

Franz Ziegler

Mechanics of Solids and Fluids



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Mechanics of Solids and Fluids

To the Students Who Accept the Challenge of Science
and to My Children Robert and Eva C.

Preface

This book offers a unified presentation of the concepts and most of the practicable principles common to all branches of solid and fluid mechanics. Its design should be appealing to advanced undergraduate students in engineering science and should also enhance the insight of both graduate students and practitioners. A profound knowledge of applied mechanics as understood in this book may help to cultivate the versatility that the engineering community must possess in this modern world of high-technology.

This book is, in fact, a reviewed and extensively improved second edition, but it can also be regarded as the first edition in English, translated by the author himself from the original German version, "Technische Mechanik der festen und flüssigen Körper," published by Springer-Verlag, Wien, in 1985.

Although this book grew out of lecture notes for a three-semester course for advanced undergraduate students taught by the author and several colleagues during the past 20 years, it contains sufficient material for a subsequent two-semester graduate course. The only prerequisites are basic algebra and analysis as usually taught in the first year of an undergraduate engineering curriculum. Advanced mathematics as it is required in the progress of mechanics teaching may be taught in parallel classes, but also an introduction into the art of design should be offered at that stage. The book is divided into 13 chapters that are arranged in such a way as to preserve a natural sequence of thought and reflections. Within a single chapter, however, the presentation, in general, proceeds from the undergraduate to intermediate level and eventually to the graduate level.

The first three chapters are devoted to the basic components of the mechanical modeling of systems at rest or in motion. They are followed by a chapter on constitutive relations ranging from Hooke's law and Newtonian fluids to that of visco-plastic materials. Vector and cartesian tensor notation is applied consequently, but one-dimensional relations of standard material testing are always given priority. Students familiar with the programming techniques of field variables will have no difficulty following this text.

Kinematics of material points and particle fields including both pathlines and streamlines, as well as the conservation of mass; statics, providing experience with forces and stresses including

hydrostatic pressure fields; mechanical work, and force potentials, are the prerequisites that, in Chap. 5, allow early exposure to the powerful principle of virtual work. Since force is recognized to be flux in the potential field, even an extension to nonmechanical systems with other driving agents becomes evident.

In the lengthy Chap. 6, material on linearized elastostatics is assembled but selected to serve the needs of structural mechanics. Thermal effects in beams, plates, and shells are given special attention in relation to load stresses in order to prepare the student for design considerations of the high-temperature environment of modern structures. The elastic visco-elastic correspondence principle serves as the vehicle for applying elastic solutions in order to determine stationary creep and solve the associated lifetime problems. At this stage, the Laplace transformation enters like the previous considerations of Chap. 4 in the form of operational calculus. Straight and curved beams, simple frames, plates, and shells of revolution are considered; torsion and the Hertz theory of contact are given special attention.

While disks and rings in stationary rotation are already considered in elastostatics, the Euler-Cauchy equations of motion are the starting point of Chap. 7 and highlight the transition from the statics of a material point at rest to the dynamics of a moving point. The conservation of both momentum and angular momentum is formulated for moving bodies enclosed in a material volume, and for the flow through a control volume that is fixed in space, or for the case in which the control surface is in prescribed motion. Prerequisites are the Lagrangean and Eulerian kinematics of Chap. 1. The control volume concept is immediately applied to determine the guiding forces of stationary flow and to explain the thrust of rockets and jet engines. Euler's turbine equation is derived, and the drag and propelling forces in a parallel viscous main stream are determined. The vector of angular momentum is further defined for rotating rigid bodies, and the generally valid formula for taking the time derivative of a vector with respect to an intermediate (rotating) reference frame is quite naturally derived. Euler's equations of gyros are discussed as well. In connection with the material on rigid-body kinematics of Chap. 1, the important field of (nonlinear) multibody dynamics (eg of vehicles and satellites) is addressed. Sections on linear and nonlinear vibrations present not only useful integration techniques in both the time and frequency domain, but also illustrate pure dynamic phenomena, like resonance and phase shift. Blake's logarithmic diagram is introduced.

