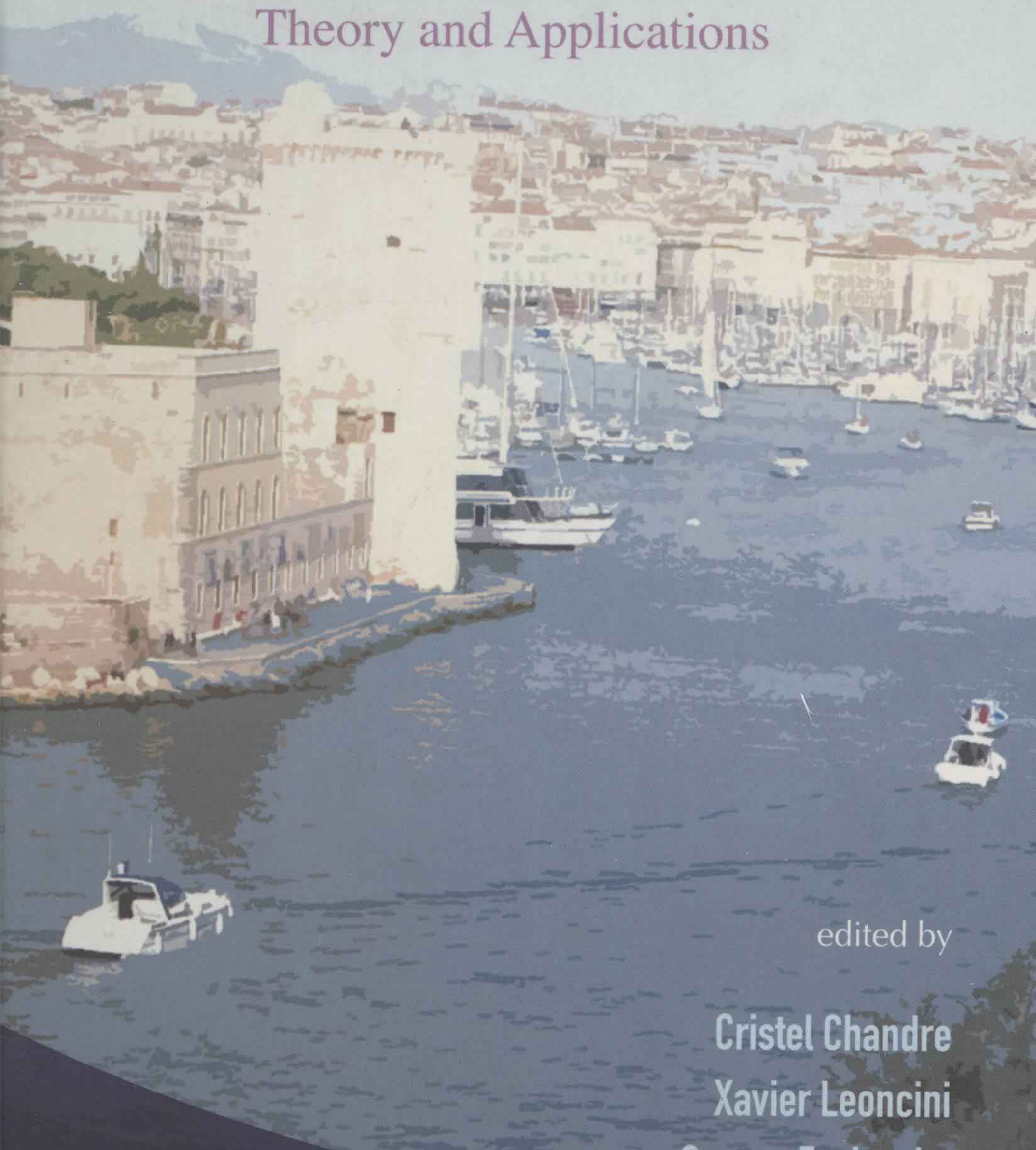
CHAOS, COMPLEXITY AND TRANSPORT

Theory and Applications



edited by

Cristel Chandre
Xavier Leoncini
George Zaslavsky

World Scientific

CHAOS, COMPLEXITY AND TRANSPORT

Theory and Applications

Proceedings of the CCT '07

Marseille, France 4 – 8 June 2007

edited by

Cristel Chandre
CNRS, France

Xavier Leoncini
Aix-Marseille Université, France

George Zaslavsky
New York University, USA

World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

CHAOS, COMPLEXITY AND TRANSPORT
Theory and Applications
Proceedings of the CCT'07

