combinatorial mathematics, optimal designs and their applications

Editor:
J. Srivastava

ANNALS OF DISCRETE MATHEMATICS 6

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COMBINATORIAL MATHEMATICS, OPTIMAL DESIGNS AND THEIR APPLICATIONS

Edited by
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PREFACE

A Symposium on Combinatorial Mathematics and Optimal Design was held at Colorado State University (CSU), Fort Collins, Colorado, on June 5–9, 1978. The symposium was international in scope. Both the speakers and the audience ranged from all over the world.

The present volume contains the contributions of the invited speakers. The papers are both of survey and research types.

This symposium was actually a "State of the Art" conference, and was similar to the one held here in September, 1971. The purpose was to help disseminate knowledge and stimulate research by bringing together top ranking workers from diverse areas of the above fields. These include Foundations, Enumerative Techniques, various branches of Graph Theory, Coding Theory, Combinatorial Problems of Designs, Optimal Design Theory, Finite Geometries, Number Theory, Combinatorial games, Computer problems, etc.

The conference was jointly sponsored by the U.S. Air Force Office of Scientific Research, and the Office of Naval Research. Dr. I.N. Shimi, of the Air Force, particularly helped in the same. On behalf of the Organizing Committee, the participants, and the scientific community, I wish to express my deep appreciation and gratitude to them.

The Organizing Committee of the Symposium consisted of Professors R.C. Bose (Colorado State University), Paul Erdös (Hungarian Academy of Sciences), Frank Harary (University of Michigan), G.C. Rota (Massachusetts Institute of Technology), Esther Seiden (Michigan State University), W.T. Tutte (University of Waterloo), and myself. The presence of these people on the organizing committee helped a great deal towards the success of the conference. I am deeply grateful to each and every one of them for being on the committee, and for the tremendous cooperation that I always received from them.

Professors Erdös, Harary, Rota and Tutte were particularly helpful in developing the program of the Conference. This time money was not available for payment for overseas travel. However, these people, particularly Professor Erdös, helped find many outstanding people from abroad who were planning to visit the United States on their own. I am thankful to them for their help in this regard. Several other distinguished foreign scientists were invited by me with the request that they arrange for their overseas travel. I am happy that almost all of them had success. Most sincere thanks go to these foreign governments and organizations for their cooperation.

As is well known now, I have been passing through severe personal problems for the past 15 years, which finally culminated in the events during the last two years. In this connection, a very difficult sequence of situations began just six weeks before the Conference. During these times, the university authorities (including Dr. Williams, Chairman of the Statistics Department, Dean Cook of the College of Natural Sciences, Vice-Presidents Olson and Neidt, and President Chamberlain) extended their understanding and support. Professor Bose informed the other Organizing Committee members about me and they joined in. I am extremely thankful to all of these people; without their encouragement the Conference would not have been held.

Thanks also go to many local people for their help. Among these, particular mention must be made of (i) the secretaries Waydene Casey and Joanne Moynihan, (ii) my then student W. Ariyaratna, and (iii) my esteemed colleagues Professors Manvel and Bose. Finally, to this list, must be added the name of Usha Srivastava, now my sister-in-law. Along with me, Usha also was going through agonies. In spite of this she helped me run things smoothly thus making a great (though indirect) contribution to the success of the Conference.

I am thankful to the authors for the many excellent papers in this volume, and also to the referees for their help.

As in the earlier conferences, the various local arrangements were made by the C.S.U. Department of Conferences and Institutes. This time, unfortunately, some participants suffered inconveniences. I wish to apologize for the same.

I am thankful to North Holland (particularly, the desk editor Aad Thoen) for their promptness in handling the manuscripts, and producing this volume.

Last, but not the least, my thanks go to all the participants in the Symposium for it was their participation which truly made it a success.

Jaya Srivastava
Symposium Director

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CONTENTS

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RESULTS AND PROBLEMS IN GALOIS GEOMETRY

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1. Introduction

Galois geometry, in its broader sense, is the study of the nonlinear sets of points in finite spaces (including here also finite spaces over nonfield structures). References on the early history of Galois geometry can be found in [29]. In this survey the main research areas in which Galois geometry may be divided were clearly indicated:

(a) To offer pure geometric interpretations of algebraic and number theoretic properties.

(b) To give estimates for the number of points lying on certain algebraic varieties. Here the solution may follow from algebraic results or from purely combinatorial and geometric methods.

