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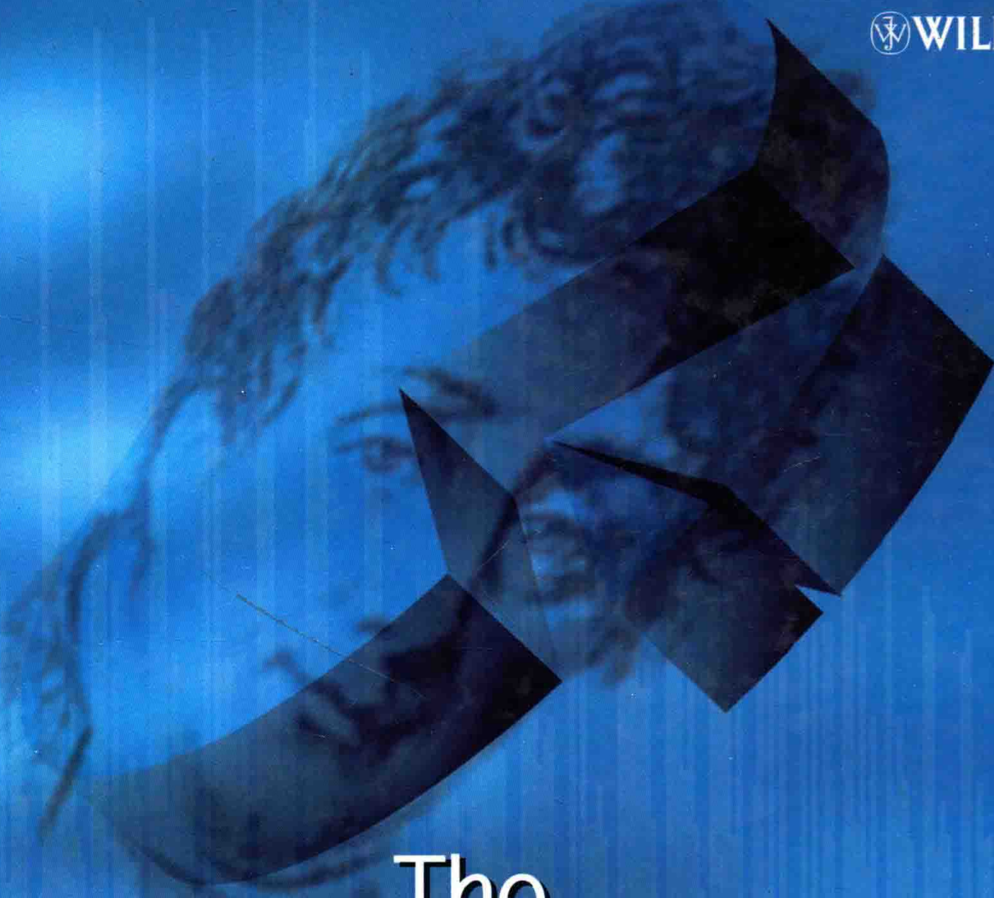

The Fractional Fourier Transform

with Applications in Optics and Signal Processing

Haldun M. Ozaktas | Zeev Zalevsky | M. Alper Kutay

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Haldun M. Ozaktas
Bilkent University, Ankara, Turkey

Zeev Zalevsky
Tel Aviv University, Tel Aviv, Israel

M. Alper Kutay
TÜBİTAK-UEKAE, Ankara, Turkey

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To our families
Past, present, and future

Preface

The fractional Fourier transform has received considerable interest since the early nineties, finding itself a place in standard texts and handbooks such as Bracewell 1999 and *The Transforms and Applications Handbook* 2000. Our primary purpose in writing this book has been to provide a widely accessible account of the transform covering both theory and applications.

Little need be said of the importance and ubiquity of the ordinary Fourier transform and frequency-domain concepts and techniques in many diverse areas of science and engineering. As a generalization of the ordinary Fourier transform, the fractional Fourier transform is only richer in theory and more flexible in applications—but not more costly in implementation. Therefore the transform is likely to have something to offer in every area in which Fourier transforms and related concepts are used. So far applications of the transform have been studied mostly in the areas of optics and wave propagation, and signal analysis and processing. These applications are discussed extensively in this book. However, we expect the transform to find applications in many other areas, and hope that this book will contribute to this end.

This text should primarily be of interest to graduate students, academics, and researchers in branches of mathematics, science, and engineering where Fourier transforms and related concepts are used. A partial list of these areas is operator theory, harmonic analysis and integral transforms, linear algebra, group representation theory, phase-space methods, time- and space-frequency representations, transform theory and techniques, signal analysis and processing, wave propagation, and many areas of optics. We have made an effort to make the book accessible to such a cross-disciplinary audience, entailing a number of compromises. The emphasis is mostly on elucidating the basic concepts from different perspectives and showing as many of the relationships between them as possible. Although most arguments and results are analytical in nature, we did not hesitate to employ suggestive physical arguments where we felt this was appropriate. Mathematical rigor is delegated to the references, as are most experimental and practical considerations. Discussion of optics has been strictly segregated so that readers with no interest in optics can simply ignore chapters 7, 8, and 9.

