

COLLOQUIA MATHEMATICA
SOCIETATIS JÁNOS BOLYAI, 10.

INFINITE AND FINITE SETS

Vol. I.

Edited by:

A. HAJNAL
R. RADO
VERA T. SÓS

NORTH-HOLLAND

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SOCIETATIS JÁNOS BOLYAI, 10.

INFINITE AND FINITE SETS

to **PAUL ERDŐS**
on his 60th birthday

Vol. I.

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R. RADO
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PREFACE

In the summer of 1972 the János Bolyai Society decided to organize a "Colloquium on Infinite and Finite Sets" in honour of Professor Paul Erdős for his 60th birthday. These volumes contain the papers presented at the Colloquium which was held at Keszthely, Hungary from June 25th to July 1st, 1973.

It is beyond the scope of this short introduction to describe adequately, for those who do not already know it, either the significance of Professor Erdős' own mathematical work or of his enormous influence on the work and development of other, especially younger, mathematicians. We are particularly aware and appreciative of his help to Hungarian mathematics, but we also know that he has become something of a legend throughout the whole mathematical world. We can only hope that the contents of these volumes will prove to be an adequately significant birthday present from us.

There were 218 participants at the Colloquium from 19 different countries, including 124 from outside Hungary. Apart from a one-hour address by Paul Erdős, there were 107 other talks. These were all restricted to 20 minutes duration and were given in three parallel sections. Section A was devoted to infinite sets and Sections B and C dealt mainly with finite combinatorics. However, when organizing the material for these volumes we decided not to classify papers in this way and these appear according to the alphabetical order of the names of the authors. The Proceedings contain the written versions of most of the talks which were given at the conference. We have also included papers from authors who could not for some reason or other attend the Colloquium. Since the whole meeting was dedicated to Paul Erdős we decided to omit all the personal dedications by the authors of the papers.

All papers appearing in these volumes were refereed. It was our policy to consider for publication only papers on *finite* or *infinite sets* and as a result of this some good papers dedicated to Professor Erdős could not be accepted.

The local organizing committee consisted of A. Hajnal (Chairman), G. Fodor and V.T. Sós (Co-Chairmen), L. Babai (Secretary), A. Ádám, P. Hamburger, I. Juhász, G. Katona, M. Makkai, A. Máté, E. Szemerédi.

We would like to express our thanks to the I.M.U. for cosponsoring the meeting and for delegating to our Organizing Committee Professors R. Rado (Reading) and E. Specker (Zurich).

The Editor

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SCIENTIFIC PROGRAM

June 25. Monday

7.⁰⁰ p.m.

Opening ceremonies

June 26. Tuesday

Morning

Section A.

9.⁰⁰ – 9.²⁰ a.m.

J. Myhill: Constructive set theory

9.³⁰ – 9.⁵⁰ a.m.

E. Specker: Combinatorial problems

Section B.

9.⁰⁰ – 9.²⁰ a.m.

E.G. Whitehead, jr.: Locally maximal sum-free sets

9.³⁰ – 9.⁵⁰ a.m.

R. Faudree, jr. – R. Schelp: Ramsey-type problems

10.⁰⁰ – 10.²⁰ a.m.

H.L. Abbott: Lower bounds for certain Ramsey numbers

11.⁰⁰ – 11.²⁰ a.m.

J. Yackel: Inequalities and asymptotic bounds for Ramsey numbers II.

11.³⁰ – 11.⁵⁰ a.m.

A. Gyárfás: Ramsey-type problems

Section C.

9.⁰⁰ – 9.²⁰ a.m.

N.W. Sauer: Derivatives of n -graphs (Hypergraphs)

9.³⁰ – 9.⁵⁰ a.m.

Zs. Baranyai: On the factorization of complete hypergraphs

10.⁰⁰ – 10.²⁰ a.m.

A. Ádám: On some combinatorial questions

11.⁰⁰ – 11.²⁰ a.m.

A. Racsmany: On the maximal number of lattice points with distances d .

11.³⁰ – 11.⁵⁰ a.m.