(c) To present graphic characterizations of algebraic varieties.

(d) To study $(k; n)$-arcs, $(k; n)$-caps and more generally $(k; n)$-sets. Of particular interest in this study is the "packing problem", i.e. the problem of finding in a given space the maximum number of points which can belong to a $(k; n)$-set, for a given value of n.

In the last 20 years the study of Galois geometry has developed in a quite remarkable way. In 1974 in some lectures we presented a brief survey on this field (see [2, n. 3]). We shall exhibit here some of the progress done since 1974, particularly in the above section (d), and some (old and new) open problems.

2. Basic notions on arcs and caps

$PG(r, q)$ will denote (if $r > 2$) a finite r-dimensional projective space of order q. If $r = 2$, the symbol $PG(2, q)$ will be used only for a desarguesian plane, whereas $\pi(q)$ will denote any projective plane of order q.

A $(k; n)$-arc of $\pi(q)$ is a set of k points of $\pi(q)$ such that n is the largest number of them which are collinear. The $(k; 2)$-arcs are simply called k-arcs. In a given plane a $(k; n)$-arc is "complete" if there does not exist a $(k'; n)$-arc which contains it (with $k' > k$).

A $(k; n)$-cap of $PG(r, q)$, where $r \geq 3$, is a set of k points of $PG(r, q)$ such that n

is the largest number of them which are collinear. The $(k; 2)$-caps are simply called k-caps.

A line g is an s-secant of a $(k; n)$-arc, K, if g contains s points of K; the 1-secants and 0-secants will be called respectively tangent and external lines to K. Let t_s denote the total number of s-secants to K; the numbers t_s are called the characters of K, and K is said to have p characters if exactly p among its characters are different from zero. The arc K is of type $(s_1, s_2, \ldots, s_h)$, where $s_1 < s_2 < \cdots < s_h = n$, if only the characters $t_{s_1}, t_{s_2}, \ldots, t_{s_h}$ are different from zero.

It has been proved that if there exists a k-arc in $\pi(q)$, then $k \leq q+1$ when q is odd and $k \leq q+2$ when q is even (see [9]).

In $\pi(q)$, with q even, every $(q+1)$-arc is incomplete and can be uniquely completed to form a $(q+2)$-arc.

3. A generalization of a theorem by Buekenhout

In 1966 Buekenhout proved the following theorem (see [10]):

Theorem. *If in a projective plane $\pi(q)$ there is a $(q+1)$-arc K such that every hexagon whose vertices are points of K is pascalian, then $\pi(q)$ is pappian and K is a conic.*

This theorem shows that a geometrical property of a single arc can determine the type of the plane in which the arc can be embedded. Alternative proofs for this theorem were given by several authors (see [1, 14, 19, 20, 26]).

In a paper due to appear (see [21]) Korchmaros has obtained a generalization of the above theorem for finite planes by weakening the hypothesis in the following way: the hexagons inscribed in K which are required to be pascalian are only those for which at least one of the lines joining two by two the three diagonal points is a chord or a tangent of the oval.

4. On abstract arcs of type (m, n)

Abstract ovals were defined by Buekenhout [11] and their study led to many interesting problems of classification and existence.

An example of an infinite abstract oval which is not embeddable in a projective plane has been given by Krier [22]. It is still an open question whether there exist or not finite abstract ovals which are not embeddable in a plane.

Barnabei and Zucchini [6] defined abstract arcs of type (m, n) which generalize Buekenhout abstract ovals. Their definition includes as particular cases the $(k; n)$-arcs of types $(0, m, n)$, $(0, n)$ or (m, n) of a finite projective plane.

The notion of abstract arc of type (m, n) is founded on the definition of generalized involution of type (m, n). Let K be a finite set, whose elements we shall call points. A generalized involution of type (m, n) on K (with $1 \leq m < n$) is any partition of the points of K in subsets of cardinality m or n, which are called respectively the blocks of type m or of type n.

An abstract arc of type (m, n) is given by a pair $(K, \mathcal{I})$, where K is a finite set and $\mathcal{I}$ is a set of generalized involutions of type (m, n) on K such that if $\mathcal{V}$ is the set of blocks of all involutions of $\mathcal{I}$, the following axioms hold:

(1) two distinct points of K belong to exactly one block of $\mathcal{V}$;

(2) two blocks of $\mathcal{V}$, with empty intersection, belong to exactly one involution of $\mathcal{I}$;

(3) in $\mathcal{V}$ there is at least one block of type n.

