ the timescale of the random forcing motion. This is precisely the disparity of particle and molecular timescales for B.M. Thus together with the fluctuation dissipation relationships equation (27) is entirely compatible with B.M. We note however that equation (30) for T.M. is particle interia dependent which equation (19) clearly is not.

Employing the same argument for equilibrium under an applied constant external force as Einstein did for B.M., we find in this instance

$$\varepsilon = \int_0^\infty \langle u(o)u(s)\rangle ds \qquad (31)$$

entirely consistent with equation (1).

THE TRANSPORT EQUATION FOR PARTICLES IN T.M.

What makes B.M. in general different from T.M. is that it is an example of a Markov Process. This stems from the fact that the timescale of particle motion is so much greater than the timescale of molecular motion. We may generally assume therefore that the same will hold for T.M. if that same disparity of timescales exists between particle and fluid motion. We may argue that the average phase space distribution $\overline{W}(\underline{x},\underline{v},t+\Delta t)$ is related to the distribution $\overline{W}(\underline{x}-\Delta\underline{x},\underline{v}-\Delta\underline{v},t)$ in the following manner (Chandrasekhar, 1943)

$$\overline{W}(\underline{x},\underline{v},t+\Delta t) = \int_{\text{all }\Delta\underline{x}} d\Delta\underline{x} \int_{\text{all }\Delta\underline{v}} d\Delta\underline{v}\;\overline{W}(\underline{x}-\Delta\underline{x},\underline{v}-\Delta\underline{v},t)\,\Psi(\underline{x}-\Delta\underline{x},\underline{v}-\Delta\underline{v};\Delta\underline{x},\Delta\underline{v},\Delta t) \qquad (32)$$

where $\Psi(\underline{x}-\Delta\underline{x},\underline{v}-\Delta\underline{v};\Delta\underline{x},\Delta\underline{v}\,\Delta t)$ is the transition probability that a particle with velocity $\underline{v}-\Delta\underline{v}$ and position $\underline{x}-\Delta\underline{x}$ at time t will have velocity $\underline{v}$ and position $\underline{x}$ at time $t+\Delta t$ later. We note it depends upon Δt in time and not upon t. Δt is considered sufficiently short that we can expand to first order in Δt on the L.H.S. and to first order in $\Delta\underline{x}$ and $\Delta\underline{v}$ on the R.H.S. of (32). Furthermore we assume Δx+$\underline{v}$ are perfectly correlated so that

$$\Psi(\underline{x},\underline{v};\Delta\underline{x},\Delta\underline{v},\Delta t) = \Psi(\underline{x},\underline{v};\Delta\underline{v},\Delta t)\;\delta(\Delta\underline{x}-\underline{v}\Delta t) \qquad (33)$$

Because of the Central Limit Theorem Ψ is Gaussian in $\Delta\underline{v}+\beta\underline{v}$ (Chadrasekhar, 1943). By so doing, we eventually obtain the Fokker Planck equation