ESSENTIAL MATHEMATICS FOR COLLEGE PHYSICS

MICHAEL RAM

A Self Study Guide

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to Edna

PREFACE

College physics students quite often have difficulty with the mathematical requirements of the course. Hopefully, the present book will alleviate the problem. Opinions on how this can best be achieved vary. Some feel that mathematical questions are best resolved in the context of the physics course. Others are of the opinion that students should have good mathematical backgrounds prior to taking the course. The latter, would, naturally, be ideal but, unfortunately, it is in general not true.

With this review, we have opted for the middle ground. The book is intended to be used in conjunction and together with the physics text. Since it is directly geared to just the mathematics of college physics, students can quickly and efficiently patch up mathematical deficiencies while keeping up with the physics. With this in mind, we review the contents of the book in chapter 1 and recommend a study schedule guide showing where in the physics course the various mathematical topics are of importance and need to be reviewed. The book is intended as a self study guide and students should be able to review material on their own without teacher assistance. Numerous worked out examples can be found throughout the book with solutions outlined in great detail. Since it is impossible to learn without drilling, many exercises are also included and students are urged to work out as many as possible. The more, the better. Answers are provided in appendix D.

Our discussions of trigonometry and vectors are reasonably comprehensive and no prior knowledge has been assumed. Since trigonometry is essential for vectors, chapter 5 should be reviewed prior to studying chapter 7. Otherwise, all other chapters are quite independent and can be studied in any order you please. A summary of important formulas is given in appendix C.

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Buffalo, New York
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Michael Ram

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CHAPTER ONE

INTRODUCTION
Who Needs the Book and How To Use It

The purpose of the book and how to use it are discussed. The contents of the chapters are summarized and a recommended study schedule is given.

My experience with introductory college physics has shown me that a serious problem faced by many students is lack of adequate mathematical preparation. True, the course is a non-calculus one, but trigonometry, algebra and geometry are still very essential. Unfortunately, many students who need the course do not meet those minimum mathematical pre-requisites, and it is to those students that this book is mainly addressed. Most college physics texts do devote a few pages to so called "Mathematical Review", but it is quite symbolic, no more than lip service really, and does not recognize the severity of the problem.

The math requirements for high school graduation in the United States are usually quite low, and many students who are later faced with the necessity of a college physics course for pursuit of their career goals, quickly discover that they lack many of the needed mathematical tools. There is little precious time available within the already tight physics course schedule to develop those skills, and what the student needs is a quick self study review of the essential mathematics. Hence the title of the book. We do not give a comprehensive presentation, but only review those topics that are absolutely essential for a college physics course. Exceptions, are the three chapters covering "Scientific Notation, Significant Figures and Rounding","Trigonometry" and "Vectors". These subjects are dealt with extensively since we believe that prior exposure of students to the material is quite minimal and inadequate.

The book has been designed as a self-study guide, and the student should have no difficulty covering the material at a reasonable pace, without teacher assistance. With this in mind, considerable effort has been devoted making sure that the discussion is clear and self sufficient. Many examples are included and worked out in great detail. We have tried to anticipate and answer every possible question the student might have. The presentation is precise and, in fact, quite rigorous, even though few proofs are given. Explanations are ellaborate, and little is left to the imagination. Also included are numerous drilling exercises which are an essential ingredient of the study and review process and must not be skipped. As a practical matter, we expect that students will use this guide in parallel with the physics course. The instructor can now teach the physics and refer mathematics questions to this study guide.

The chapters have been written as self contained units, and do not have to be studied in order of presentation but can be covered as required. In fact, since a lot of the material is needed right at the beginning of the physics course, the student may find it useful to review some chapters concurrently. Chapter 2 is quite general and, we believe, so important for calculations, that we recommend it be studied by all students right at the beginning. Notation has been kept close to that used in college physics courses and many examples are, intentionally, similar in mathematical content and form to ones encountered when solving physics problems.

We now give brief summaries of the various chapters, indicating at what stage in the physics course the material is essential. For easy reference, this information is also summarized in Table 1-1 at the end of the chapter.

CHAPTER 2. SCIENTIFIC NOTATION, SIGNIFICANT FIGURES AND ROUNDING:
A very important component of the college physics course consists of solving many exercises and problems. This chapter teaches students how to handle numbers and carry out calculations with less chance of error. The importance of significant figures and rounding is discussed. Scientific (powers of 10) notation is covered in detail, and simple rules are given for converting decimal numerals to scientific notation, and vice versa. Poor understanding of this can be a serious source of error

in calculation. Finally, we give a simple rule for carrying out "order of magnitude" estimates. This is an essential tool. All too often, students accept any number they obtain from a calculation as correct without bothering to check whether the result is at all reasonable. Our rule, when applied, allows the student to quickly determine how big (within a factor of less than 10) the answer should be.