ISBN-13 978-981-281-879-9
ISBN-10 981-281-879-0

Printed in Singapore by World Scientific Printers

CHAOS, COMPLEXITY AND TRANSPORT

Theory and Applications

Proceedings of the CCT '07

PREFACE

Chaos and turbulence are ubiquitous features of physical systems. Their manifestations are very diverse and not always well understood. Improving knowledge in this field is not only important for our apprehension of non-linear physics but also essential to tame or control their behaviours. Moreover the interdisciplinary character of such phenomena which were observed notably in fluid dynamics, atomic physics, plasma physics, accelerator physics, celestial mechanics, condensed matter, among others, makes research in this field quite peculiar as any advance in one direction may have strong repercussions and consequences in others (Chaos!).

As a shared interest one can notably consider transport properties. These are often characterised by Lévy-type processes, strange (fractal) kinetics, intermittency, etc. Typically one finds that portions of the trajectories are almost regular for quite a long time. This phenomenon (Lévy flight) gives rise to strong memory effects. History comes into play, and thus rules out the traditional use of Markov processes to model transport. What makes these properties so special is that they are associated with rare events in time but are crucial for the physical behaviour of the system. One may also emphasise on the important role played by coherent structures and their impact on transport.

The classical approach to study transport dynamics has been complemented by various novel approaches, based on either the development of new physical and mathematical ideas and on the implementation of sophisticated numerical codes. The concept of Lévy processes, fractional kinetics and anomalous transport have proved to be extremely important from a conceptual point of view indicating a new direction in the non-linear dynamics. However, many questions are still open, from both a conceptual and an applied point of view. For example, the role of chaotic advection in complex situations has still to be properly addressed. The aim is to understand which (if any) of the properties commonly attributed to the processes of turbulent dispersion may be accounted for by the basic non-linear mechanisms encountered in chaotic advection. Analogously, the non-perfect nature of the tracers used in geophysical measurements and/or the possible "active" nature of some constituents turns out to be very important, determining a different behaviour of the advected particles and of true fluid particles.

Further on, it is now clear that there exist regimes of anomalous transport, which may lead to a faster spreading and escape of advected quantities. Such a phenomenon is especially important in plasma dynamics, as well as in turbulent flows due to the action of coherent structures. The regimes of anomalous transport may have a truly asymptotic nature or they may be intermediate regimes encountered in proximity of significant time scales of the system; a better comprehension of these regimes is necessary for various applications. One of them being the transport in magnetised fusion plasmas, understanding transport in these systems as well as characterising the origin of anomalous behaviour is essential not only to define proper control strategies to obtain better confinement, but also to monitor what may happen near the plasma edge, where the energy is collected.

All these anomalous phenomena arise once we accept the fact that uniform chaos is not often realistic. In the early days of the study of chaos, ergodic theory provided an adequate support for the kinetic approach. This is no longer the case. If we are to describe new experimental observations, data from simulations, and to develop new applications, a significantly broader notion of transport is required as well as an expanded arsenal of mathematical tools. The phase space is divided between regions where the motion is regular or irregular. Such diversity in the dynamical landscape makes transport properties more subtle than initially anticipated. In fact, many difficulties are already present in the case of few degrees of freedom Hamiltonian systems. Typically the phase space of smooth Hamiltonian systems is not ergodic in a global sense, due to the presence of islands of stability, the rate of phase space mixing in the chaotic sea is not uniform due to the phenomenon of "stickiness", and the Gaussian nature of transport is generally lost, due to the so-called flights and trappings and the associated power-law tails observed in probability distribution functions. This last feature is also shared with most systems dealing with complexity. Understanding the paths from dynamics to kinetics and from kinetics to transport and complexity involves a strong interdisciplinary interaction among experts in theory, experiments and applications.