Clearly, (k, n)-arcs of types $(0, m, n)$, $(0, n)$ or (m, n) of any finite projective plane can be seen as abstract arcs of type (m, n) if we consider as elements of I the partitions induced on the points of the arc by the lines of the pencils whose centers do not belong to the arc.

It can be proved that if $|K| := k > 2$ every point of K belongs to the same number, say d, of blocks.

Let $(K, \mathcal{I})$ be any abstract arc of type (m, n). To this we can associate an incidence structure $(\mathcal{P}, \mathcal{L})$ as follows:

(i) $\mathcal{P} = K \cup \mathcal{I}$;

(ii) $\mathcal{L} = \mathcal{V}$;

(iii) incidence is defined in a natural way: a point A of K is incident with $v \in \mathcal{V}$ when $A \in v$ and a point A of $\mathcal{I}$ is incident with $v \in \mathcal{V}$ when $v \in A$.

The problem arises to decide whether or not the above structure can be completed in a suitable way to a projective plane. Examples of sufficient conditions for embeddability are the following:

(A) every involution of $\mathcal{I}$ has a number d of blocks (where d is equal to the number of blocks through any point of K);

(B) $|\mathcal{V}| = d^2 - d + 1$.

5. The packing problem

We denote by $m(r, q)$ the maximum number of points in PG(r, q) that belong to a k-cap if $r \geq 3$ or to a k-arc if $r = 2$. The problem of finding the values of $m(r, q)$, known as the "packing problem", seems to be very difficult and only the following results are known to date:

$m(2, q) = q + 1$,	q odd,	Bose [9],
$m(2, q) = q + 2$,	q even,	Bose [9],
$m(3, q) = q^2 + 1$,	q odd,	Bose [9],
$m(3, q) = q^2 + 1$,	q even, $\neq 2$,	Qvist [25],
$m(r, 2) = 2^r$,	$r \geq 2$,	Bose [9],
$m(4, 3) = 20$,		Pellegrino [24]
$m(5, 3) = 56$,		Hill [16].

The knowledge of $m(r, q)$ is important for applications to statistic and to coding theory, and a considerable amount of work has also been done to provide upper and lower bounds on $m(r, q)$ for $q \geq 4$ (see [15, 16, 17, 18, 23]).

In [24] is proved the existence of only two non-isomorphic 20-caps in PG(4, 3).

The packing problem arises (and in general is far from being solved) also for non desarguesian planes, for k-sets of kind s and for (k, n)-arcs and (k, n)-caps.

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COMBINATORIAL PROBLEMS OF EXPERIMENTAL DESIGN II: FACTORIAL DESIGNS*

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The designs used for statistically controlled experiments fall into two important classes: Incomplete block designs and factorial designs. The combinatorial problems arising in connection with incomplete block designs were surveyed in an earlier paper, Bose [13]. This is a companion paper which surveys the combinatorial problems of factorial designs.

1. Introduction

The principles of experimental design as we know it today were formulated by R.A. Fisher in his famous book, *Statistical Methods for Research Workers* [25] and in his paper on "The Arrangement of Field Experiments" [26]. They arose out of his own attempts and those of his precursors to increase the precision of field experiments. The object of the experimenter is to investigate the response of experimental units to a set of treatments. The experimental units are divided into sets called *blocks* which are relatively homogeneous. A certain set of treatments is applied to the units (*plots*) in a given block and the response observed. Statistical analysis is then used to estimate the effects of treatments. The three principles formulated by Fisher are *randomization, replication* and *local control.* Randomization dictates that once the set of treatments which are to be applied to the units of a given block have been chosen they should be applied randomly to the different units. The principle of replication requires that each treatment be applied to more than one experimental unit. In general, these units belong to different blocks. Due to the variability of the experimental material, the responses of the units to which the same treatment has been applied are not identical. Statistical analysis allows us to estimate the precision of the estimated treatment effects. If the number of treatments is large and experimental material is not homogeneous, precision can be increased by not accommodating all treatments in a given block. Thus blocks are incomplete. The problem of design is to select sets of treatments to be applied to the units of different blocks. The selection is subject to combinatorial constraints, since apart from the question of precision, it is desirable to have a design such that statistical analysis is as simple as possible, and such that its results are easily interpreted. There are two important classes of designs. In incomplete block designs (so called) the treatments are simple; for example, we may be interested in comparing the yields or some other characteristic of a large number of varieties

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of corn. On the other hand, treatments may be complex. Thus each treatment may consist of a number of factors, each at a certain level. For example, in studying effects of fertilizers, each treatment may contain nitrogen, potash and sulphate at different levels. Designs with complex treatments are called *Factorial Designs*. In this paper, we shall attempt a survey of combinatorial problems related to Factorial Designs. The combinational problems related to Incomplete Block Designs have been surveyed in an earlier paper, Bose [13].