The coupled equations of motion of an unbranched chain of spring-mass systems are derived by means of Newton's law, and the modal properties of its natural vibrations are determined by means of the Holzer-Tolle procedure. Stodola's matrix iteration scheme is sketched in Exercise A 11.11. The practically important design of

vibrational absorbers is considered in general in Chap. 7 and for torsional vibrations in Chap. 10.

Using the free-body diagram of a high beam element, the partial differential equations of a vibrating Timoshenko beam are derived. Plane body waves and the associated linear eigenvalue problems and the Rayleigh surface wave are included in this chapter to enhance the understanding, eg of ultrasonic techniques. They are applied in material testing and for medical diagnosis. Seismic waves, namely, the loadings in earthquake engineering, should be mentioned here, as well as the water hammer in hydraulic engineering. Some illustrations are given in Chap. 11 and 12.

A first integral of the equations of motion is useful for both solids and fluids. A first example, the planar pendulum, is already analyzed along these lines of general validity in Chap. 7. Thus, in Chap. 8 not only the work theorem of dynamics and its special form of conservation of mechanical energy is derived by proper integration over the material volume, but also integration performed along a given streamline, keeping the time constant, renders the generalized Bernoulli equation of fluid dynamics. The latter is specialized to the stationary flow of ideal fluid and recognized to be the law of conservation of the specific mechanical energy of a particle moving along the streamline and pathline. Interpretations of hydraulic measurements of stationary flow are given by means of that original Bernoulli equation. Generalizations to include the power supply of a stationary stream and, thus, the loss of pressure head of a guided ideal flow through a turbine or the pressure gain of a flow through a pump and the loss of energy head in a viscous stream are discussed in some detail. Relative streamlines with respect to a stationary rotating reference frame are considered, and a proper form of a Bernoulli-type equation is derived to further ease the application of fluid dynamics to rotating machines. The extension to the first law of thermodynamics (of material and control volumes) concludes that introductory chapter on energy conservation, and it is hoped that the gap between the mechanics course and a parallel course on thermodynamics is thereby somewhat narrowed. The Clausius-Duhem inequality is only mentioned. Fourier's law of heat conduction, however, as an outcome of an irreversible process is stated and applied in Chap. 6.

Chapter 9 on stability starts out with the derivation of the Dirichlet criterion in an energy norm by means of small perturbations applied to a conservative mechanical system at rest. Thus, the dynamic nature of instability is stressed from the very beginning. The conservation of mechanical energy of the perturbing motion and the assumption of minimal potential energy of the equilibrium configuration render the proper inequalities that bound the kinetic and potential energy in the nearfield of the equilibrium configuration. Bifurcation, snap-through, and imperfection

sensitivity are discussed and well illustrated in the load factor deformation diagram. The Euler buckling of slender columns and the buckling of plates are generally discussed; further examples are given in Chap. 11. In addition, the method of small perturbations is applied to consider the stability of a principal motion.

The limits of stability of ductile structures are discussed and the ultimate loads of simple beams and frames determined. Safety analysis within structural mechanics like a cross-sectional and plastic system reserve follows quite naturally. Melan-Koiter's shake-down theorems are formulated and applied to a plastic thick-walled spherical pressure vessel under lifepressure loading. Consideration of the stability of an open-channel flow and of instability as a result of flutter round off this first overview based on phenomena, rather than mathematics.

The knowledge of dynamics of MDOF-systems is further expanded in Chap. 10, where the Lagrange equations of motion (of the second kind), the outcome of the more general D'Alembert's principle are presented. The latter is derived in quite the same fashion as the principle of virtual work in statics. Only dynamic systems under holonomic constraints are considered and a few applications to vibrational systems given. A spring-mounted foundation in coupled translational-rotational motion is treated in some detail to illustrate the dangerous beat phenomenon. Parametric excitation is shown to occur in a pendulum with a periodically moving support. Matrix structural dynamics is encountered when considering a simple beam with lumped masses.

