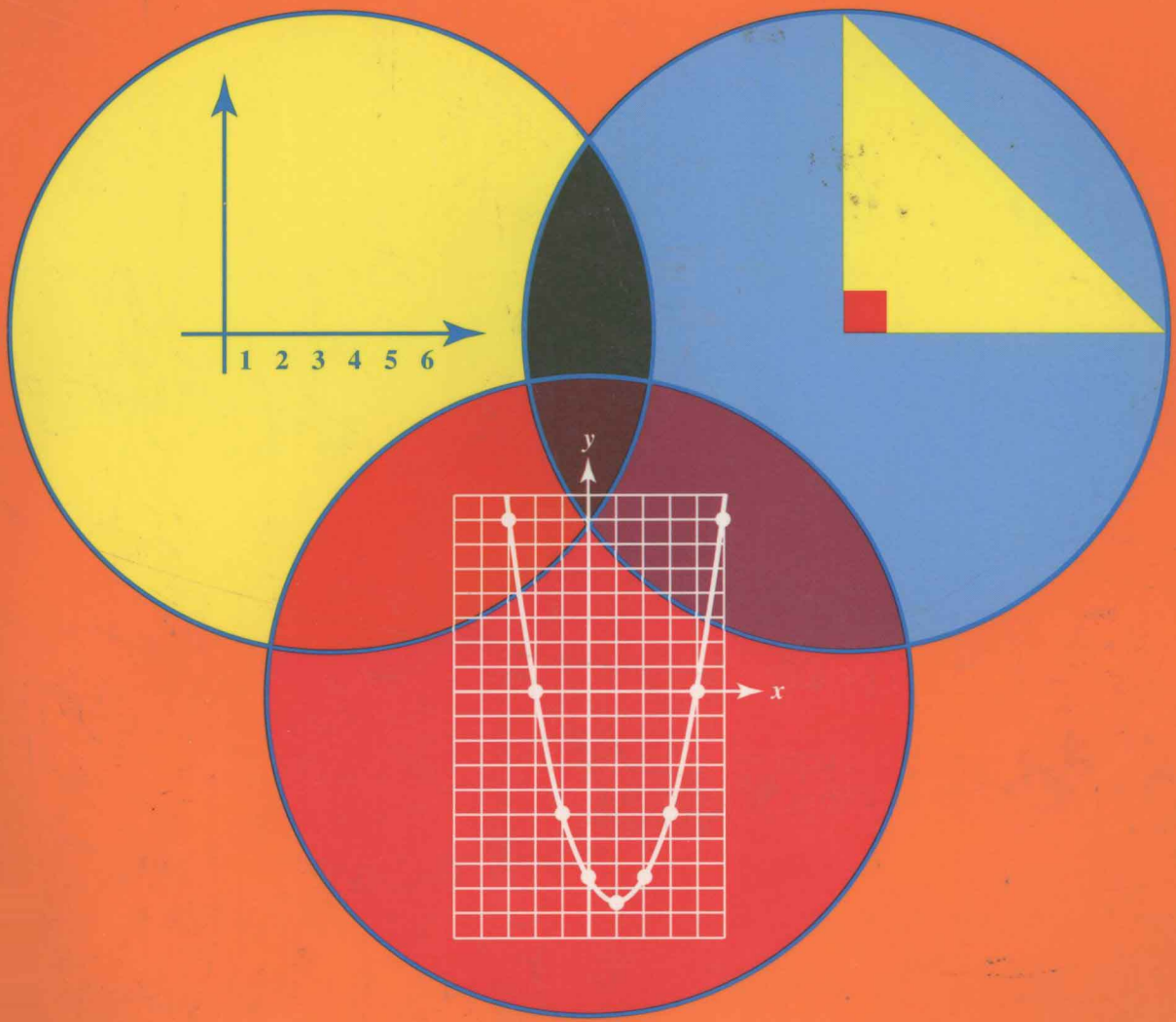
REVISED

ALGEBRA I

ISIDORE DRESSLER



AN AMSCO PUBLICATION

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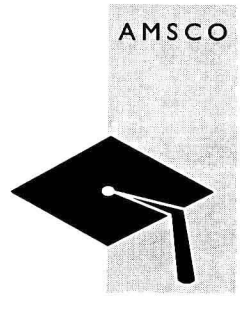

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PREFACE

Today, groups of professional mathematicians, as well as teachers of mathematics, are suggesting that significant changes be made in the elementary algebra course. These changes concern themselves not only with the content of the course, but also with its organization and the methods to be used in teaching it.

