ST(P) Technology Today Series
A Series for Technicians

Mathematics for Technicians

Scotec Edition

Level II Mechanical Engineering Mathematics

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AUTHORS' NOTE ON THE SERIES

Arising from the recommendations of the Hudson Report, with particular reference to the Haslegrove Report, the Scottish Technical Education Council was set up in 1973. The Council have established six Sector Committees, each responsible for a group of courses.

A major change of emphasis in the educational approach adopted in each SCOTEC course has been introduced by the use of 'objectives' in the syllabus which allows student and lecturer to achieve planned progress through the course on a step-by-step basis.

This set of books starts with *An Introductory Course* which provides the mathematics required by level I Technicians and continues at level II with *Mechanical Engineering Mathematics* (this volume) together with the sister book *Electrical Engineering Mathematics*. Each book follows a standard pattern, and each chapter opens with the words 'On reaching the end of this chapter you should be able to:-' and this statement is followed by the objectives for that particular topic as laid down in the syllabus. Thereafter each chapter contains explanatory text, worked examples, and copious supplies of further exercises. As planned at present the series comprises:-

AN INTRODUCTORY COURSE	Level I
MECHANICAL ENGINEERING MATHEMATICS	Level II
ELECTRICAL ENGINEERING MATHEMATICS	Level II
MATHEMATICS FOR ENGINEERING TECHNICIANS	Level III

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CONTENTS

1. SIMULTANEOUS LINEAR EQUATIONS

After reaching the end of this chapter you should be able to:-
Solve a pair of simultaneous equations by substitution and by elimination.

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

Consider the equations:

$$3x+2y = 7$$

$$4x+y = 6$$

The unknown quantities x and y appear in both equations. To solve the equations we have to find values of x and y so that *both* equations are satisfied. Such equations are called *simultaneous equations.*

Three methods are available for solving simultaneous equations.

1. Substitution Method

EXAMPLE 1

Solve the equations:

$$2x+y = 10 \qquad (1)$$

$$3x+2y = 17 \qquad (2)$$

We can write equation (1) above as:

$$y = 10-2x \qquad (3)$$

and, substituting this value of y into equation (2), we have:

$$3x+2(10-2x) = 17$$

and we now have an equation with x the only unknown.

$$\therefore \qquad 3x+20-4x = 17$$

$$\therefore \qquad x = 3$$

Substituting this value for x in equation (3),

$$y = 10-2(3)$$

$$\therefore \quad y = 4$$

The solutions are therefore

$$x = 3 \quad \text{and} \quad y = 4$$

The solutions should always be checked by substituting the values found into each of the original equations:

Equation (1) has:

$$\text{L.H.S.} = 2(3)+4 = 10 = \text{R.H.S.}$$

and equation (2) has:

$$\text{L.H.S.} = 3(3)+2(4) = 17 = \text{R.H.S.}$$

EXAMPLE 2

Solve the equations:

$$2x+3y = 16 \qquad (1)$$

$$3x+2y = 14 \qquad (2)$$

From equation (1):

$$3y = 16-2x$$

$$\therefore \quad y = \frac{16-2x}{3} \qquad (3)$$

Substituting this value in equation (2),

$$3x+\frac{2(16-2x)}{3} = 14$$

and, multiplying through by 3, we get:

$$9x+2(16-2x) = 42$$

from which $\quad x = 2$

Substituting this value for x in equation (3), we have:

$$y = \frac{16-2(2)}{3} = \frac{16-4}{3}$$

$$\therefore \quad y = 4$$

The solutions are therefore:

$$x = 2 \quad \text{and} \quad y = 4$$

Checking these values by substituting into the original equations we have:

Equation (1) has:

$$\text{L.H.S.} = 2(2)+3(4) = 16 = \text{R.H.S.}$$

Equation (2) has:

$$\text{L.H.S.} = 3(2)+2(4) = 14 = \text{R.H.S.}$$

2. Elimination Method

This method is most generally used in solving equations which contain the first power only of the unknown quantities.

EXAMPLE 3

Solve the equations:

$$3x+4y = 11 \qquad (1)$$

$$x+7y = 15 \qquad (2)$$

If we multiply equation (2) by 3 we shall have the same coefficient of x in each of the equations:

$$3x+21y = 45 \qquad (3)$$

We can now eliminate x by subtracting equation (1) from equation (3).

$$(3x+21y)-(3x+4y) = 45-11$$

$$\therefore \quad 17y = 34$$

$$\therefore \quad y = 2$$

To find x we may substitute in either of original equations.