The principle of virtual work as presented in Chap. 5 and 10 is the basis for the important approximation techniques and discretization procedures associated with the names of Rayleigh, Ritz, and Galerkin. Chapter 11 offers a rather complete account from a purely mechanical standpoint and also gives a short introduction to the finite-element method (FEM). Convergence in the mean square of the outcome of the Galerkin procedure is conserved if the equations of equilibrium or motion containing the forces are subject to approximations. A generalized form is discussed that makes application as convenient as the original Ritz approximation. Any practical application of some complexity, however, requires further consultation of the specialized literature and a gradual buildup of experience. Priority is given in this context to examples in which additional mechanical insight can be gained: For example, the buckling of a slender rod under the influence of a Winkler foundation exhibits mode jumping, and flexural vibrations under moving load excitation illustrate another type of effective structural bending stiffness in addition to critical speeds, to name just two illustrations of considerable engineering importance. The reduction in time of a nonlinear ordinary differential equation of motion is shown by the Ritz-Galerkin approximation of the Duffing oscillator,

by harmonic balance, and by means of the Krylow-Bogoljubow approximation.

In Chap. 12, which deals with impact dynamics, most of the material is related to the simplest possible modeling. The exchange of momentum is assumed to be a sudden process, ie the velocity fields of the colliding masses suffer a jumplike redistribution. Only the two extreme physical cases of idealized elastic impact with conservation of total mechanical energy and the inelastic impact of extreme dissipation are elaborated within that context. To illustrate and justify some of these assumptions, a thin elastic rod of finite length is considered, taking into account the back and forth running waves following a short and hard impact. Thereby, the sound speed of rods is introduced, in addition to the wave speeds derived in Chap. 7. Similarly, water hammer in a suddenly shut down lifeline is reconsidered, examining the compressibility of the fluid and the elasticity of the walls of a cylindrical pipe. The crude approximation derived in Chap. 7 for a draining pipe may be seen as limited to the quasistatic closure of the gate. The sound speed in that case depends on the stiffness of springs in a series connection.

The last, Chap. 13, completes to some extent the discussion of fluid dynamics. More important, the lift exerted on a body in an ideal flow is related to circulation, in addition to being just the result of the surface tractions. The Navier-Stokes equations of viscous flow and their nondimensional form are explored, providing further motivation for the introduction of the Reynolds' and Froude's numbers. Similarity solutions with respect to the drag coefficient, the viscous flow through a pipe, and the boundary layer of a flow along a semiinfinite plate are the few applications possible due to the limited length of this book. Since the singular nature of perturbation of the ideal flow through viscosity becomes evident in many cases to be limited to the boundary layer, the importance of the outer flow that may be assumed the ideal is recognized. Therefore, potential flows are discussed and some fields of streamlines derived. The singularity method is a major tool of analysis and its basic idea is sketched; the formulas of Blasius are then derived. The force exerted on a body by a von Karman street of vortices is calculated and the Strouhal number introduced. The boundary value problem of kinematic waves excited in a fluid strip of finite depth by a moving rigid and linear elastic wall is solved to illustrate one of the important interaction problems and to show mode coupling. The Mach number and supersonic flow of gas dynamics are discussed by considering the isentropic outflow of gas through a nozzle from a pressure vessel.

Each chapter not only contains more or less concise derivations, but also exclusively shows practical applications. A book on mechanics cannot be read like a novel. The reader is expected to work out examples with paper and pencil or a personal

computer and to have a good command of a suitable programming language, eg FORTRAN. Exercises are presented in an appendix to each chapter. Some test general problem-solving skills, some contain new material. However, hints when possibly needed are always given, together with complete solutions. Other collections of examples should be consulted.