CHAPTER 3. ALGEBRA: This is strictly a review of those aspects of algebra that are of importance for college physics, and is in no way comprehensive. Some of the topics that are covered, however, such as solution of linear algebraic equations in one or two unknowns, are presented quite rigorously. Concepts are carefully defined. The chapter will prove useful in mechanics (statics, kinematics and applications of Newton's laws of motion), electric circuits and in geometrical optics (finding position of images and magnification). In fact, many of the examples and exercises use both notation and data of the kind encountered in mechanics and optics.

CHAPTER 4. GEOMETRY: Again, this is strictly a review chapter, emphasizing only those aspects that are of importance to the college physics course. The presentation is rigorous, but proofs are, in general, avoided. The concept of RADIAN is covered in detail since many may be unfamiliar with it and it is of great importance when discussing circular motion. Students should review this chapter when studying mechanics and optics. Sections 1 through 9 are useful for optics and applications of Newton's laws of motion. Section 10 and 13 are important when considering motion in a circle. The student may find the discussion of ellipses (section 14) helpful when studying Kepler's laws of planetary motion.

CHAPTER 5. TRIGONOMETRY: This chapter is a must. Many students come out of high school not knowing how to handle the basic trigonometric functions. Even though the subject is really elementary, it is not always given sufficient attention in schools, and students often go through the material without really understanding what it's all about. All students must review this chapter before, or concurrently, with mechanics. Many simple exercises are given, and the serious student should work diligently through all these drills, which is really what the exercises are. The student who masters the chapter will find the going much easier when studying vectors, mechanics, electric fields and optics. Also mentioned at the end of the chapter are the useful approximations $\sin\theta \cong \tan\theta \cong \theta$ valid for small angles θ when expressed in radians.

CHAPTER 6. ANALYTICAL GEOMETRY: Graphs are very important in illustrating the behaviour of functions. What graphs mean and how that are plotted is carefully reviewed. The important equation of a straight line is discussed in detail. Also included is the solution of simultaneous equations by graphical methods. The student taking the introductory physics lab course will find this chapter particularly useful.

CHAPTER 7. VECTORS: This is a very crucial chapter. The material is not usually covered in high school, and the college physics course may be the student's first exposure to it. The subject is not difficult, but it requires considerably more coverage than the handful of pages devoted to it in college physics texts. The extensive and detailed discussion we present essentially removes this deficiency. Particular emphasis is given to resolution of vectors into cartesian components, and the important concept of unit vector is discussed. Students must study this chapter right at the beginning of the year since the material is essential for mechanics. Understanding and learning to use vectors will pay

dividends throughout the physics course.

CHAPTER 8. SPECIAL FUNCTIONS: Students should review the section on periodic functions when learning about simple harmonic motion and waves. All the mathematics needed is clearly detailed there. The section on exponential functions is useful when covering RC and RL circuits in electricity. Logarithms are important in sound when the decibel is discussed. Special emphasis is given to graphical representation of the functions.

CHAPTER 9. BINOMIAL THEOREM. ARITHMETIC AND GEOMETRIC PROGRESSIONS: This material is useful in many applications encountered through the college physics course. Of special interest are the discussions of the important approximations $(1+x)^b \cong 1+bx$ and $\frac{1}{1+x} \cong 1-x$, valid for small values of x. Students should become familiar with the contents of this chapter for easy reference when needed in the course.

We have not reviewed the subject of error analysis since our experience is that the material is usually adequately covered in lab manuals. Also, little is said about scientific calculators since students do not seem to have problems using them, and it would be hard to improve on the instruction manuals provided by manufacturers of these wonderful devices.

TABLE 1-1. STUDY SCHEDULE GUIDE

CHAPTER	WHEN TO STUDY
2	General; right at the beginning of course. Essential for all calculations.
3	Particularly important for problems in kinematics, statics, electric circuits and optics where solutions of simultaneous algebraic equations in more than one unknown are encountered.
4	Important in mechanics and optics. Section 1 through 9 are useful in mechanics when dealing with applications of Newton's laws and in optics. Sections 10 and 13 are very important when circular motion is covered. Section 14 will be useful in discussions of Kepler's laws of planetary motion.
5	This material is essential for vectors, which are usually introduced at the beginning of the course. Since time is very short, the student may wish to cover chapters 5 and 7 concurrently. Also important for electric fields and optics.
6	Graphical representations are important throughout physics, and it would be wise to review this chapter as soon as possible. Especially useful for lab part of course.
7	Study right at the beginning of the course with mechanics, and review again when learning about electric fields.
8	Section 1 is important for simple harmonic motion and waves. Section 2 is useful for RC and RL circuits in electricity, and section 3 will be useful in sound when the decibel is discussed.
9	Uses of this material are scattered in many applications throughout the physics course. Students should become familiar with contents for easy reference when needed.