Algebra I was written to meet the changing needs of both the student and the teacher of elementary algebra. The book, therefore, is modern in concept and includes topics not previously included in a traditional elementary algebra course: the basic concepts and language of sets, the properties of numbers, absolute value, the function concept, inequalities, a broader role of the coordinate plane, and deductive proof of algebraic theorems.

The major purpose of the book is twofold: (1) to aid the student in understanding the basic concepts of elementary algebra, and (2) to help the student acquire the important manipulative algebraic skills. Although the text presents a modern structural approach, the author has avoided an extremely rigorous treatment of the subject matter. The topics are developed on a level appropriate to the mathematical maturity and sophistication of the average student.

Algebra I, which treats fully the topics included in a modern elementary algebra course, is organized so that it can be used by the teacher in conjunction with the various textbooks now in use. The methods used in presenting the subject matter will help the student to understand and master the material studied. The book makes available to both the student and the teacher an abundance of exercises, model problems, and instructional materials. These may be used as a supplement to the class textbook, and may also be used as a source book for remedial and review work.

An unusual feature of the book is its method of organization. Each chapter contains a series of learning units which, with proper application, the student can master alone. The basic concepts of the unit are carefully developed with the use of simple language and symbolism. New words are clearly defined. Explanations and teaching problems lead to the statement of important general principles and procedures. These principles and procedures are then stated clearly and concisely. Model problems, whose solutions are accompanied by detailed, step-by-step explanations, teach the student how to apply the principles and follow the procedures. The student completes the unit by doing a series of carefully graded exercises. The exercises cover almost every type of difficulty and test the student's understanding of basic concepts as well as the student's mastery of manipulative skills.

—I. D.

CONTENTS

Chapter 1

SYMBOLS, NUMBERS, AND NUMERALS

1. Symbols and Numerals

SYMBOLS FOR OBJECTS, PERSONS, AND IDEAS

A ***symbol*** may be a picture or a written word that is used to represent an object, a person, or an idea. For example,  represents a glass, "George Washington" represents a person, and "democracy" represents an idea.

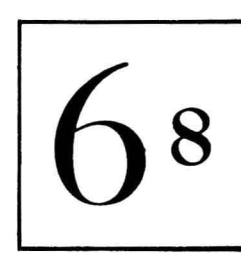
We generally use quotation marks, " ", when we refer to the symbol for an object, a person, or an idea rather than to the thing itself. For example, a bird has a head, whereas "bird" has four letters. However, when there can be no confusion as to whether we are talking about the object itself or about the symbol for the object, we omit the quotation marks. Thus, when George Washington is written on a blackboard, it is obvious that the name of a person, not the person himself, appears there.

It may be possible to represent the same object, person, or idea by several different symbols. For example, "George Washington" and "the father of our country" are different symbols for the same person.

SYMBOLS FOR NUMBERS

In your study of mathematics, you used symbols to represent numbers. In algebra, we will work with the numbers of mathematics and we will use their customary names and symbols. Number names or number symbols are called ***numerals*** or ***numerical expressions.***

Remember that a numeral and the number it represents are not the same thing. A number is an idea that we can talk about and think about; a numeral is the name or symbol for a number. We know, for example, that the number 8 is larger than the number 6. In the figure, however, the numeral "6" is larger than the numeral "8."



SYMBOLS FOR OPERATIONS

The symbol "+" is used to indicate the operation of *addition*. The numeral "6 + 2" represents the result obtained when 2 is added to 6. The result, 8, is called the ***sum***.

The symbol "×" is used to indicate the operation of *multiplication*. The numeral "6 × 2" represents the result obtained when 2 is multiplied by 6. The result, 12, is called the ***product***.

The symbol "−" is used to indicate the operation of *subtraction*. The numeral "6 − 2" represents the result obtained when 2 is subtracted from 6. The result, 4, is called the ***difference***.

The symbol "÷" is used to indicate the operation of *division*. The numeral "6 ÷ 2" represents the result obtained when 6 is divided by 2. The result, 3, is called the ***quotient***. The fraction "$\frac{6}{2}$" also indicates a quotient which is the result of dividing 6 by 2.