This book was influenced by the standards of teaching mechanics at Northwestern University, Stanford University, and Cornell University, schools the author visited several times and in the order cited. It is a textbook designed for classroom teaching or self-study, not a treatise reporting new scientific results. The author is obviously indebted to many investigators over a period of more than two centuries, as well as earlier books on mechanics. Extensive bibliographies may be found in the *Encyclopedia of Physics*, published by Springer-Verlag, Berlin, and various handbooks of engineering mechanics and fluid mechanics. Also, the regular volumes of *Applied Mechanics Reviews*, published by ASME, New York, should be consulted. The classic textbooks written by the late Professor Timoshenko must be mentioned here. The first edition of this book published in German by Springer-Verlag Wien, had its roots in H. Parkus's *Mechanics of Solids* (in German). Most of the figures of the first edition have been used to illustrate this book. The author is indebted to several colleagues at the Technical University of Vienna for promoting this book in their classes and to a number of former graduate students, including Dr. P. Fotiu (UC-San Diego), Dr. N. Hampl (now with Getzner Chemie), Dr. R. Hasenzagl (now with Control Data), Dr. H. Hasslinger (now with AMAG), Dr. H. Hayek, Dr. R. Heuer, Dr. F. Höllinger (now with the Danube-Power), Dr. H. Irschik (Professor of Mechanics, U-Linz), Dr. F. Rammerstorfer (Professor of Light-Weight Structures and Aircraft Design), Dr. W. Scheidl (chief engineer of Elin). Their encouragement and helpful criticism and the numerous other contributions by students of civil and mechanical engineering made this book possible.

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Please forward any suggestions that might lead to an improvement of the text to the author or Springer-Verlag.

Franz Ziegler

Vienna, Austria

Contents

Preface.....	vii
1. Kinematics	1
1.1. Point Kinematics.....	1
1.1.1. Example: The Trajectory in a Homogeneous Gravity Field Above a "Flat Planet".....	5
1.1.2. Example: Guided Motion of a Point.....	5
1.1.3. The Natural Coordinates of the Trajectory.....	7
1.2. Kinematics of Rigid Bodies.....	8
(§) Show ω to be a Free Vector.....	9
(§) Reduction of Angular Velocity Vectors.....	9
1.2.1. Special Motions of a Rigid Body	11
(§) Pure Translation.....	11
(§) Rotation About a Fixed Point.....	11
(§) Plane Motion of Rigid Bodies.....	13
(§) Example: The Wheel in a Straight Rolling Motion.....	14
(§) Acceleration	14
1.3. Kinematics of Deformable Bodies	15
1.3.1. Elongation and Shear.....	18
1.3.2. Dilatation and Strain Deviations.....	21
1.3.3. Streamlines and Streamtubes: Local and Convective Acceleration	21
1.3.4. Kinematic (Geometric) Boundary Conditions.....	26
1.4. Supplements to and Applications of Point and Rigid-Body Kinematics	27
1.4.1. The Velocity Diagram of Plane Motion.....	27
1.4.2. Kinematics of the Planetary Gear Train.....	28
1.4.3. The Universal Joint (after Kardan).....	29
1.4.4. Central Motion (The Kepler Problem): Polar Coordinates.....	31
1.5. Supplements to and Applications of Deformation Kinematics	33
1.5.1. The Uniaxial Homogeneous Deformation.....	33
1.5.2. The Natural Coordinates of the Streamline.....	34
1.5.3. The Strain Tensor. The Plane Strain State	36
(§) The Tensor Property of the Strain Matrix.....	36
(§) The Principal Axes Transformation, Mohr's Circle.....	37
1.6. Conservation of Mass: The Continuity Equation	38
1.6.1. Stationary Flow Through a Conical Pipe: Eulerian and Lagrangean Representations.....	44
1.7. Exercises A 1.1 to A 1.8 and Solutions.....	46
2. Statics, Systems of Forces, Hydrostatics.....	55
2.1. Forces, Body-Forces, Tensions, Stresses, Equilibrium.....	55
2.1.1. Stresses in a Tensile Rod: Mohr's Circle.....	57
2.1.2. Plane State of Stress: Mohr's Circle.....	61
2.1.3. General State of Stress.....	64
2.1.4. Mean Normal Stress and Stress Deviations.....	68

