

DIFFERENTIAL EQUATIONS
& APPLICATIONS TO
BIOLOGY & TO INDUSTRY

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dedicated to the memory of

Stavros Busenberg (1941-1993)

Differential Equations and Applications to Biology and to Industry

Mario Martelli (California State University, Fullerton)

Kenneth Cooke (Pomona College, Claremont)

Ellis Cumberbatch (Claremont Graduate School, Claremont)

Betty Tang (Arizona State University, Tempe)

Horst Thieme (Arizona State University, Tempe)



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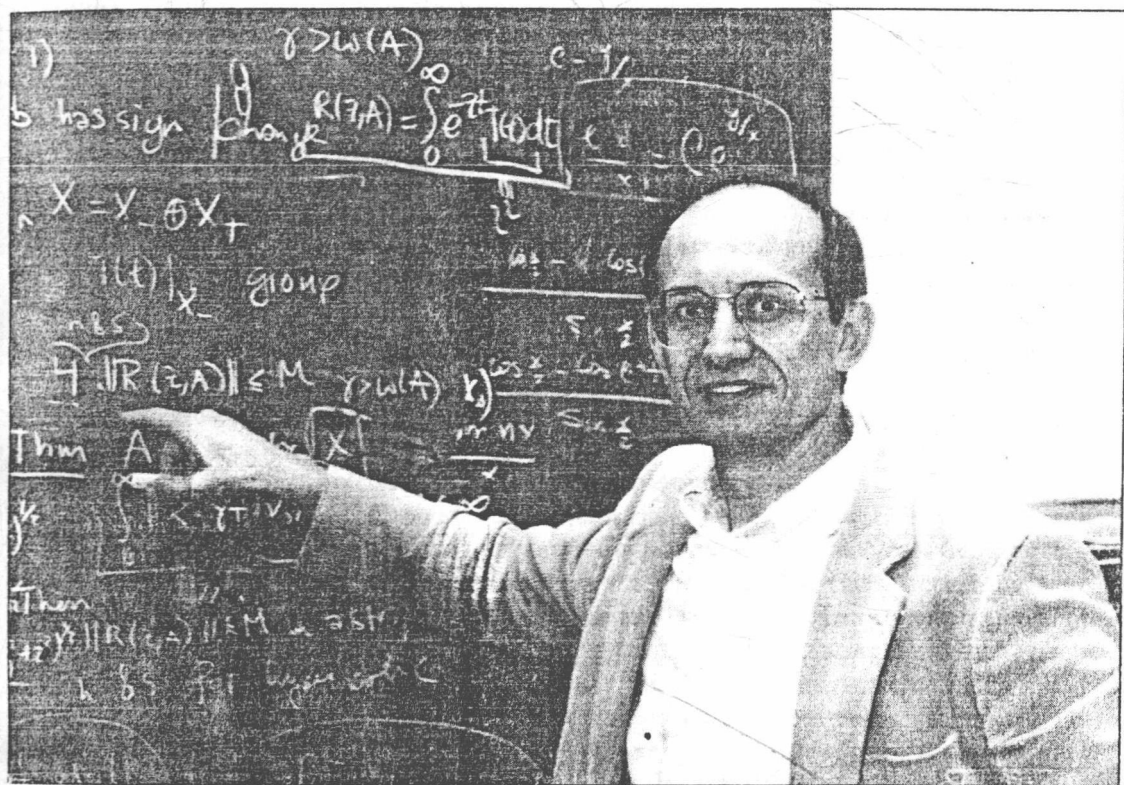
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**Differential Equations
and Applications to
Biology and to Industry**



Stavros Busenberg
1941–1993

PREFACE

On April 3, 1993 the intense and far reaching life of Stavros Busenberg was brought to a premature end by amyotrophic lateral sclerosis (ALS), also known as Lou Gehrig's disease. He had developed the disease approximately ten months before, and after a courageous battle, which challenged the inner strengths of his personality and the exceptional qualities of his mind, Stavros left his friends and colleagues with a legacy which will never be forgotten. His keen sense of friendship, his enthusiasm for scientific discovery, and his gifts as a teacher are there to be admired and remembered, but will be very hard to emulate.