The fractional Fourier transform is intimately related to several indispensable concepts appearing in diverse areas. We have tried to present the transform in a broad context, showing its relationship to as many of these different concepts as possible. This has required the inclusion of a considerable amount of background and review material to ensure that the book is reasonably self-contained. Nevertheless, we have assumed the reader has at least elementary undergraduate-level exposure to signals and systems and linear algebra. A similar exposure to optics is assumed for those wishing to study the chapters on optics. For instance, we define the ordinary Fourier transform and list its properties, but we do not attempt to develop the insight and intuition that would constitute the focus of an elementary text. Specific suggestions for background reading are provided in the introductory chapter, as well as at the end of certain chapters.

The background material contained in chapters 2 and 7, and especially chapters 3 and 8, is an important feature of the book which we hope readers not very familiar with these topics will find useful in their own right. In these chapters, we occasionally go beyond providing background and preliminary material, to present a self-complete exposition of certain topics which have been neglected in other texts. As such, these chapters may be useful as primary or supplementary material for a variety of different courses. Substantial parts of this book have been used as the primary material for courses on time-frequency analysis, advanced signal processing, and as the theoretical core material of a course on optical information processing at Bilkent University. The material may be especially useful for courses in advanced Fourier optics or information optics emphasizing phase-space concepts and the Wigner distribution, or as supplementary material for introductory courses in these areas. The book can also form the basis of a specialized course on the fractional Fourier transform or the fractional Fourier transform and time-frequency representations, and their applications in optics and/or signal processing. However, depending on the emphasis of the course, and especially if optics is excluded, it may be useful to use the book together with one of the excellent tutorials or books on time-frequency representations (see the end of chapter 3).

A detailed overview of the book, including the relationships of the chapters to each other and suggestions for using the book for self-study, is presented in section 1.3.

Acknowledgments

One summer day in 1992, Adolf W. Lohmann, David Mendlovic, and Haldun M. Ozaktas were having a meeting at the Physics Institute of the University of Erlangen-Nürnberg. Haldun M. Ozaktas had been pondering the fact that in an optical system with several lenses employing a point source for illumination, one observes the Fourier transform of the object at the images of the point source (section 8.4.3). In such a system one typically observes first a Fourier transform, then an inverted image, then an inverted Fourier transform, then an erect image, then another Fourier transform, and so on. Inspired by other fractional operations he was familiar with, this led him to propose that the distributions of light at intermediate planes may be interpreted as "fractional Fourier transforms" of continually increasing orders, only if one could figure out how to define such a transform. David Mendlovic noted the irregular and nonuniform nature of such optical systems, and suggested that it might be more illuminating to consider propagation in quadratic graded-index media. Since a certain length d_0 of such a medium was known to produce an ordinary Fourier transform, and since the medium is uniform in the direction of propagation, he suggested that a length ad_0 should produce a distribution of light which can be interpreted as the ath order "fractional Fourier transform." This led us to define the transform as an operator power, essentially according to definition B in chapter 4. It was known that rays propagating in quadratic graded-index media exhibit circular trajectories when viewed in geometrical-optical phase space. Grasping the implications of this from a wave-optical perspective, Adolf W. Lohmann suggested that since ordinary Fourier transformation corresponds to $\pi/2$ rotation of the Wigner distribution, then rotation of the Wigner distribution by $a\pi/2$ should correspond to ath order fractional Fourier transformation. Although the initial motivation came from optics, mathematical and signal processing aspects of the transform were soon to intrigue us. Our journey exploring the transform and its applications, soon joined by M. Alper Kutay and Zeev Zalevsky, was a memorable one. Here we would like to express our most heartfelt and warm appreciation of having had the privilege of interacting and collaborating with David Mendlovic and Adolf W. Lohmann. The idea of writing a book on the fractional Fourier transform was originally conceived together with David Mendlovic; unfortunately, his obligations prevented him from actively participating in the preparation of this book.

We have had the benefit of collaborating and interacting with many students and colleagues throughout the years. We would like to thank the following both for their contributions to the area and for many fruitful discussions: Özer K. Akdemir, Orhan Arıkan, Orhan Aytür, Laurence Barker, Billur Barshan, A. Ümit Batur, Yigal Bitran, Gözde Bozdağı, Çağatay Candan, John H. Caulfield, Rainer G. Dorsch, Nilgün Erkaya, Carlos Ferreira, Javier García, Ruşen Güldoğan, Özgür Güleryüz, Tuğrul Hakioğlu, Levent Onural, Hakan Özaktaş, Ayşegül Şahin, Hakan Ürey, K. Bernardo Wolf, and İ. Şamil Yetik. M. Fatih Erden deserves to be singled out for his many contributions to the area and the many inspiring discussions we have had.

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