2. Factorial designs

Suppose there are m factors; $A_1, A_2, \ldots, A_m$ in a factorial experiment and the factor A_i can be applied at any one of s_i levels. A treatment in which the factor A_i has been applied at the level x_i $(i = 1, 2, \ldots, m)$ may be denoted by $a_1^{x_1} a_2^{x_2} \cdots a_m^{x_m}$. We say that the experiment is of the type $s_1 \times s_2 \times \cdots \times s_m$. The number of different treatments is obviously $N = s_1 s_2 \cdots s_m$. If $s_1 = s_2 = \cdots = s_m$ we have a symmetric factorial experiment of the type s^m.

The effect of the treatment $a_1^{x_1} a_2^{x_2} \cdots a_m^{x_m}$ can be denoted by $t(x_1, x_2, \ldots, x_m)$. There are $N = s_1 s_2 \cdots s_m$ treatment effects. We consider all possible linear functions (with real coefficients) of the treatment effects. A typical linear function is

$$L = \sum C_{x_1 x_2, \ldots, x_m} t(x_1, x_2, \ldots, x_m) \tag{2.1}$$

where the summation is over all possible levels, $1 \leqslant x_i \leqslant s_i$, $i = 1, 2, \ldots, m$.

Orthogonality and independence of these linear functions are defined in the usual way. The mean of the treatment effects is given by

$$\mu = \frac{1}{N} \sum t(x_1, x_2, \ldots, x_m) \tag{2.2}$$

A linear function is said to be a contrast if it is orthogonal to the mean. Hence a necessary and sufficient condition for L given by (2.1) to be a contrast is

$$\sum C_{x_1 x_2 \cdots x_m} = 0. \tag{2.3}$$

Each independent linear function of the treatment effects can be regarded as carrying a *degree of freedom* (d.f.). One of these linear functions can be taken as the mean μ. Hence $N-1$ independent contrasts carry $N-1$ degrees of freedom. These degrees of freedom can be further subdivided into main effects and interactions as follows: A contrast is said to belong to the main effect A_i if the coefficients in the linear function constituting the contrast are independent of the levels of the factors other than A_i. Thus there are $s_i - 1$ independent contrasts which belong to the main effect A_i, and they carry $s_i - 1$ d.f.

A contrast is said to belong to the interaction of the factors A_i and A_j $(i \neq j)$, if it is orthogonal to any main effect, and the coefficients of the linear function constituting the contrast are independent of the levels of the factors other than A_i

and A_j. Thus there are $(s_i - 1)(s_j - 1)$ independent contrasts which belong to the interaction of A_i and A_j, and they carry $(s_i - 1)(s_j - 1)$ d.f.

In general, a contrast is said to belong to the interaction of the factors $A_{i_1}, A_{i_2}, \ldots, A_{i_r}$ if the linear function constituting the contrast is orthogonal to any main effect or to any k-factor interaction $k < r$, and the coefficients of the linear function constituting the contrast are independent of the levels of the factors other than $A_{i_1}, A_{i_2}, \ldots, A_{i_r}$. Thus there are $(s_{i_1} - 1)(s_{i_2} - 1) \cdots (s_{i_r} - 1)$ independent contrasts which belong to the interaction $A_{i_1}, A_{i_2}, \ldots, A_{i_r}$ and they carry as many d.f. Now

$$1 + \sum_{r=1}^{m} \binom{m}{r} (s_{i_1} - 1)(s_{i_2} - 1) \cdots (s_{i_r} - 1) = s_1 s_2 \cdots s_m = N.$$

Hence the mean, main effects and the various interactions carry the N d.f. which is the number of independent linear functions.

In a complete factorial experiment with r replications, there are r blocks, and each block has N plots (experimental units) to which the treatments are applied at random. The main effects and various interactions can then be estimated.