On June 1-4, 1994 an International Conference took place in Claremont, California in memory of Stavros. About 200 scientists from all over the world came to pay their tribute to his accomplishments. Besides the editors of this volume, many other people gave invaluable time and efforts to the success of the meeting. Each one of them deserves our gratitude. Among them special recognition goes to Stavros' wife, Bonnie Busenberg, who coordinated most of the non-programmatic organizational details of the event. A sincere thanks goes to Courtney Coleman who handled most of the financial problems connected with the meeting. Our appreciation to Henry Riggs, President of Harvey Mudd College, for his generous hospitality and to Hank Krieger, Chair of the HMC Mathematics Department, for underwriting the early expenses of the meeting. We are also much indebted to Sue Cook, Barbara Schade and Selina Zerbel for their invaluable secretarial assistance.

The meeting highlighted the three major areas of research in which Stavros had made outstanding contributions: Differential Equations, Mathematical Biology and Industrial Mathematics. All three areas were represented at the meeting by top researchers, who communicated their most recent discoveries. To accommodate the large number of participants and to give everyone the opportunity to present their ideas, there were also two well-attended poster sessions. The names of many participants, speakers, and presenters of posters are listed in this volume. Friends in fields other than science attended the meeting as well. A piano concert was offered by Valeria Profeta Romano, who travelled from Italy with her uncle and aunt, Dr. Ennio and Pina Romano, dear friends of Stavros, to honor his memory. The Conference was financially supported by the National Science Foundation, Rocketdyne, Harvey Mudd College, and other donors who asked to remain anonymous.

This volume contains a selection of papers which were presented at the conference and accepted after peer review. To the many researchers who contributed their time in refereeing the papers goes our deepest appreciation. This book could not have been realized without their help. Each one of the editors handled the articles appropriate to their area of expertise. M. Martelli acted as coordinating editor and put together the entire volume with the invaluable technical support of Elizabeth and Josh Hodas.

The articles are presented in alphabetical order, with the name of the person who presented the paper at the conference being listed as the first author. In the

papers, however, the authors are listed in the order they requested. The large number of contributors, together with cost and space considerations, forced the editors to impose strict page limitations. Therefore many articles are necessarily succinct and do not contain complete proofs. We apologize to the reader for this inconvenience, which we feel is a minor price to pay for the advantage of having in a single volume, and for a reasonable price, the latest developments in at least three different areas of research.

This volume will bring back, at least in spirit, the person we gathered in Claremont to remember as a dear friend, an outstanding mathematician, and a remarkable human being. Each one of us, reading this book, will surely be overwhelmed at times by the memory of better days when Stavros was with us. We should remember, however, that the thirst for knowledge and the drive for discovery are the truest legacy of our friend. A new idea coming from this book, a better method developed after reading these pages will make him smile, and will be the best way of paying him the tribute he deserves.

M. Martelli, K. Cooke, E. Cumberbatch, B. Tang, H. Thieme
July 1995

IN MEMORY OF STAVROS BUSENBERG*

Stavros Busenberg was born on October 16, 1941, in Jerusalem to Greek parents. The family moved to the United States in 1958. Stavros obtained his bachelor's degree in mechanical engineering from the Cooper Union in New York City, and master's degrees in mathematics and in mechanical engineering from the Illinois Institute of Technology. In 1967 he received his Ph.D. in mathematics from the same institution, under the supervision of John DeCicco. After spending a year on a post-doctoral position at the North American Science Center of the Rockwell Corporation, he joined the faculty of Harvey Mudd College in Claremont, California in 1968. He held several visiting positions at prestigious institutions such as Stanford University, Oak Ridge National Laboratory, Caltech, Oxford University, the University of Trento (Italy), the University of Victoria (Canada), Massey University (New Zealand) and the University of Bordeaux II (France).

Stavros met his wife Bonnie Egan in 1966. They were married in 1969 at Sage Chapel of Cornell University where Bonnie was a graduate student in Biology. Their union was blessed by the birth of two children: George, born in 1971 and John born a year later. A strong believer in the Greek and Latin principle of developing mind and body (*"mens sana in corpore sano"*), Stavros played varsity volleyball in college, earned a black belt in judo, hiked, bicycled, played tennis and squash all his adult life. From his multicultural background Stavros learned how to speak English, Greek, French, Italian, and Arabic. He could understand German and Russian, and was an avid reader of novels and poems in their original languages. He loved plants and took pride in the fruit trees and vines that he and Bonnie cultivated in their yard.