T. Tarján: On complexity of graphs realizing truth functions

June 26. Tuesday

Afternoon

Section A.

- 4.⁰⁰ — 4.²⁰ p.m. J. Baumgartner: Ineffability properties of cardinals I.
4.³⁰ — 4.⁵⁰ p.m. A. Hajnal: Weak partition relations
5.⁰⁰ — 5.³⁰ p.m. J.A. Larson: Partition theorems for certain ordinal products
6.⁰⁰ — 6.²⁰ p.m. E.C. Milner: Denumerable partition relations
6.³⁰ — 7.⁰⁰ p.m. F. Galvin: An elementary proof of $\omega_1 \rightarrow (\alpha)_k^2$

Section B.

- 4.⁰⁰ — 4.²⁰ p.m. A. Etourneau: Construction et connexité des graphes planaires ayant 12 sommets de degré 15 et $n - 12$ sommets de degré 5
5.⁰⁰ — 5.²⁰ p.m. I.T. Jakobsen: On graphs critical with respect to edge coloring
6.⁰⁰ — 6.²⁰ p.m. H. Sachs: Über einige Untersuchungen in der Theorie der Vierfarbenprobleme
6.³⁰ — 6.⁵⁰ p.m. H.J. Presia: Ein Satz aus der Theorie der Vierfarbenprobleme

Section C.

- 4.⁰⁰ — 4.²⁰ p.m. M. Rupp: Application of graphs to matrix inversion
4.³⁰ — 4.⁵⁰ p.m. B. Zelinka: Tolerance relation on sets
5.⁰⁰ — 5.²⁰ p.m. G.J. Simmons: Command graphs
6.⁰⁰ — 6.²⁰ p.m. H.J. Walther: Einige Anmerkungen zu einer Arbeit von B. Grünbaum
6.³⁰ — 6.⁵⁰ p.m. J. Sheehan: An external problem in finite graph theory

June 27. Wednesday

Morning

Section A.

- 9.³⁰ – 9.⁵⁰ a.m. R. Laver: On the consistency of Borel's conjecture
10.⁰⁰ – 10.²⁰ a.m. K. Devlin: A generalization of Martin's axiom
11.⁰⁰ – 11.²⁰ a.m. L. Bukovszky: Strong partition relations, iterated
ultrapowers and the distributivity in
Boolean algebras
11.³⁰ – 11.⁵⁰ a.m. J.D. Halpern – P.E. Howard: Cardinal addition and
the axiom of choice

Section B.

- 9.⁰⁰ – 9.²⁰ a.m. E.G. Straus: Some combinatorial theorems in
Euclidean geometry
9.³⁰ – 9.⁵⁰ a.m. B.L. Rothschild: Euclidean Ramsey theorems II.
10.⁰⁰ – 10.²⁰ a.m. A. Schinzel: Points and lines in the n -dimensional
space
11.⁰⁰ – 11.²⁰ a.m. J. Nešetřil: Ramsey type theorems
11.³⁰ – 11.⁵⁰ a.m. W. Deuber: Generalization of Ramsey's theorem for
structures

Section C.

- 9.⁰⁰ – 9.²⁰ a.m. A. Bouchet: Realizability in the plane of abstract
intersection sequences
9.³⁰ – 9.⁵⁰ a.m. R.B. Eggleton: Graphic sequences and graphic
polynomials
10.⁰⁰ – 10.²⁰ a.m. J.M. dos S. Simoes-Pereira: On uniquely (m, n) -
partitionable graphs
11.⁰⁰ – 11.²⁰ a.m. A. Recski: On partitional matroids with applications
11.³⁰ – 11.⁵⁰ a.m. G. Székely: Random walks and random trees

June 27. Wednesday
Afternoon

Section A.