The contributions are the proceedings of the conference Chaos, Complexity and Transport which was held in Marseilles (France) from June 4th to June 8th 2007. Due to the interdisciplinary character of the problem the conference made a point on balancing theoretical, numerical and experimental contributions in order to encourage the interactions between experimentalists and theoreticians in the same fields but also cross-disciplinary contributions.

This book is organised into two parts. In the first part, we gather what we consider more general or theoretical contributions, while the second part is dedicated to applications.

In the first part, some features of the dynamics for large N systems with long range interactions and a large number of degree of freedom giving rise to out of equilibrium phase transitions are presented in detail. One may also discover how stochastic webs in multidimensional systems can be used as a way for tiling the plane with specific symmetries. At the same time one discusses the phenomenon of chaotic transport and chaotic mixing through the course of geodesics or the construction of mixing flows using knots. Then one can learn about entropy and complexity, or about Bose-Einstein condensation of classical waves, as well as transport in deterministic ratchets. Regarding Hamiltonian systems, some new approaches to the theoretical treatment of separatrix chaos are discussed. The possibility of having a giant acceleration and about a control technique in area preserving maps are explored.

The second part covers mainly applications. To facilitate reading, we have created two subdivisions. The first one deals with plasmas and fluids, while the second concerns more fields. In the plasmas and fluid subdivision, one will be able to learn in some detail, the implication of topological complexity and Hamiltonian chaos in fusion plasmas, as well as precise experimental studies of advection-reaction diffusion systems. And also non-diffusive transport observed in simulations of plasmas and the problem of solving numerically rotating Rayleigh-Bénard convection in cylinders. Then there are the experimentally observed self-excited instabilities in plasmas containing dust particles and the clustering properties of plasma turbulence signals as well as intermittency scenario of transition to chaos in plasma. Finally, one can read about magnetic reconnection in collisionless plasmas as well as the complexity of the neutral curve of oscillatory flows.

In the second subdivision, an overview of chemotaxis models using an interesting analogy with non-linear mean-field Fokker-Planck equations is presented. Then one shall learn about switchability of a flow, before moving to celestial mechanics and the formation of spiral arms and rings in barred galaxies or learning about Lévy walks for energetic electrons in space. The phenomenon of wave chaos in an underwater sound-channel is then discussed, followed by problem of Fermi acceleration in randomised driven billiards. Finally one shall learn about memory regeneration phenomenon in fractional depolarisation of dielectrics, as well as nodal pattern analysis for conductivity of quantum ring and the application of the GALI method to the dynamics of multidimensional symplectic maps.

As already mentioned, this book reflects to some extent the presentations and the resulting discussions carried out during the conference Chaos, Complexity and Transport: Theory and Applications, which was held in the Pharo site of the Université de la Méditerranée, Marseilles, France, in June 2007. In these regard, we would like to thank all participants and express our sincere gratitude to the contributing authors. We also take this opportunity to express our debt and gratitude for the support to sponsors: Centre National de la Recherche Scientifique, the GDR Phenix et GDR Dycoec, the Commissariat à l'Energie Atomique (CEA), the Conseil Général des Bouches du Rhône, the Ministère Délégué à la Recherche, the Ville de Marseille, the GREFI-MEFI, the European Physical Society, the University de Provence and University de la Méditerranée, The US department of Naval Research, the Delegation Generale de l'Armement and the Centre de Physique Théorique (UMR 6207). We also would like to thank Mrs A. Elbaz, V. Leclercq-Ortal and M-T Donel (from the Centre de Physique Théorique) for their help before, during and after the workshop. We would like also to thank M. Mancis and S. Foulu from Protisvalor Méditerranée for their help.