Since addition, multiplication, subtraction, and division are operations which are performed on two numbers, each of these operations is called a ***binary operation***.

A number may be represented by many numerals. For example, each of the following numerals may be used as a number name for a collection of eight objects:

"8"	"eight"	"VIII"	"ocho"
"6 + 2"	"18 − 10"	"4 × 2"	"6400 ÷ 800"

Note that "8" is probably the simplest looking of all the preceding numerals, each of which represents the number eight.

Exercises

1. Select three different objects and give a symbol for each object.
2. Select three different persons and give a symbol for each person.
3. Select three different ideas and give a symbol for each idea.

In 4–19, each sentence contains an underlined symbol. State whether the sentence refers to the object, person, or idea, or whether the sentence refers only to the symbol itself.

4. A house has a roof.
5. House ends with an *e*.
6. Buffalo is a word which has two *f*'s.
7. A buffalo is an animal.

8. Pen is used to represent two different things.
9. He wrote with the pen.
10. Franklin D. Roosevelt was a President of the United States.
11. Franklin D. Roosevelt appears in many books.
12. Ruth is a short girl.
13. Ruth is a short name.
14. Mr. Crane selected Tom as his companion.
15. Mr. Crane selected Tom as his son's name.
16. Honesty is the best policy.
17. He wrote honesty in his notebook.
18. Robert spelled beauty incorrectly.
19. She walks in beauty like the night.

20. Give two different symbols for the same (*a*) object (*b*) person (*c*) idea.

In 21–24, tell which number every numeral in each exercise represents.

21.	$3 + 1$	$6 - 2$	4×1	$12 \div 3$	IV
22.	$12 - 3$	$36 \div 4$	$8\frac{1}{2} + \frac{1}{2}$	$36 \times \frac{1}{4}$	IX
23.	$5 - 5$	7×0	$\frac{1}{2} - \frac{1}{2}$	$.7 - .7$	0×1
24.	.5	$\frac{1}{4} + \frac{1}{4}$	$1\frac{1}{2} - 1$	$\frac{25}{50}$	50%

25. Write five different numerals for each of the following numbers:
a. 6 *b.* 20 *c.* 100 *d.* 1 *e.* 0 *f.* $\frac{1}{4}$

In 26–33, state whether the two numerals represent the same number.

26. $5 + 3$, $16 \div 2$ **27.** 8×1, 12×1 **28.** 8×0, 4×0 **29.** $\frac{1}{3} + \frac{1}{3}$, $\frac{1}{2} + \frac{1}{4}$
30. $6 + 4$, $4 + 6$ **31.** 5×7, 7×5 **32.** $8 \div 2$, $2 \div 8$ **33.** 7×0, $7 - 0$

In 34–37, four of the five numerals in each exercise represent the same number. Select the numeral which represents a number different from this number.

34.	$8 + 4$	$36 - 24$	12×0	$24 \div 2$	$6 \div \frac{1}{2}$
35.	8×1	$8 - 0$	$1.5 + 6.5$	$80 \div 100$	800%
36.	$5 \times \frac{1}{5}$	$\frac{1}{2} + \frac{1}{2}$	$3\frac{2}{3} - 2\frac{2}{3}$	$1.25 \div 1.25$	1%
37.	$2 \times .15$	$\frac{1}{10} + \frac{1}{5}$	$.03 \times 10$	$\frac{4}{15} - \frac{1}{5}$	30%

In 38–53, write a simpler looking numeral for each given numeral.

38. $\frac{3}{5} + \frac{7}{5}$	**39.** $75 + 1.25$	**40.** $\frac{3}{4} + \frac{5}{8}$	**41.** $35 + 1.05$
42. $\frac{12}{7} - \frac{5}{7}$	**43.** $4.65 - 2.25$	**44.** $\frac{7}{8} - \frac{2}{8}$	**45.** $\frac{15}{3} - \frac{1}{2}$
46. $12 \times \frac{1}{3}$	**47.** $\frac{21}{2} \times \frac{1}{7}$	**48.** $\frac{1}{4} \times 4$	**49.** $2.5 \times .64$
50. $6 - .35$	**51.** 1.25×4	**52.** $6.28 \div 4$	**53.** $60 \div 1.25$