2.2.	Systems of Forces	70
2.2.1.	The Plane Force System: Computational and Graphic Reduction, Conditions of Equilibrium.....	76
(§)	Example: Support Reactions of an In-Plane Loaded Structure	78
2.2.2.	Symmetry of the Stress Tensor.....	79
2.2.3.	The Parallel Force System: Center of Forces, Center of Gravity (Centroids), Static Moments.....	80
2.3.	Hydrostatics.....	84
2.3.1.	Fluid Under Gravity.....	85
(§)	Incompressible Fluid.....	85
(§)	A Linear Compressive Spring.....	87
(§)	A Nonlinear Spring	88
2.3.2.	Pressurized Fluids.....	88
(§)	Principle of the Hydraulic Pump.....	89
(§)	Vessels and Pipes	90
2.3.3.	The Gravitational Hydrostatic Pressure in Open Containers.....	92
(§)	A Flat Horizontal Base of Area A	92
(§)	A Plane Retaining Wall of Area A	92
(§)	Circular Cylindrical Surface (Fig. 2.23).....	95
(§)	Hydrostatic Pressure on a Spherical Surface (Fig. 2.24) ...	96
(§)	Hydrostatic Loading of a Doubly Curved Surface.....	97
(§)	Illustrative Example: Uplift.....	98
2.3.4.	The Hydrostatic Buoyancy.....	99
(§)	The Upright Floating Cylinder: Stability (Fig. 2.26).....	101
(§)	Floating Cylinder with Horizontal Axis: Stability.....	102
(§)	Buoyancy	103
2.4	Moments of Inertia of a Plane Area A and Their Rules of Transformation.....	104
(§)	Moments of Inertia About Parallel Axes.....	105
(§)	Moments of Inertia About Rotated Axes (Mohr's Circle)	106
(§)	The Ellipse of Inertia.....	107
(§)	Example: The Central Ellipse of a Rectangle, $A = B \times H$	108
2.5.	Statics of Simple Structures	108
2.5.1.	Beams and Frames.....	109
2.5.1.1.	Local Equilibrium of a Plane Arch and Plane Beam Element (Fig. 2.31).....	114
2.5.1.2.	Straight Beams, Force and Funicular Polygon.....	119
(§)	The Cantilever Beam.....	119
(§)	The Hinged-Hinged Beam.....	121
(§)	Illustrative Example: Eccentric Axial Force.....	123
(§)	The Graphic Solution by Means of the Force and Funicular Polygon	125
(§)	Continuous (Multispan) Beams.....	129
2.5.1.3.	Influence Lines	129
2.5.1.4.	Plane Frames and the Three-Hinged Arch.....	132
2.5.1.5.	Two Statically Determinate Stress States.....	134
(§)	Bending Stresses in a Sandwich Cross-Section.....	134
(§)	Torsional Shear Stresses in a Thin-Walled Tube	135
2.5.2.	Trusses.....	136
2.5.2.1.	Planar Trusses.....	140
2.5.3.	Statics of Flexible Cables (and Chains).....	143
2.6.	Exercises A 2.1 to A 2.15 and Solutions.....	147
3.	Mechanical Work, Power, Potential Energy.....	163
3.1.	Work and Power of Single Forces and Couples	163
3.1.1.	Example: The Work of Gravity Forces.....	165