Substituting in equation (1):

$$3x+4(2) = 11$$

$$\therefore \quad x = 1$$

Therefore the solutions are:

$$x = 1 \quad \text{and} \quad y = 2$$

To check these values substitute them in equation (2). (There would be no point in substituting them in equation (1) for this was used in finding x from the y value.) Substituting in equation (2),

$$\text{L.H.S.} = 1+7(2) = 15 = \text{R.H.S.}$$

EXAMPLE 4

Solve the equations:

$$5x+3y = 29 \qquad (1)$$

$$4x+7y = 37 \qquad (2)$$

The same coefficient of x can be obtained if equation (1) is multiplied by 4 and equation (2) by 5. As before, we may then subtract and x will disappear.

Multiplying equation (1) by 4,

$$20x+12y = 116 \qquad (3)$$

Multiplying equation (2) by 5,

$$20x+35y = 185 \qquad (4)$$

Subtracting equation (3) from equation (4),

$$(35-12)y = 185-116$$

$$\therefore \quad y = 3$$

Substituting in equation (1),

$$5x+3(3) = 29$$

$$\therefore \quad x = 4$$

Therefore the solutions are:

$$x = 4 \quad \text{and} \quad y = 3$$

A check on these values is made by substituting them into equation (2):

$$\text{L.H.S.} = 4(4)+7(3) = 37 = \text{R.H.S.}$$

Frequently, in practice, the coefficients of the unknowns are not whole numbers. The same methods apply but care must be taken with the arithmetic.

EXAMPLE 5

Solve the equations:

$$3.175x+0.238y = 6.966 \qquad (1)$$

$$2.873x+4.192y = 11.804 \qquad (2)$$

To eliminate, say, x we must arrange for x to have the same coefficient in both equations. To achieve this we multiply equation (1) by the coefficient of x in equation (2) and then equation (2) by the coefficient of x in equation (1).

Multiplying equation (1) by 2.873,

$$9.122x+0.683\,8y = 20.02 \tag{3}$$

Multiplying equation (2) by 3.175,

$$9.122x+13.31y = 37.48 \tag{4}$$

Subtracting equation (3) from equation (4),

$$12.63y = 17.46$$

$$\therefore \quad y = 1.383$$

Substituting this value in equation (1),

$$3.175x+0.238(1.383) = 6.966$$

$$\therefore \quad x = \frac{6.966-0.329\,7}{3.175}$$

$$\therefore \quad x = 2.089$$

Therefore the solutions are:

$$x = 2.089 \quad \text{and} \quad y = 1.383$$

A check on these values may be made by substituting them into equation (2):

$$\text{L.H.S.} = 2.873(2.089)+4.192(1.383) = 11.804 = \text{R.H.S.}$$

EXAMPLE 6

Solve the equations:

$$\frac{2x}{3}-\frac{y}{4} = \frac{7}{12} \tag{1}$$

$$\frac{3x}{4}-\frac{2y}{5} = \frac{3}{10} \tag{2}$$

In this example it is best to clear each equation of fractions before attempting to solve simultaneously.

Multiplying equation (1) by 12,

$$8x-3y = 7 \tag{3}$$

Multiplying equation (2) by 20,

$$15x-8y = 6 \tag{4}$$

We can now proceed in the usual way.

Multiplying equation (4) by 8,

$$120x-64y = 48 \tag{5}$$

Multiplying equation (3) by 15,

$$120x - 45y = 105 \qquad (6)$$

Subtracting equation (5) from equation (6),

$$-45y - (-64)y = 105 - 48$$

$$\therefore \qquad y = 3$$

Substituting in equation (3),

$$8x - 3(3) = 7$$

$$\therefore \qquad x = 2$$

Hence solution is:

$$x = 2 \quad \text{and} \quad y = 3$$

A check on these values will necessitate substitution into both equations (1) and (2) since both were modified before any elimination took place:

Equation (1) has:

$$\text{L.H.S.} = \frac{2(2)}{3} - \frac{3}{4} = \frac{7}{12} = \text{R.H.S.}$$

Equation (2) has:

$$\text{L.H.S.} = \frac{3(2)}{4} - \frac{2(3)}{5} = \frac{3}{10} = \text{R.H.S.}$$

3. Graphical Method

Simultaneous linear equations may be solved by plotting the graphs of the two equations and finding where they intersect. This method is explained fully in Chapter 2.

PROBLEMS INVOLVING SIMULTANEOUS EQUATIONS

In problems which involve two unknowns it is necessary to form two separate equations from the given data and then to solve these as shown above.

EXAMPLE 7

In a certain lifting machine it is found that the effort (E) and the load (W) which is being raised are connected by the equation $E = aW + b$. An effort of 3.7 units raises a load of 10 units whilst an effort of 7.2 units raises a load of 20 units. Find the values of the constants a and b and hence find the effort needed to lift a load of 12 units.