Cristel Chandre
Xavier Leoncini
George Zaslavsky
Editors

CONTENTS

THEORY

OUT–OF–EQUILIBRIUM PHASE TRANSITIONS IN MEAN-FIELD HAMILTONIAN DYNAMICS

PIERRE-HENRI CHAVANIS

Laboratoire de Physique Théorique, Université Paul Sabatier, 118, route de Narbonne
31062 Toulouse, France
E-mail: chavanis@irsamc.ups-tlse.fr

GIOVANNI DE NINNO

Sincrotrone Trieste, S.S. 14 Km 163.5, Basovizza, 34012, Trieste, Italy
University of Nova Gorica, Vipavska 13, POB 301, SI-5000, Nova Gorica, Slovenia
E-mail: giovanni.deninno@elettra.trieste.it

DUCCIO FANELLI

Theoretical Physics, School of Physics and Astronomy, University of Manchester,
Manchester M13 9PL, United Kingdom
E-mail: Duccio.Fanelli@manchester.ac.uk

STEFANO RUFFO

Dipartimento di Energetica "S. Stecco" and CSDC, Università di Firenze, and INFN,
via S. Marta, 3, 50139 Firenze, Italy
E-mail: stefano.ruffo@unifi.it

Systems with long-range interactions display a short-time relaxation towards Quasi–Stationary States (QSSs), whose lifetime increases with system size. With reference to the Hamiltonian Mean Field (HMF) model, we here review Lynden-Bell's theory of "violent relaxation". The latter results in a maximum entropy scheme for a water-bag initial profile which predicts the presence of *out–of–equilibrium phase transitions* separating homogeneous (zero magnetization) from inhomogeneous (non–zero magnetization) QSSs. Two different parametric representations of the initial condition are analyzed and the features of the phase diagram are discussed. In both representations we find a second order and a first order line of phase transitions that merge at a tricritical point. Particular attention is payed to the condition of existence and stability of the homogeneous phase.

Keywords: Quasi-stationary states, Hamiltonian Mean-Field model, Out–of–equilibrium phase transitions.

1. Introduction

Hamiltonian systems arise in many branches of applied and fundamental physics and, in this respect, constitute a universal framework of extraordinary conceptual importance. Spectacular examples are undoubtedly found in the astrophysical context. The process of hierarchical clustering via gravitational instability, which gives birth to the galaxies,[1] can in fact be cast in a Hamiltonian setting. Surprisingly enough, the galaxies that we observe have not yet relaxed to thermodynamic equilibrium and possibly correspond to intermediate Quasi–Stationary States (QSSs). The latter are in a long-lasting dynamical regime, whose lifetime diverges with the size of the system. The emergence of such states has been reported in several different domains, ranging from charged cold plasmas[2] to Free Electron Lasers (FELs),[3] and long-range forces have been hypothesized to be intimately connected to those peculiar phenomena.

Long-range interactions are such that the two-body interaction potential decays at large distances with a power–law exponent which is smaller than the space dimension. The dynamical and thermodynamical properties of physical systems subject to long-range couplings were poorly understood until a few years ago, and their study was essentially restricted to astrophysics (e.g., self-gravitating systems). Later, it was recognized that long–range systems exhibit universal, albeit unconventional, equilibrium and out–of–equilibrium features.[4] Besides slow relaxation to equilibrium, these include ensemble inequivalence (negative specific heat, temperature jumps), violations of ergodicity and disconnection of the energy surface, subtleties in the relation of the fluid (i.e. continuum) picture and the particle (granular) picture, new macroscopic quantum effects, etc..

While progress has been made in understanding such phenomena, an overall interpretative framework is, however, still lacking. In particular, even though the ubiquity of QSSs has been accepted as an important general concept in non-equilibrium statistical mechanics, different, contrasting, attempts to explain their emergence have catalysed a vigorous discussion in the literature.[5]

To shed light onto this fascinating field, one can resort to toy models which have the merit of capturing basic physical modalities, while allowing for a dramatic reduction in complexity. This is the case of the so-called Hamiltonian Mean Field (HMF) model which describes the evolution of N rotators, coupled through an equal strength, attractive or repulsive, cosine interaction.[6] The Hamiltonian, in the attractive case, reads

$$H = \frac{1}{2}\sum_{j=1}^{N} p_j^2 + \frac{1}{2N}\sum_{i,j=1}^{N} [1 - \cos(\theta_j - \theta_i)] \ , \tag{1}$$

where θ_j represents the orientation of the j-th rotator and p_j stands for the conjugated momentum. To monitor the evolution of the system, it is customary to introduce the magnetization, an order parameter defined as $M = |\mathbf{M}| = |\sum \mathbf{m}_i|/N$, where $\mathbf{m}_i = (\cos\theta_i, \sin\theta_i)$ is the magnetization vector. The HMF model shares many similarities with gravitational and charged sheet models[7,8] and has been extensively studied[9] as a paradigmatic representative of the broad class of systems with long-range interactions. The equilibrium solution is straightforwardly worked out[6] and reveals the existence of a second-order phase transition at the critical energy density $U_c = 3/4$: below this threshold value the Boltzmann-Gibbs equilibrium state is inhomogeneous (magnetized).

