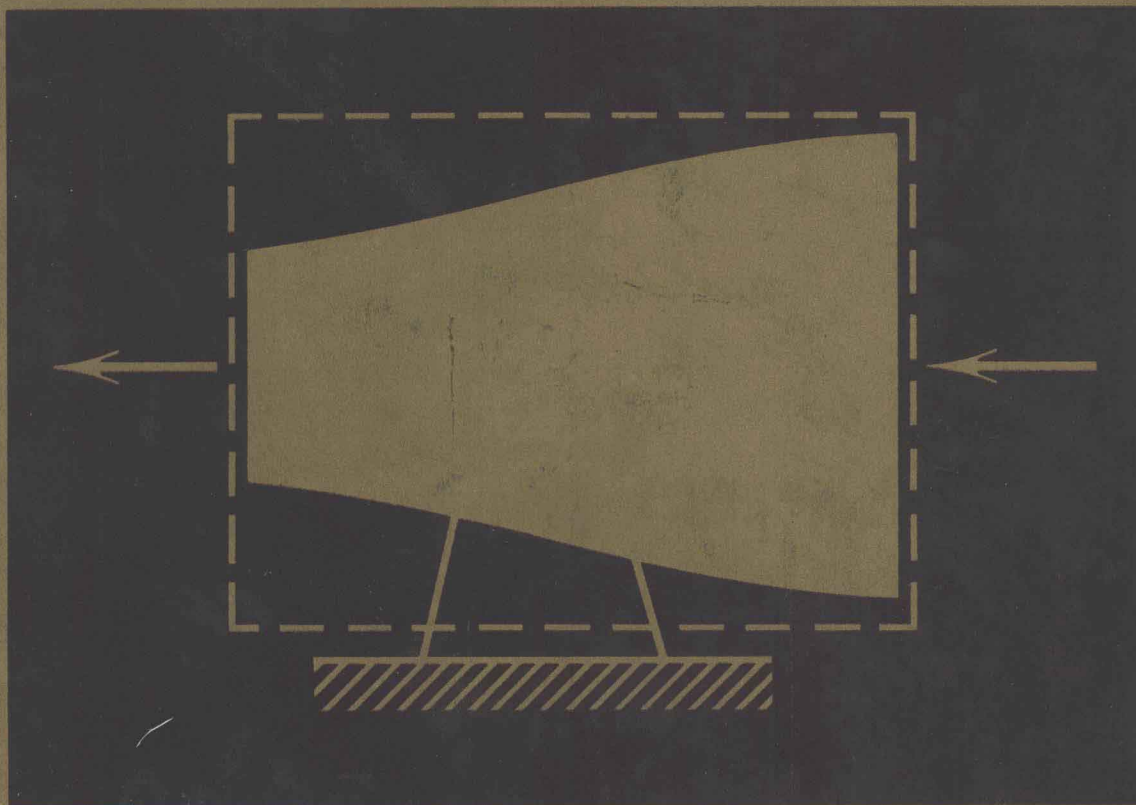


fox &
mcdonald

introduction
to
fluid
mechanics



introduction to fluid mechanics

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introduction to fluid mechanics

preface

This textbook was written for an introductory fluid mechanics course at the junior level. The decision to begin writing our own book reflected our dissatisfaction with existing textbooks. The decision to complete it resulted from our experience with earlier versions that were used in our own classes at Purdue University during the past three years.

Our objective has been to make the material interesting, easy to read, and easy to understand. All of it has been tested in the classroom by two generations of students to ensure that we have reached these goals. Thus we are convinced that students who read the book will understand it.

Because the material is easy to understand, the instructor can depart from conventional “lecture” teaching techniques. He can use classroom time to bring in current outside material, to solve example problems, and to explain the complexities of the assigned homework problems. Thus he can use each class period in the manner that seems most appropriate. Since there is no quantum of material that must be covered during a certain hour, the instructor can spend time in reviewing or amplifying any area in which he senses that students are having difficulty.

This flexibility permits the instructor to have much more interaction with the class than is possible through rigid adherence to a fixed schedule of formal lectures. It also permits the student to relate to the instructor and the material, since the

student knows that class time is available to answer questions that are currently “bugging” him. These positive features are explained in Section 1–5, “Note to Students.”

The material has been carefully selected. It includes all of the topics that we feel can be covered thoroughly in a 15-week (one-semester) course with three 50-minute lectures per week. At Purdue, we cover the entire book, with the exception of the starred sections. A laboratory (meeting two hours each week) is used for demonstrations, films, and simple experiments. A sample course outline is included in the *Solutions Manual* that accompanies this book. Desirable prerequisites are introductory courses in rigid body dynamics and heat power or thermodynamics, and mathematics through integral calculus. The thermodynamics course may be taken concurrently.