3. Confounding and fractional replication

In a complete factorial the block size tends to be large. Within block variability increases the error and reduces the efficiency. Yates [48, 49] developed the concept of confounding to overcome this difficulty. In many practical situations higher order interactions tend to be negligible. Yates method takes advantage of this fact to increase the efficiency of the experiment. We illustrate this concept by considering a symmetrical 2^3 experiment with three factors A_1, A_2, A_3 each at two levels 0 and 1.

In Yates' notation the treatment $a_1^{x_1} a_2^{x_2} \cdots a_m^{x_m}$ is regarded as a formal algebraic product. Since $a_i^0 = 1$ and $a_i^1 = a_i$, so for any treatment in which the level $x_i = 0$, the factor $a_i^{x_i}$ is dropped, and is replaced by a_i if $x_i = 1$. When all the levels are zero, the treatment is denoted by 1. Thus the eight treatments of a 2^3 experiment can be denoted by

$$1, a_1, a_2, a_3, a_2a_3, a_1a_3, a_1a_2, a_1a_2a_3.$$

The mean can then be written as the formal product

$$\mu = \tfrac{1}{8}(a_1 + 1)(a_2 + 1)(a_3 + 1) \tag{3.1}$$

since the linear function constituting the mean is obtained by formally developing the product on the right-hand side of (3.1).

The main effects and interactions can be written as

$$\tfrac{1}{4}(a_1 \pm 1)(a_2 \pm 1)(a_3 \pm 1) \tag{3.2}$$

where the +ve or the −ve sign is taken in $a_i \pm 1$ according as the factor A_i occurs at the level 0 or 1 in the effect. The numerical factor is conventionally taken as the reciprocal of the number of positive terms in any main effect or interaction. Thus the interaction between the three factors A_1, A_2, A_3, is written as

$$A_1A_2A_3 = \tfrac{1}{4}(a_1 - 1)(a_2 - 1)(a_3 - 1). \tag{3.3}$$

The treatments can be split in two groups $a_1a_2a_3$, a_1, a_2, a_3, and 1, a_2a_3, a_3a_1, a_1a_2. The treatments of the first (second) group carry a positive (negative) sign in the formal expansion of the right-hand side of (3.3).

If the treatments of different groups are assigned to different blocks as in Fig. 1 and we take the usual linear model

$$E\{y(a_1^{x_1}\, a_2^{x_2}\, a_3^{x_3})\} = t(x_1, x_2, x_3) + b_i + g \tag{3.4}$$

where $y(a_1^{x_1}\, a_2^{x_2}\, a_3^{x_3})$ denotes the observed response from the plot to which the treatment $a_1^{x_1}\, a_2^{x_2}\, a_3^{x_3}$ has been applied, b_i is the effect of block which contains this plot and g is the general effect, then we can estimate the main effects and two factor interaction, but the estimate of the three factor interaction is confounded with blocks since we can only estimate

$$\tfrac{1}{4}\{(a_1a_2a_3 + a_1 + a_2 + a_3) - (1 + a_1 + a_2 + a_3) + b_1 - b_2\} \tag{3.5}$$

where b_i is the effect of the ith block.

The general problem of confounding in the symmetrical case can be stated as follows:

Let $s = p^n$ where p is a prime. Consider a symmetric factorial experiment s^m. It is required to split the s^m treatments into s^{m-r} blocks, each of size s^r, so that as far as possible lower order interactions are unconfounded.

A general theory of confounding was developed by Bernard [7] for the 2^m factorial experiment. We can write the r factor interaction

$$A_{i_1}A_{i_2}\cdots A_{i_r} = \prod_{j=1}^{m}(a_j \pm 1)/2^{m-1} \tag{3.6}$$

where the negative sign is taken in $a_j \pm 1$ if j belongs to the set $\{i_1, i_2, \ldots, i_r\}$ and the positive sign is taken otherwise. Consider the Abelian group G generated by the symbols $A_1, A_2, \ldots, A_m$ with the relations

$$A_1^2 = A_2^2 = \cdots = A_m^2 = I,$$

where I is the identity element of the group. Bernard shows that any confounded arrangement corresponds to a subgroup of G. If we take any r independent

a_3	a_1
a_2	$a_1a_2a_3$

a_2a_3	1
a_2a_3	a_1a_2

Fig. 1

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