The first symptoms of the disease which would finally take his life appeared in June of 1992 while Stavros was visiting the University of Bordeaux. At that time he started limping slightly. Over the next two months the condition worsened slowly. After many tests he was finally diagnosed in October as suffering from amyotrophic lateral sclerosis, a degenerative nerve disease for which there is no known therapy. In November the disease started spreading very fast to other parts of his body, but Stavros never gave up. Despite being eventually confined to a wheelchair and unable to make even simple movements, he continued teaching until the very last week of his life. He also continued doing research with many of his collaborators, including Betty Tang, Carlos Castillo-Chavez, Kenneth Cooke, Mimmo Iannelli, Mario Martelli and Horst Thieme. His battle with the disease ended in defeat in the morning hours of April 3, 1993.

With his departure the mathematical community lost a truly remarkable individual, who distinguished himself in scientific productivity, teaching ability, and professional and organizational skills.

Stavros wrote more than 75 papers, published by the most prestigious interna-

*We wish to thank Bonnie Busenberg for providing some of this biographical information.

tional journals. His research efforts were mainly concentrated on differential equations and their applications to biology and to industrial problems. Stavros was one of the leading experts in the area of vertically transmitted diseases. He included age structure in the mathematical models of these diseases and proved that making the transmission rates age-dependent does not change the global stability of an endemic system with susceptibles and infectives only. He also provided a general way to understand all possible mixing structures in stratified populations. In collaboration with David Fisher, one of his former students at Harvey Mudd, and with Mario Martelli, Stavros obtained a very beautiful result on the period T of periodic orbits of an autonomous system governed by a function which is Lipschitz with constant L in some normed space E . The result states that $TL \geq 6$ and this inequality is optimal in its stated generality. He also wrote or edited five books. The most recent, co-authored with Kenneth Cooke, was on Vertically Transmitted Diseases. It was published by Springer-Verlag in the Biomathematics Series in January 1993, just three months before his premature death.

Stavros liked to have bright undergraduates work with him. One of them, Melissa Aczon, worked with him on a research project until the very last weeks of his life. Stavros guided his students with consummate skills, making sure that they did by themselves all steps they could master. He was a natural teacher, always well organized, clear and challenging. His philosophy was to push his students to their limits, to give them a feeling of accomplishment, and to make them realize that they have achieved remarkable results. He was a co-director and a driving force behind the Harvey Mudd College Mathematics Clinic, a program in which small teams of students study open-ended problems of interest to sponsoring companies. One of the projects he supervised resulted in the procurement of a patent for a high-resolution video camera.

His research efforts and his teaching duties did not stop him from other professional activities. For several years he was associate editor of the Journal of Mathematical Analysis and Applications and a member of the editorial board of the Journal of Mathematical Biology. He was the driving force in the organization of several national and international meetings and special sessions. The list includes, but it is not limited to, the 1977 NSF CBMS conference on Topological Degree Methods in Nonlinear Boundary Value Problems which Stavros organized with Kenneth Cooke and which took place in Claremont, California. Again with Kenneth Cooke, Stavros organized the 1981 International Conference on Differential Equations and Applications to Ecology, Epidemics and Population Problems. This conference also took place in Claremont. The proceedings were collected in a volume edited by Stavros and Kenneth and published by Academic Press. In 1988 Stavros and Mario Martelli organized a special session on Differential and Difference Equations in conjunction with the 846th meeting of the American Mathematical Society in Claremont. In 1990 Stavros and Mario organized the Claremont International Conference on Differential Equations and Applications to Biology and Population Dynamics to honor Kenneth Cooke on his 65th birthday. Two companion volumes of proceedings of the conference were edited by Stavros and Mario and published

by Springer-Verlag, one in the series Lecture Notes in Mathematics and the other in the series Lecture Notes in Biomathematics. On the last day of the conference Stavros acknowledged the praise for the outstanding organization, coming from all participants, and said: "See you again in Claremont in the year 2000!" If his destiny had been different, his friends and colleagues would have gathered in Claremont to celebrate the very best beginning of the new millennium.

The great Italian poet Giuseppe Ungaretti wrote:

*Si sta
come d'autunno
sugli alberi
le foglie*

*We are
like leaves
on trees
in the Fall*

Stavros' leaf was a beautiful one, with delicate colors and striking patterns. He was a creative, generous, and inspiring collaborator, mentor, and teacher. His early departure has been an incredible loss, but in a short time he left an indelible trace in the life of those who knew him. Thank you, our dear friend, for the incredible gift of your presence among us.

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Differential Equations and Applications to Biology and to Industry