We believe that people—students and teachers alike—learn by doing; that is, by applying the principles they have learned to a variety of problems. Throughout the book, the emphasis is on starting from fundamentals to develop solutions. This approach is also emphasized for student problem solving, and we recommend a standard format for homework and examination solutions (see Section 1–5). This format requires that the problem statements be read and understood before the solution is attempted. It requires that basic equations be written and assumptions listed before numerical calculations are started. Thus, the student is forced to think about both the problem and the equations before he becomes involved in the detailed work of numerical substitution.

The text contains many example problems. We selected them carefully to illustrate matters that, in our experience, have troubled students. Solutions to these examples have been prepared to demonstrate good solution technique and to explain troublesome points. A different physical layout with wider margins has been used to make these solutions as visible and legible as possible.

For further clarification and demonstration of basic principles in fluid mechanics, many fine instructional films and film loops are available. We have referred to these films in the text where their use is appropriate; a complete list of suppliers and titles is included in Appendix C.

When our students have finished this course, we expect them to be able to apply the basic equations to a variety of problems, including new problems that they have not encountered before. We also emphasize physical understanding throughout, in order to make students aware of the variety of phenomena that can occur in fluid flow situations. By minimizing the number of “magic formulas” and emphasizing the fundamental approach, we believe that students will feel more confident in their ability to apply the material and will be able to reason out solutions to rather challenging problems.

Our experience with this material (in the form of notes) over a three-year period at Purdue University has been very encouraging. However, we recognize that no

single approach can satisfy all needs. Therefore we welcome criticisms and suggestions from interested readers or users of this book.

We are indebted to several of our colleagues, especially to John A. Brighton of the Pennsylvania State University and James P. Johnston of Stanford University, for providing constructive and cogent reviews of an earlier version of this book. We have incorporated many of their suggestions. However, we are responsible for errors of fact or omission.

ALAN T. McDONALD
ROBERT W. FOX

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introduction

In beginning the study of any subject, a number of questions come to mind immediately. Among those which a student in the first course in fluid mechanics may well wonder about are the following:

What is fluid mechanics all about?

Why do I have to study it?

Why should I want to study it?

How does it relate to subject areas with which I am already familiar?

In this chapter we will try to provide at least a qualitative answer to these and similar questions and thus an introduction to the subject.

1-1 Definition of a Fluid

Since fluid mechanics is a science dealing with the behavior of fluids at rest and in motion, it is logical to begin our study of the subject with a definition of the term fluid.

A fluid is a substance that deforms continuously under the application of a shearing (i.e. tangential) stress no matter how small the shearing stress.

Thus, according to the physical forms in which matter exists, fluids comprise the liquid and gas (or vapor) phase. The distinction between a fluid and the

remaining possible state of matter (i.e. the solid state) is clear if one compares a fluid as defined above with the behavior of a solid. A solid is a substance that deforms when a shear stress is applied, but it does not continue to deform.

In Fig. 1.1 the behavior of a solid (Fig. 1.1a) and a fluid (Fig. 1.1b) under the action of a constant shear force are contrasted. In Fig. 1.1a the shear force is applied to the solid through the upper of two plates to which the solid has been bonded. When the shear force is applied to the plate, the block is deformed as shown. From our previous work in mechanics, we know that, provided the elastic limit of the solid material is not exceeded, the deformation is directly proportional to the applied shear stress, τ , where $\tau = F/A$ and A is the area of the surface in contact with the plate.

Now let us repeat the experiment using a fluid between the plates. In order to observe the behavior of the fluid we use a dye marker to outline a fluid element as shown by the solid lines (Fig. 1.1b). Upon application of the force, F , to the upper plate, we notice that the fluid element continues to deform as long as the force is applied. The shape of the fluid element, at successive instants of time, $t_2 > t_1 > t_0$, is shown by the dashed lines in Fig. 1.1b, which represent the positions of the dye markers. Note also that the fluid in direct contact with the solid boundary has the same velocity as the boundary itself; that is, there is no slip at the boundary. This is an experimental fact based on numerous observations of fluid behavior.¹

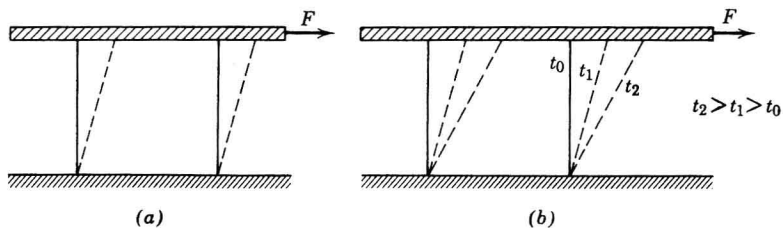


Fig. 1.1 Behavior of (a) solid and (b) fluid, under the action of a constant shear force.