Substituting $E = 3.7$ and $W = 10$ into the given equation, we have

$$3.7 = 10a+b \tag{1}$$

Substituting $E = 7.2$ and $W = 20$ into the given equation, we have

$$7.2 = 20a+b \tag{2}$$

Subtracting equation (1) from equation (2),

$$3.5 = 10a$$

$$a = 0.35$$

Substituting for a in equation (1),

$$3.7 = 10\times 0.35+b$$

$$\therefore \quad 3.7 = 3.5+b$$

$$\therefore \quad 3.7-3.5 = b$$

$$\therefore \quad b = 0.2$$

The given equation therefore becomes:

$$E = 0.35W+0.2$$

When $W = 12$,

then $E = 0.35\times 12+0.2 = 4.2+0.2 = 4.4$ units

Hence an effort of 4.4 units is needed to raise a load of 12 units.

EXAMPLE 8

The currents I_1 and I_2 in a certain circuit are connected by the following equations:

$$0.4I_1-0.3I_2 = 3 \tag{1}$$

$$1.1I_1-0.2I_2 = 5 \tag{2}$$

Find I_1 and I_2.

Multiplying equation (1) by 1.1, we get

$$0.44I_1-0.33I_2 = 3.3 \tag{3}$$

Multiplying equation (2) by 0.4, we get

$$0.44I_1-0.08I_2 = 2.0 \tag{4}$$

Subtracting equation (4) from equation (3),

$$-0.25I_2 = 1.3$$

$$\therefore \quad I_2 = \frac{1.3}{-0.25}$$

$$\therefore \quad I_2 = -5.2$$

Substituting for I_2 in equation (1), we get

$$0.4I_1 - 0.3\times(-5.2) = 3$$

$$\therefore \quad 0.4I_1 + 1.56 = 3$$

$$0.4I_1 = 3 - 1.56$$

$$\therefore \quad 0.4I_1 = 1.44$$

$$\therefore \quad I_1 = \frac{1.44}{0.4}$$

$$\therefore \quad I_1 = 3.6$$

EXAMPLE 9

Two equations connecting resistances R_1 and R_2 in an electric circuit are:

$$\frac{3}{R_1} + \frac{4}{R_2} = 1.6$$

$$\frac{5}{R_1} + \frac{8}{R_2} = 3.0$$

Find the values of R_1 and R_2.

Let $x = \dfrac{1}{R_1}$ and $y = \dfrac{1}{R_2}$, then

$$3x + 4y = 1.6 \qquad (1)$$

$$5x + 8y = 3.0 \qquad (2)$$

Multiplying (1) by 2 $\qquad 6x + 8y = 3.2 \qquad (3)$

Subtracting (2) from (3), $\qquad x = 0.2$

Substituting for x in (2), $\qquad 5\times 0.2 + 8y = 3.0$

$$y = 0.25$$

$$\therefore \quad R_1 = \frac{1}{0.2} = 5$$

and, $\qquad R_2 = \dfrac{1}{0.25} = 4$

EXAMPLE 10

A heating installation for one house consists of 5 radiators and 4 convector heaters and the cost of the installation is £270. In a second house 6 radiators and 7 convector heaters are used and the cost of this installation is £402. In each house the installation costs are £50. Find the cost of a radiator and the cost of a convector heater.

For the first house the cost of the hardware is:

$$£270-£50 = £220$$

For the second house the cost of the hardware is:

$$£402-£50 = £352$$

Let £x be the cost of a radiator and £y the cost of a convector heater.

For the first house, $5x+4y = 220$ (1)

For the second house, $6x+7y = 352$ (2)

Multiplying (1) by 6: $30x+24y = 1320$, and (3)

Multiplying (2) by 5: $30x+35y = 1760$ (4)

Subtracting equation (3) from equation (4) then

$$11y = 440$$

$$\therefore \quad y = 40$$

and substituting for $y = 40$ in equation (1) then

$$5x+4(40) = 220$$

$$\therefore \quad 5x = 60$$

$$\therefore \quad x = 12$$

Therefore the cost of a radiator is £12 and the cost of a convector heater is £40.

Exercise 1

Solve the following simultaneous equations:

1) $3x+2y = 14$
$2x+5y = 24$

2) $7x-3y = -2$
$8x-2y = 2$

3) $\frac{x}{4}+\frac{y}{5} = \frac{3}{2}$
$2x+3y = 19$

4) $\frac{x}{2}+\frac{y}{3} = \frac{13}{6}$
$\frac{2x}{7}-\frac{y}{4} = \frac{5}{14}$