

LECTURE NOTES  
IN PHYSICS

E. Papantonopoulos  
(Ed.)

# The Physics of the Early Universe



Springer

E. Papantonopoulos (Ed.)

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 Springer

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# Preface

This book is an edited version of the review talks given in the Second Aegean School on the Early Universe, held in Ermoupolis on Syros Island, Greece, in September 22-30, 2003. The aim of this book is not to present another proceedings volume, but rather an advanced multiauthored textbook which meets the needs of both the postgraduate students and the young researchers, in the field of Physics of the Early Universe.

The first part of the book discusses the basic ideas that have shaped our current understanding of the Early Universe. The discovering of the Cosmic Microwave Background (CMB) radiation in the sixties and its subsequent interpretation, the numerous experiments that followed with the enumerable observation data they produced, and the recent all-sky data that was made available by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite, had put the hot big bang model, its inflationary cosmological phase and the generation of large scale structure, on a firm observational footing.

An introduction to the Physics of the Early Universe is presented in K. Tamvakis' contribution. The basic features of the hot Big Bang Model are reviewed in the framework of the fundamental physics involved. Shortcomings of the standard scenario and open problems are discussed as well as the key ideas for their resolution.

It was an old idea that the large scale structure of our Universe might have grown out of small initial fluctuations via gravitational instability. Now we know that matter density fluctuations can grow like the scale factor and then the rapid expansion of the universe during inflation generates the large scale structure of our Universe. R. Durrer's review offers a systematic treatment of cosmological perturbation theory. After the introduction of gauge invariant variables, the Einstein and conservation equations are written in terms of these variables. The generation of perturbations during inflation is studied. The importance of linear cosmological perturbation theory as a powerful tool to calculate CMB anisotropies and polarisation is explained.

The linear anisotropies in the temperature of CMB radiation and its polarization provide a clean picture of fluctuations in the universe after the big bang. These fluctuations are connected to those present in the ultra-high-energy universe, and this makes the CMB anisotropies a powerful tool for constraining the fundamental physics that was responsible for the generation of structure. Late time effects also leave their mark, making the CMB tem-

perature and polarization useful probes of dark energy and the astrophysics of reionization. A. Challinor's contribution discusses the simple physics that processes primordial perturbations into the linear temperature and polarization anisotropies. The role of the CMB in constraining cosmological parameters is also described, and some of the highlights of the science extracted from recent observations and the implications of this for fundamental physics are reviewed.

It is of prime interest to look for possible systematic uncertainties in the observations and their interpretation and also for possible inconsistencies of the standard cosmological model with observational data. This is important because it might lead us to new physics. Deviations from the standard cosmological model are strongly constrained at early times, at energies on the order of 1 MeV. However, cosmological evolution is much less constrained in the post-recombination universe where there is room for deviation from standard Friedmann cosmology and where the more classical tests are relevant. R. Sander's contribution discusses three of these classical cosmological tests that are independent of the CMB: the angular size distance test, the luminosity distance test and its application to observations of distant supernovae, and the incremental volume test as revealed by faint galaxy number counts.

The second part of the book deals with the missing pieces in the cosmological puzzle that the CMB anisotropies, the galaxies rotation curves and microlensing are suggesting: dark matter and dark energy. It also presents new ideas which come from particle physics and string theory which do not conflict with the standard model of the cosmological evolution but give new theoretical alternatives and offer a deeper understanding of the physics involved.

Our current understanding of dark matter and dark energy is presented in the review by V. Sahni. The review first focusses on issues pertaining to dark matter including observational evidence for its existence. Then it moves to the discussion of dark energy. The significance of the cosmological constant problem in relation to dark energy is discussed and emphasis is placed upon dynamical dark energy models in which the equation of state is time dependent. These include Quintessence, Braneworld models, Chaplygin gas and Phantom energy. Model independent methods to determine the cosmic equation of state are also discussed. The review ends with a brief discussion of the fate of the universe in dark energy models.

The next contribution by A. Lukas provides an introduction into time-dependent phenomena in string theory and their possible applications to cosmology, mainly within the context of string low energy effective theories. A major problem in extracting concrete predictions from string theory is its large vacuum degeneracy. For this reason M-theory (the largest theory that includes all the five string theories) at present, cannot provide a coherent picture of the early universe or make reliable predictions. In this contribution particular emphasis is placed on the relation between string theory and inflation.

In another development of theoretical ideas which come from string theory, the universe could be a higher-dimensional spacetime, with our observable part of the universe being a four-dimensional “brane” surface. In this picture, Standard Model particles and fields are confined to the brane while gravity propagates freely in all dimensions. R. Maartens’ contribution provides a systematic and detailed introduction to these ideas, discussing the geometry, dynamics and perturbations of simple braneworld models for cosmology.

The last part of the book deals with a very important physical process which hopefully will give us valuable information about the structure of the Early Universe and the violent processes that followed: the gravitational waves. One of the central predictions of Einstein’s general theory of relativity is that gravitational waves will be generated as masses are accelerated. Despite decades of effort these ripples in spacetime have still not been observed directly.

As several large scale interferometers are beginning to take data at sensitivities where astrophysical sources are predicted, the direct detection of gravitational waves may well be imminent. This would (finally) open the long anticipated gravitational wave window to our Universe. The review by N. Andersson and K. Kokkotas provides an introduction to gravitational radiation. The key concepts required for a discussion of gravitational wave physics are introduced. In particular, the quadrupole formula is applied to the anticipated source for detectors like LIGO, GEO600, EGO and TAMA300: inspiralling compact binaries. The contribution also provides a brief review of high frequency gravitational waves.

Over the last decade, advances in computer hardware and numerical algorithms have opened the door to the possibility that simulations of sources of gravitational radiation can produce valuable information of direct relevance to gravitational wave astronomy. Simulations of binary black hole systems involve solving the Einstein equation in full generality. Such a daunting task has been one of the primary goals of the numerical relativity community. The contribution by P. Laguna and D. Shoemaker focusses on the computational modelling of binary black holes. It provides a basic introduction to the subject and is intended for non-experts in the area of numerical relativity.

The Second Aegean School on the Early Universe, and consequently this book, became possible with the kind support of many people and organizations. We received financial support from the following sources and this is gratefully acknowledged: National Technical University of Athens, Ministry of the Aegean, Ministry of the Culture, Ministry of National Education, the Eugenides Foundation, Hellenic Atomic Energy Committee, Metropolis of Syros, National Bank of Greece, South Aegean Regional Secretariat.

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## VIII Preface

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Athens, May 2004

*Lefteris Papantonopoulos*

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Part I

**The Early Universe  
According to General Relativity:  
How Far We Can Go**