- 4.⁰⁰ — 4.²⁰ p.m. R. McKenzie — J.D. Monk: On the automorphism groups of Boolean algebras
- 4.³⁰ — 4.⁵⁰ p.m. I. Juhász: Ramification systems in topology
- 5.⁰⁰ — 5.³⁰ p.m. A. Pultr: Big cardinals and some problems of representations of categories
- 6.⁰⁰ — 6.²⁰ p.m. Z. Hedrlin: Results different in finite and infinite algebra
- 6.³⁰ — 6.⁵⁰ p.m. A. Máté — A. Hajnal: Embedding disjoint stationary sets into stationary sets

Section B.

- 4.⁰⁰ — 4.²⁰ p.m. M.D. Plummer — L. Lovász: On bi-critical graphs
- 4.³⁰ — 4.⁵⁰ p.m. M. Las Vergnas: Sur les facteurs d'un graphe
- 5.⁰⁰ — 5.²⁰ p.m. S. Porubsky: On exactly covering systems
- 6.⁰⁰ — 6.²⁰ p.m. M. Sulyok — J. Komlós — E. Szemerédi: A general method in combinatorial number theory
- 6.³⁰ — 6.⁵⁰ p.m. J.E. Graver: Maximum depth problem for indecomposable exact covers

Section C.

- 4.⁰⁰ — 4.²⁰ p.m. D. Newman: Waring problem for polynomials
- 4.³⁰ — 4.⁵⁰ p.m. E. Szemerédi: On combinatorics
- 5.⁰⁰ — 5.²⁰ p.m. S.L.G. Choi: Problems and results on finite sets of integers
- 6.⁰⁰ — 6.²⁰ p.m. J. Wallis: Supplementary difference sets
- 6.³⁰ — 6.⁵⁰ p.m. V. Neumann-Lara: On regular graphs

June 29. Friday

Morning

Section A.

9. ⁰⁰ – 9. ²⁰ a.m.	N.G. de Bruijn: Set theory with type restrictions
9. ³⁰ – 9. ⁵⁰ a.m.	J. Stern: Large cardinals
10. ⁰⁰ – 10. ²⁰ a.m.	R. Rado: Anti-Ramsey theorems
11. ⁰⁰ – 11. ²⁰ a.m.	C.St.J.A. Nash-Williams: Transversals of infinite set systems
11. ³⁰ – 11. ⁵⁰ a.m.	L. Surányi: On the complement of n -saturated graphs

Section B.

9. ⁰⁰ – 9. ²⁰ a.m.	K. Harting – J. Surányi: Combinatorial and geometrical investigations in elementary number theory
9. ³⁰ – 9. ⁵⁰ a.m.	L. Babai: Automorphism groups of graphs
10. ⁰⁰ – 10. ²⁰ a.m.	D.A. Holton: Vertex stable graphs
11. ⁰⁰ – 11. ²⁰ a.m.	H.A. Jung: Über die Zusammenhangstruktur symmetrischer graphen
11. ³⁰ – 11. ⁵⁰ a.m.	W. Imrich: On graphical regular representation of groups

Section C.

9. ⁰⁰ – 9. ²⁰ a.m.	W. Dörfler: Some results on the reconstruction of graphs
9. ³⁰ – 9. ⁵⁰ a.m.	D.A. Klarner: Sets generated by iteration of operations
10. ⁰⁰ – 10. ²⁰ a.m.	E. Köhler: Über die Konstruktion neuer balanced incomplete block-designs und ihrer graph-theoretischen Verallgemeinerung
11. ⁰⁰ – 11. ²⁰ a.m.	J.A. Bondy: Pancyclic graphs-results and conjectures
11. ³⁰ – 11. ⁵⁰ a.m.	K. Leeb: A Ramsey theorem for Deuber's Z_{mpc} -s.

June 29. Friday

Afternoon

Section A.

- 4.⁰⁰ — 4.⁵⁰ p.m. Problem section
5.⁰⁰ — 5.²⁰ p.m. P. Vojtás — B. Balcar: Remarks on free algebras
5.³⁰ — 5.⁵⁰ p.m. G. Elekes: Partition theorems on set structures

Section B.