3.1.2. Example: The Work of a Couple.....	165
3.2. Power Density, Stationary and Irrotational Forces, Potential Energy.....	166
3.3. Potential Energy of External Forces.....	169
3.3.1. Homogeneous and Parallel Gravity, Potential of the Dead Weight.....	169
3.3.2. Central Force Field with Point Symmetry.....	170
3.4. Potential Energy of Internal Forces.....	170
3.4.1. The Elastic Potential of the Hookean Solid (Linear Spring).....	173
(§) Example: A Simple Truss.....	175
3.4.2. The Barotropic Fluid.....	177
3.5. Lagrangean Representation of the Work of Internal Forces, Kirchhoff's Stress Tensor.....	177
3.6. Exercises A 3.1 to A 3.2 and Solutions.....	180
4. Constitutive Equations.....	183
4.1. The Elastic Body, Hooke's Law of Linear Elasticity.....	183
4.1.1. The Linear Elastic Body, Hooke's Law.....	184
(§) The Bending Test.....	189
(§) The Torsional Test.....	191
4.1.2. A Note on Anisotropy.....	193
(§) Plane Stress State.....	193
(§) Transverse Isotropy Lateral to the x Axis.....	194
4.1.3. A Note on Nonlinearity.....	194
4.2. The Visco-Elastic Body.....	197
4.2.1. Newtonian Fluid.....	198
(§) Illustrative Example.....	199
(§) The One-Dimensional Viscous Model.....	201
4.2.2. Linear Visco-Elasticity.....	202
(§) The Kelvin-Voigt Body.....	202
(§) Maxwell Fluid.....	204
(§) The Multiple-Parameter Linear Visco-Elastic Body.....	206
(§) General Linear Viscoelasticity.....	208
4.2.3. A Nonlinear Visco-Elastic Material.....	209
(§) Example: The Creep Collapse of a Tensile Rod.....	211
4.3. The Plastic Body.....	212
4.3.1. The Rigid-Plastic Body.....	212
4.3.2. The Elastic-Plastic Body.....	215
4.3.3. The Visco-Plastic Body.....	220
4.4. Exercise A 4.1 and Solution.....	222
5. Principle of Virtual Work.....	225
5.1. Example: The Three-Hinged Arch.....	228
5.2. Influence Lines of Statically Determinate Structures.....	230
5.3. Conservative Mechanical Systems.....	233
5.3.1. Differential Equation of the Deflection of a Linear Elastic Beam.....	234
5.3.2. The von Karman Plate Equations.....	237
5.4. Principle of Complementary Virtual Work.....	240
5.4.1. Castigliano's Theorem and Menabrea's Theorem.....	242
(§) The Linear Elastic, Thin, and Straight Rod.....	244
(§) A Plane Truss with a Single Internal Static Indeterminacy (Fig. 5.7).....	245
(§) A Double-Span Beam Loaded According to Fig. 5.9.....	246
(§) Deflection of a Uniformly Loaded Cantilever.....	247
(§) The Plane Snap Under the Action of Tip Forces.....	249
5.4.2. Betti's Method.....	249

(§)	Thin-Walled Structures Free of Any Torsion	250
(§)	The Cantilever of Fig. 5.11	250
(§)	The Cantilever with an Additional Simple Support, Fig. 5.13	251
5.4.3.	Transformation of the Principles of Minimum Potential and Complementary Energy	252
5.5.	Exercises A 5.1 to A 5.4 and Solutions	253
6.	Selected Topics of Elastostatics	257
6.1.	Continuum Theory of Linearized Elastostatics	259
(§)	The One-Dimensional Problems of Linear Elasticity	261
(§)	Shrink Fit	262
6.1.1.	Thermoelastic Deformations	263
(§)	The Complementary Energy of a Thermally Loaded Rod	265
(§)	Example: A Single-Span Redundant Beam	267
(§)	Maysel's Formula of Thermoelasticity	268
(§)	A Hollow Sphere with Point Symmetry and a Thick- Walled Cylinder with Axial Symmetry, Maysel's Formula	269
6.1.2.	Saint Venant's Principle	270
6.1.3.	Stress and Strain Hypotheses	272
(§)	Principal Normal Stress Hypothesis	272
(§)	Hencky-von Mises Energy Hypothesis	272
(§)	Mohr-Coulomb Stress Hypothesis	273
(§)	The Concept of Allowable Stress	273
6.2.	Rods and Beams with Straight Axes	273
(§)	Normal Force and Bending Moments	274
6.2.1.	Shear Stresses and Deformations due to a Shear Force	276
(§)	Rectangular Cross-Section	278
(§)	Maximal Shear in an Elliptic or Circular Cross-Section	278
(§)	Equation (6.58) when Applied to the T Cross-Section	278
(§)	Example: Deflection of a Cantilever in Shear Bending	282
6.2.2.	Mohr's Method of Calculating Deflections	282
(§)	Mohr's Analytic Method Applied to the Cantilever of Fig. 6.9	284
(§)	Mohr's Graphic Method Applied to the Cantilever of Fig. 6.9	285
(§)	Influence Lines of Deformations by Mohr's Method	286
(§)	Mohr's Method	287
(§)	The Multispan Beam of Fig. 6.13	288
6.2.3.	Thermal Stresses in Beams	289
(§)	The Single-Span, Simply Supported Beam	291
(§)	A Redundant Single-Span Beam	292
6.2.4.	Torsion	293
6.2.4.1.	Thin-Walled, Single- and Multiple-Cell Cross- Sections	293
6.2.4.2.	Thin-Walled Open Cross-Sections	299
(§)	Torsion of a Thin-Walled Bar with Rectangular Cross-Section	300
(§)	Generalization of Eq. (6.134)	303
(§)	Constrained Warping	304
(§)	The Cantilever with a C-Profile of Fig. 6.17	307
6.2.4.3.	Torsion of Elliptic and Circular, Full and Hollow Cylinders	310
(§)	Elliptic Cross-Section	312
6.2.4.4.	Torsion of a Notched Circular Shaft	314
6.2.4.5.	Prandtl's Membrane and an Electric Analogy	317
6.3.	Multispan Beams and Frames	318
(§)	The Force Method of the Multispan Beam	318