2. The Numbers of Arithmetic

Let us recall the numbers with which you became familiar when you studied arithmetic. We shall refer to these numbers as the ***numbers of arithmetic.***

COUNTING NUMBERS OR NATURAL NUMBERS

In your study of arithmetic, you learned how to count. You know that when we count, we start with a first number which is named "one," or "1." The number which follows 1 is named "two," or "2." The number 2 is called the ***successor*** of the number 1. The number 2 in turn has a successor which is named "three," or "3." Each counting number has a successor which is 1 more than the number. Because of this, the process of counting is endless; there is no last counting number.

The ***counting numbers,*** which are also called ***natural numbers,*** are represented by the symbols:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \ldots$$

The three dots after the 12 indicate that the numbers continue in the same pattern without end.

The first ten natural numbers may be represented by the symbols:

$$1, 2, 3, 4, \ldots, 10$$

The three dots after the 4 indicate that the numbers continue in the same pattern until 10 is reached.

WHOLE NUMBERS

The number zero is not one of the counting numbers. Therefore, 0 is not a natural number. The number zero, together with all the natural numbers, forms a new collection of numbers called ***whole numbers.*** The whole numbers are represented by the symbols:

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \ldots$$

FRACTIONS

In arithmetic you learned several meanings of a ***fraction.*** We shall say that *a fraction is a symbol which indicates the quotient of two numbers.* You are familiar with fractions like $\frac{1}{2}$, $\frac{3}{4}$, $\frac{12}{5}$, $\frac{8}{8}$, and $\frac{20}{10}$.

There are many different fractions which name the same number. For example, each of the different symbols $\frac{6}{2}$, $\frac{9}{3}$, $\frac{12}{4}$, and $\frac{36}{12}$ is a fraction which names the number 3.

Mixed numbers are numbers that are named by symbols such as $1\frac{1}{4}$, $4\frac{2}{3}$, $5\frac{3}{8}$. Note that a mixed number is the sum of a whole number and a proper fraction. A mixed number may be named by many different fractions. For example, the number named by $1\frac{1}{4}$ can be represented by the fractions $\frac{5}{4}$, $\frac{10}{8}$, $\frac{15}{12}$, etc.

Decimal fractions are numbers which are named by symbols such as .4, .23, .035, 2.5. They can also be represented by quotients. For example, the number named by .4 can be represented by $\frac{4}{10}$, $\frac{2}{5}$, $\frac{8}{20}$, etc.

RATIONAL NUMBERS

If a number can be represented by a fraction which indicates the quotient of a whole number divided by a natural number, the number is called a ***rational number.***

For example, $\frac{1}{2}$, $\frac{2}{3}$, $1\frac{1}{4}$, .4, 3, and 0 are rational numbers.

The whole numbers are included among the rational numbers because any whole number can be represented as a fraction which indicates the quotient of that number and 1. For example:

$$0 = \frac{0}{1} \qquad 1 = \frac{1}{1} \qquad 2 = \frac{2}{1} \qquad 3 = \frac{3}{1} \qquad 5 = \frac{5}{1} \qquad 10 = \frac{10}{1}$$

Therefore, we see that all the numbers of arithmetic which we have discussed are rational numbers.

For reasons that we shall learn later, we may not divide a number by zero. For example, $\frac{5}{0}$ has no value because it does not name any number. We say that division by zero is meaningless.

We shall learn later that not all fractions represent rational numbers. We shall also learn to work with additional rational numbers which are not among the numbers of arithmetic we have studied.

Exercises

1. Name the first counting number.
2. Name the successor of each of the following natural numbers:
   a. 75 *b.* 120 *c.* 999 *d.* 514,621 *e.* 64,499,999
3. State a rule for finding the successor of a given natural number.
4. Name a number which is a whole number but is not a natural number.
5. Write four fractions that are different names for each of the following numbers:
   a. 4 *b.* 9 *c.* 12 *d.* 50 *e.* 0
6. Write four fractions that are different names for each of the following fractions:
   a. $\frac{1}{2}$ *b.* $\frac{3}{4}$ *c.* $\frac{4}{6}$ *d.* $\frac{8}{5}$ *e.* $\frac{7}{1}$