- 4.⁰⁰ — 4.²⁰ p.m. E. Jucovic — F. Olejnik: On chromatic and
achromatic numbers of uniform hyper-
graphs
4.³⁰ — 4.⁵⁰ p.m. J.C. Bermond: The circuit hypergraph of a tourna-
ment
5.⁰⁰ — 5.²⁰ p.m. C. Berge: Coloring hypergraphs
6.⁰⁰ — 6.²⁰ p.m. A.A. Zykov: Farbungen und Zusammenziehungen
von Hypergraphen
6.³⁰ — 6.⁵⁰ p.m. B. Toft: Color critical hypergraphs

Section C.

- 4.⁰⁰ — 4.²⁰ p.m. D. Kleitman: Some new results on Dedekind prob-
lems
4.³⁰ — 4.⁵⁰ p.m. G.O.H. Katona: Set systems
5.⁰⁰ — 5.²⁰ p.m. W. Taylor: Combinatorial convexity theorems
6.⁰⁰ — 6.²⁰ p.m. D.E. Daykin: Finite sets
6.³⁰ — 6.⁵⁰ p.m. W. Mader: Extremalprobleme des Zusammenhangs
von Graphen

June 30. Saturday
Morning

Section A.

- 9.⁰⁰ – 9.²⁰ a.m. G. Freud: On a conjecture of P. Erdős
- 9.³⁰ – 9.⁵⁰ a.m. Yu.A. Archangelskij: Application of infinite series
in solving the Euler-Poisson equations
- 10.⁰⁰ – 10.²⁰ a.m. L. Lorch: Some monotonicity properties of polynomials with equally spaced zeros
- 11.²⁰ – 11.⁵⁰ a.m. F. Bagemihl: The present state of ambiguity
- 12.⁰⁰ – 12.²⁰ a.m. E. Harzheim: A combinatorial theorem related to
the Jordan-Brouwer separation theorem

Section B-C.

- 9.⁰⁰ – 9.⁵⁰ a.m. Problem section

Section B.

- 10.³⁰ – 10.⁵⁰ a.m. R.L. Graham: Ramsey theorems for graphs using
many colors
- 11.⁰⁰ – 11.²⁰ a.m. R.K. Guy – J.L. Selfridge: Optimal covering of the
square
- 11.³⁰ – 11.⁵⁰ a.m. V.T. Sós – P. Erdős – M. Simonovits: Ramsey type
theorems for multicolored subgraphs

Section C.

- 10.³⁰ – 10.⁵⁰ a.m. A.J.W. Hilton: Simultaneously independent k -sets
of edges of a hypergraph
- 11.⁰⁰ – 11.²⁰ a.m. J.L. Jolivet: On the join of a family of graphs
- 11.³⁰ – 11.⁵⁰ a.m. J. Körner: On some properties of perfect graphs
- 12.⁰⁰ – 12.²⁰ a.m. I. Bárány: Sperner type theorems

June 30. Saturday
Afternoon

Section A.

- 3.³⁰ – 3.⁵⁰ p.m. D. Kurepa: On the system of all maximal chains
(antichains) of graph
- 4.⁰⁰ – 4.²⁰ p.m. I. Gutman: Some unsolved graph theoretical prob-
lems in quantum chemistry
- 4.³⁰ – 4.⁵⁰ p.m. A.R.D. Matthias: Analytic filters

Section B.

- 3.³⁰ – 3.⁵⁰ p.m. G. Purdy: On the planarity of three-point universal
graphs
- 4.⁰⁰ – 4.²⁰ p.m. J. Spencer: Nonconstructive methods in com-
binatorics
- 4.³⁰ – 4.⁵⁰ p.m. M. Simonovits: On a theorem of P. Erdős

Section C.

- 3.³⁰ – 3.⁵⁰ p.m. F. Escalante: On iterated clique graphs
- 4.⁰⁰ – 4.²⁰ p.m. M. Chein – M. Rivière: *P*-covering indices of graphs
- 4.³⁰ – 4.⁵⁰ p.m. M. Behzad: Two unsolved problems in graph theory

Plenary section

- 5.²⁰ – 6.⁰⁰ p.m. P. Erdős: Problems and results in finite and infinite
combinatorial analysis

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