CHAPTER TWO

SCIENTIFIC NOTATION SIGNIFICANT FIGURES AND ROUNDING

The material of this chapter is essential for computations, both in class and lab, and is needed throughout the physics course. The chapter shows how to handle numbers and carry out calculations with less chance of error. A simple, but very important, rule for "order of magnitude" estimates is given.

In science, one routinely encounters very, very small numbers and very, very large ones. For example, the mass of an electron is 0.00000000000000000000000000000091085 kg and that of the sun, 1,991,000,000,000,000,000,000,000,000 tons. In this form, these numbers are somewhat difficult to write down and handling them is also very awkward. The problem can be resolved by using SCIENTIFIC NOTATION. Since this notation is based on powers of 10, we first review that subject. A more extensive treatment of exponents and power functions is given in Chapter 8.

2-1. POWERS OF 10: By definition, if n is a positive integer,

$$10^n = 10 \cdot 10 \cdot 10 \cdot \ldots\ldots\ldots \cdot 10 \qquad (2\text{-}1)$$

n factors of 10

10^n is read "10 to the power of n". 10 is called the BASE and n, the EXPONENT. For example,

$10^1 = 10$, $10^2 = 10 \cdot 10 = 100$, $10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$, etc.

Consider

$$10^2 \times 10^3 = (10 \cdot 10) \times (10 \cdot 10 \cdot 10) = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5 = 10^{2+3}$$

and

$$10^5 \times 10^1 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \times (10) = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^6 = 10^{5+1} .$$

These are special cases of the following general rule:

IF m AND n ARE POSITIVE INTEGERS

$$10^m \times 10^n = 10^{m+n}$$

THAT IS, TO MULTIPLY POWERS OF 10, JUST ADD THE EXPONENTS.

This rule is true for multiplication of any number of powers of 10. For example

$$10^2 \times 10^4 \times 10^9 \times 10^3 = 10^{2+4+9+3} = 10^{18}$$

The concept of powers of 10 is extended to negative powers by defining

$$10^{-n} = \frac{1}{10^n} \qquad (2\text{-}2)$$

where n is a positive integer. Thus

$10^{-1} = \frac{1}{10} = 0.1$, $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$, etc.

For completeness, one also defines the zeroth power of 10 as follows:

$$10^0 = 1 \qquad (2\text{-}3)$$

With these definitions, you should have no difficulty in verifying that the rule we have given for multiplying powers of 10 holds for all integral powers, positive, negative or zero. That is[1]

$$10^m \times 10^n = 10^{m+n} \quad , \quad (m,n = 0, \pm 1, \pm 2, \pm 3, \ldots.) \qquad (2\text{-}4)$$

Examples: Using the definitions (2-1), (2-2) and (2-3) calculate $10^{-2}\times10^4$, $10^0\times10^2$ and $10^2\times10^{-3}\times10^0\times10^5$. Verify that Eq. (2-4) is satisfied in each case.

Solutions:

$$10^{-2}\times10^4 = \frac{1}{10^2}\times10^4 = \frac{10\cdot10\cdot10\cdot10}{10\cdot10} = 10\cdot10 = 10^2 = 10^{-2+4}.$$

$$10^0\times10^2 = 1\times10^2 = 10^2 = 10^{0+2}.$$

$$10^2\times10^{-3}\times10^0\times10^5 = 10^2\times\frac{1}{10^3}\times1\times10^5 = \frac{(10\cdot10)\cdot1\cdot(10\cdot10\cdot10\cdot10\cdot10)}{10\cdot10\cdot10}$$

$$= 10\cdot10\cdot10\cdot10 = 10^4 = 10^{2-3+0+5}.$$

Exercises 2-1:

Using the definitions (2-1), (2-2) and (2-3), calculate the following and verify that Eq. (2-4) is satisfied in each case:

1. $10^4\times10^3$
2. $10^4\times10^{-3}$
3. $10^{-5}\times10^5$
4. $10^0\times10^5$
5. $10^3\times10^0$
6. $10^{-7}\times10^3$
7. $10^2\times10^4\times10^3$
8. $10^6\times10^{-2}\times10^{-5}$
9. $10^{-3}\times10^{-7}\times10^{10}$

From Eq. (2-2) we get, after cross multiplication

$$\frac{1}{10^{-n}} = 10^n \qquad (2\text{-}5)$$

where n is a positive integer. In many applications, one encounters products and quotients of powers of 10. Relations (2-1) through (2-5)

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[1] The notation m,n = 0,±1, ±2, ±3, ... indicates that m and n can be any positive or negative integers, including zero.