(§)	The Deformation Method	319
(§)	The Deformation Method Applied to Frames	321
6.3.1.	The Planar Single-Story Frame	323
6.4.	Plane-Curved Beams and Arches	324
(§)	The Complementary Energy of the Curved Beam	328
6.4.1.	Slightly Curved Beams and Arches	329
(§)	The Slightly Curved Parabolic Arch of Fig. 6.27	329
(§)	The Slightly Curved Ring	331
(§)	Spinning Rings	332
6.5.	In-Plane Loaded Plates	333
6.5.1.	The Semiinfinite Plate	335
(§)	The Boussinesq Problem	337
(§)	The Stress Function in the Case of a Tangential Single Force	338
6.5.2.	Stationary Spinning Disks	339
6.5.3.	The Infinite Plate with a Circular Hole: Kirsch's Problem	341
6.5.4.	Thermal Membrane Stresses in Plates	343
6.6.	Flexure of Plates	343
6.6.1.	Axisymmetric Flexure of Circular Kirchhoff Plates	348
6.6.2.	The Infinite Plate Strip	349
6.6.3.	The Rectangular Plate with Four Edges Simply Supported	350
6.6.4.	Thermal Deflection of Plates	351
(§)	A Plate of Quadratic Planform	352
(§)	The Infinite Plate	352
6.7.	Thin Shells of Revolution	353
(§)	Membrane Stresses	354
(§)	Bending Perturbation of the Membrane State	355
6.7.1.	Thin Circular Cylindrical Shells	358
(§)	The Open Cylindrical Storage Tank	359
6.7.2.	The Spherical Dome of Fig. 6.39	360
6.7.3.	Thermal Stresses in Thin Shells of Revolution	362
(§)	The Radial Thermal Expansion of a Circular Cylindrical Shell	363
6.8.	Contact Problems (The Hertz Theory)	364
6.9.	Stress-Free Temperature Fields, Fourier's Law of Heat Conduction	368
6.10.	The Elastic-Visco-Elastic Analogy	370
6.10.1.	The Creeping Simply Supported Single-Span Beam	371
6.10.2.	The Heated Thick-Walled Pipe (Fig. 6.42)	372
6.11.	Exercises A 6.1 to A 6.22 and Solutions	375
7.	Dynamics of Solids and Fluids, Conservation of Momentum of Material and Control Volumes	397
7.1.	Conservation of Momentum	400
7.2.	Conservation of Angular Momentum	404
7.3.	Applications of Control Volumes	408
7.3.1.	Stationary Flow Through an Elbow	408
(§)	The Plane Elbow	410
(§)	A Nozzle with a Straight Axis	410
(§)	Plane U-Shaped Elbow	410
7.3.2.	Thrust of a Propulsion Engine	411
7.3.3.	Euler's Turbine Equation	413
7.3.4.	Water Hammer in a Straight Pipeline	415
7.3.5.	Carnot's Loss of Pressure Head	416
7.4.	Applications to Rigid-Body Dynamics	418
7.4.1.	The Rolling Rigid Wheel	420
7.4.2.	Cable Drive	422
7.4.3.	Dynamics of the Crushing Roller (Fig. 1.3)	424