Because a fluid deforms continuously under the application of a shear stress, we may alternatively define a fluid as a substance which, when at rest, cannot sustain a shear stress.

¹ The no slip condition is demonstrated in the film loops S-FM003, *Shear Deformation of Viscous Fluids*, and S-FM006, *Boundary Layer Formation*. These loops were produced by Educational Services, Inc., Watertown, Mass., and the National Committee for Fluid Mechanics Films. The films are distributed by Encyclopaedia Britannica Educational Corporation. (A complete list of fluid mechanics film titles and sources is given in Appendix C.)

1–2 Scope of Fluid Mechanics

Having defined a fluid and noted the characteristics that distinguish it from a solid, we might ask the question: “Why study fluid mechanics?” One answer is, “It is a required course in the curriculum!” While this may be a true and logical answer, it is not the only logical answer.

A knowledge and understanding of the basic principles and concepts of fluid mechanics are essential in the analysis and design of any system in which a fluid is the working medium. The design of virtually all means of transportation requires an application of the principles of fluid mechanics. Included are aircraft for both subsonic and supersonic flight, ground effect machines, hovercraft (now in service for channel crossings between France and England), vertical takeoff and landing aircraft requiring minimum runway length, surface ships, submarines, and automobiles. In recent years automobile manufacturers have given more consideration to aerodynamic design. This has been true for some time for the designers of both racing cars and boats. The design of propulsion systems for space flight as well as for toy rockets is based on the principles of fluid mechanics. The collapse of the Tacoma Narrows Bridge some years ago is evidence of the possible consequences of neglecting the basic principles of fluid mechanics.² It is commonplace today to perform model studies to determine the aerodynamic forces on and flow fields around buildings and structures. These include studies of skyscrapers, baseball stadiums, smokestacks, and shopping plazas.

The design of all types of fluid machinery including pumps, fans, blowers, compressors, and turbines clearly requires a knowledge of the basic principles of fluid mechanics. Lubrication is an area of considerable importance in fluid mechanics. Heating and ventilating systems for private homes, large office buildings, and underground tunnels, and the design of pipeline systems are further examples of technical problem areas requiring a knowledge of fluid mechanics. The circulatory system of the body is essentially a fluid system. It is not surprising then that the design of artificial hearts, heart-lung machines, breathing aids, and other such devices must rely on the basic principles of fluid mechanics.

Even some of our recreational endeavors are directly related to fluid mechanics. The slicing and hooking of golf balls can be explained by the principles of fluid mechanics (although they can be corrected only by a golf pro!).

The list of applications of the principles of fluid mechanics could be extended considerably. Our main point here is that fluid mechanics is not a subject studied for purely academic interest; rather, it is a subject with widespread importance both in our every-day experiences and in modern technology.

Clearly we cannot hope to consider in detail even a small percentage of these and other specific problems of fluid mechanics. Instead, the purpose of this text

² For dramatic evidence of aerodynamic forces in action, see the Ohio State University film, *Collapse of the Tacoma Narrows Bridge*.

is to present the basic laws and associated physical concepts that provide the basis or starting point in the analysis of any problem in fluid mechanics.

1-3 Basic Equations

An analysis of any problem in fluid mechanics necessarily begins, either directly or indirectly, with statements of the basic laws governing the fluid motion. These laws, which are independent of the nature of the particular fluid, are:

- (a) Conservation of mass.
- (b) Newton's second law of motion.
- (c) The first law of thermodynamics.
- (d) The second law of thermodynamics.

Clearly not all of these laws are always required in the solution of any one problem. In some problems, it is necessary to bring into the analysis additional relations, in the form of constitutive equations describing the behavior of physical properties of fluids under given conditions.

It is obvious that the basic laws with which we shall deal are the same as those used in mechanics and thermodynamics. Our task will be to formulate these laws in forms suitable for solution of fluid flow problems and to apply them to the solution of a wide variety of problems.

It should be emphasized that there are, as we shall see, many apparently simple problems in fluid mechanics that cannot be solved by totally analytical means. In such cases we must resort to experiments and experimental observations.

1-4 Methods of Analysis

As we have indicated, the basic laws that are employed in the analysis of problems in fluid mechanics are the same ones that you have used previously in your earlier studies of thermodynamics and basic mechanics. From these earlier studies you will recall that the first step in solving a problem is to define the system that you are attempting to analyze. In basic mechanics, extensive use was made of the free body diagram. In thermodynamics you referred to the system under analysis as either a closed system or an open system. In this text we shall employ the terms system and control volume. The importance of defining the system or control volume to which the basic equations are to be applied in the analysis of a problem cannot be overemphasized. At this point it is wise to review the basic difference between a system and a control volume.

1-4.1 SYSTEM AND CONTROL VOLUME

A system is defined as a fixed, identifiable quantity of mass; the system boundaries