In the following, we shall discuss the appearance of QSSs in the HMF setting and review a maximum entropy principle aimed at explaining the behaviour of out-of-equilibrium macroscopic observables. The proposed approach is founded on the observation that in the continuum limit (for an infinite number of particles) the discrete HMF equations converge towards the Vlasov equation, which governs the evolution of the single–particle distribution function (DF). Within this scenario, the QSSs correspond to statistical equilibria of the continuous Vlasov model. As we shall see, the theory allows us to accurately predict out-of-equilibrium phase transitions separating the homogeneous (non-magnetized) and inhomogeneous (magnetized) phases.[10,11] Special attention is here devoted to characterizing analytically the basin of existence of the homogeneous zone. Concerning the structure of the phase diagram, a bridge between the two possible formal settings, respectively[10] and,[12] is here established.

The paper is organized as follows. In Section 2 we present the continuous Vlasov picture and discuss the maximum entropy scheme. The properties of the homogeneous solution are highlighted in Section 3, where conditions of existence are also derived. Section 4 is devoted to analyze the stability of the homogeneous phase. A detailed account of the phase diagram is provided in Sections 5 and 6, where the case of a "rectangular" and generic water–bag initial distribution are respectively considered. Finally, in Section 7 we sum up and draw our conclusions.

2. On the emergence of quasi-stationary states: Predictions from the Lynden-Bell theory within the Vlasov picture

As previously mentioned, long-range systems can be trapped in long-lasting Quasi-Stationary-States (QSSs),[13] before relaxing to Boltzmann thermal equilibrium. The existence of QSSs was firstly recognized with reference to galactic and cosmological applications (see[7] and references therein) and then, more recently, re-discovered in other fields, e.g.two-dimensional turbulence[14] and plasma-wave interactions.[8] Interestingly, when performing the infinite size limit $N \to \infty$ *before* the infinite time limit, $t \to \infty$, the system remains indefinitely confined in the QSSs.[15] For this reason, QSSs are expected to play a relevant role in systems composed by a large number of particles subject to long-range couplings, where they are likely to constitute the solely experimentally accessible dynamical regimes.[2,3]

QSSs are also found in the HMF model, as clearly testified in Fig. 1. Here, the magnetization is monitored as a function of time, for two different values of N. The larger the system the longer the intermediate phase where it remains confined before reaching the final equilibrium. In a recent series of papers,[3,10–12,16] an approximate analytical theory based on the Vlasov equation has been proposed which stems from the seminal work of Lynden-Bell.[17] This is a fully predictive approach, justified from first principles, which captures most of the peculiar traits of the HMF out–of–equilibrium dynamics. The philosophy of the proposed approach, as well as the main predictions derived within this framework, are reviewed in the following.

In the limit of $N \to \infty$, the HMF system can be formally replaced by the following Vlasov equation

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial \theta} - \left(M_x[f]\sin\theta - M_y[f]\cos\theta\right)\frac{\partial f}{\partial p} = 0, \tag{2}$$

where $f(\theta, p, t)$ is the one-body microscopic distribution function normalized such that $\mathcal{M}[f] \equiv \int f d\theta dp = 1$, and the two components of the complex magnetization are respectively given by

$$M_x[f] = \int f \cos\theta d\theta dp, \tag{3}$$

$$M_y[f] = \int f \sin\theta d\theta dp.$$

The mean field energy can be expressed as

$$U = \frac{1}{2}\int f p^2 d\theta dp - \frac{M_x^2 + M_y^2}{2} + \frac{1}{2}. \tag{4}$$