7.4.4.	Swing Crane with a Boom.....	424
7.4.5.	Balancing of Rotors.....	426
7.4.6.	The Gyro-Compass.....	426
7.4.7.	The Linear Oscillator.....	428
(§)	Periodic Forcing Function, $F(t) = F(t + T_e)$	432
(§)	Excitation by a Nonperiodic Forcing Function.....	436
(§)	Representation of the Motion in the Phase Plane ($\xi, d\xi/dt$).....	437
(§)	Some Structural Models of the Linear Oscillator.....	442
(§)	Linear Torsional Vibrations.....	442
7.4.8.	Nonlinear Vibrations.....	443
(§)	Motion of a Planar Pendulum.....	443
(§)	SDOF-System with Dry Friction.....	446
7.4.9.	Linear Elastic Chain of Oscillators.....	449
(§)	The Residual Method of Holzer and Tolle.....	450
(§)	Dunkerly's Formula.....	452
(§)	Natural Modes.....	453
(§)	The Amplitude Frequency Response Functions of the Two- Mass System.....	453
7.5.	Bending Vibrations of Linear Elastic Beams.....	454
(§)	Natural and Forced Vibrations of a Slender, Hinged-Hinged, Single-Span Beam.....	456
7.6.	Body Waves in the Linear Elastic Solid.....	458
(§)	The Longitudinal Wave.....	459
(§)	The Shear Wave.....	460
7.7.	Exercises A 7.1 to A 7.12 and Solutions.....	460
8	First Integrals of the Equations of Motion,	
	Kinetic Energy.....	475
8.1.	The Power Theorem and Kinetic Energy.....	475
8.2.	Conservation of Mechanical Energy.....	476
8.3.	Kinetic Energy of a Rigid Body.....	477
8.3.1.	Pure Rotation of the Rigid Body About a Fixed Point O	478
8.3.2.	Rotation About an Axis e_a Fixed in Space.....	478
8.4.	Conservation of Energy in SDOF-Systems.....	479
8.4.1.	Motion of a Linear Oscillator After Impact (Fig. 8.1).....	479
8.4.2.	The Basic Vibrational Mode of a Linear Elastic Beam.....	480
8.4.3.	Acceleration of a Motorized Vehicle.....	482
8.4.4.	The Turning Points of a Nonlinear, Dry-Friction Oscillator.....	482
8.5.	Bernoulli's Equation of Fluid Mechanics.....	483
8.5.1.	Stationary Flow with Power Charging or Discharging.....	487
8.5.2.	Velocity of Efflux from a Small Aperture in an Open Vessel or a Pressurized Tank (Fig. 8.5).....	488
8.5.3.	Stationary Flow Round an Immersed Rigid Body at Rest.....	489
8.5.4.	Inviscid Flow Along a Rigid Wall.....	490
8.5.5.	Pressure in a Pipe Measured by a Gully.....	491
8.5.6.	Prandtl's Tube and Pitot's Tube.....	492
8.5.7.	Transient Flow in a Drain Pipe Controlled by a Cock.....	493
8.5.8.	Free Vibrations of a Fluid in an Open U-Shaped Pipe.....	494
8.5.9.	Lossless Flow Through a Diffusor.....	496
8.5.10.	A Bernoulli-Type Equation in a Rotating Reference System.....	497
(§)	Example: Segner's Water Wheel.....	499
8.6.	Remarks on the First Law of Thermodynamics (Conservation of Energy).....	501
8.7.	Exercises A 8.1 to A 8.5 and Solutions.....	503