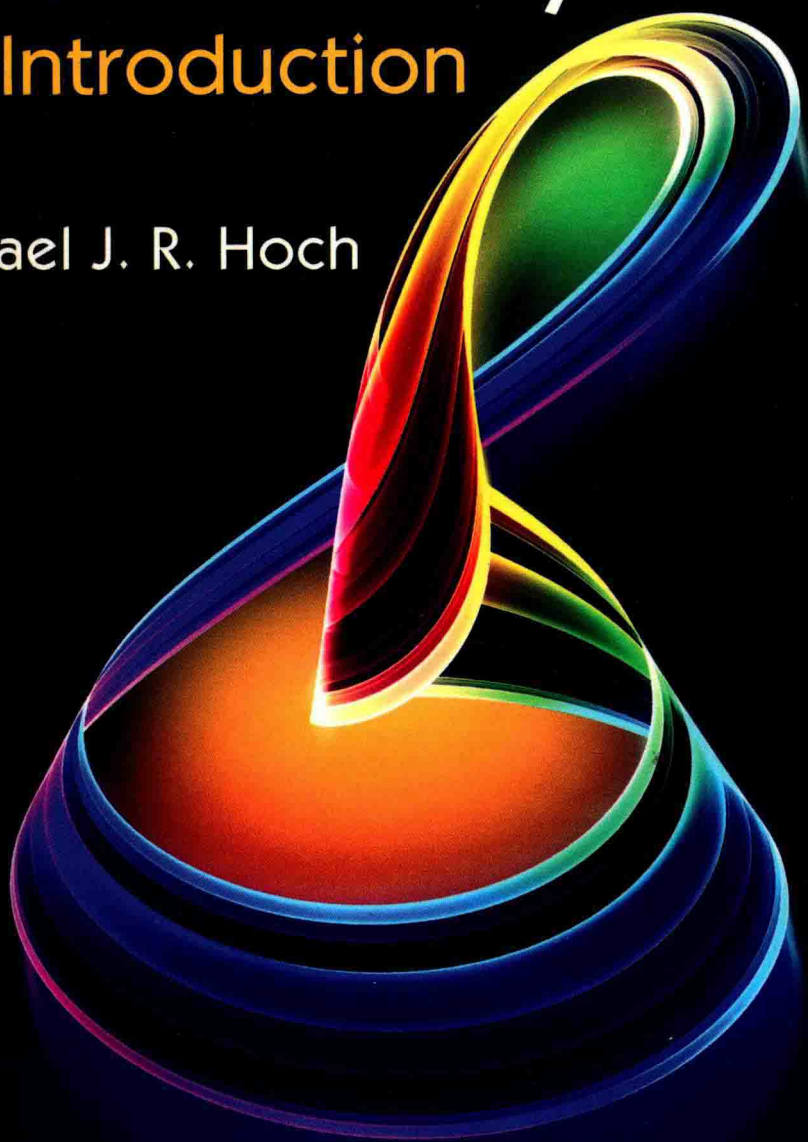


Statistical and Thermal Physics

An Introduction

Michael J. R. Hoch



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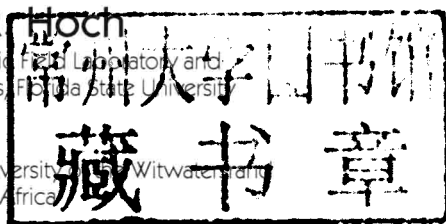
An Introduction

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Statistical and Thermal Physics

To my wife Renée

Preface

Thermal and statistical physics concepts and relationships are of fundamental importance in the description of systems that consist of macroscopically large numbers of particles. This book provides an introduction to the subject at the advanced undergraduate level for students interested in careers in basic or applied physics. The subject can be developed in different ways that take either macroscopic classical thermodynamics or microscopic statistical physics as topics for initial detailed study. Considerable insight into the fundamental concepts, in particular temperature and entropy, can be gained in a combined approach in which the macroscopic and microscopic descriptions are developed in tandem. This is the approach adopted here.

The book consists of two major parts, within each of which there are several sections, as detailed below. A flow chart that shows the chapter sequence and the interconnection of major topics covered is given at the end of this introduction. Part I is divided into three sections, each made up of three chapters. The basics of equilibrium thermodynamics and the first and second laws are covered in Section IA. These three chapters introduce the reader to the concepts of temperature, internal energy, and entropy. Two systems, ideal gases and ideal noninteracting localized spins, are used extensively as models in developing the subject. Use of ideal equations of state for gases and for paramagnetic systems allows illustrative applications of the thermodynamic method. Magnetic systems and magnetic work are dealt with in some detail. The operation of a Carnot refrigerator with an ideal paramagnet as working substance is presented along with the traditional ideal gas case. The chemical potential is introduced from a thermodynamic viewpoint in Chapter 3 and is discussed in subsequent chapters in terms of the microscopic statistical approach.

Chapters 4, 5, and 6 in Section IB provide a complementary microscopic statistical approach to the macroscopic approach of Section IA. Considerable insight into both the entropy and temperature concepts is gained, and the general expression for the entropy is given in terms of

the number of accessible microstates in the fixed energy, microcanonical ensemble approach. This relationship is of central importance in the development of the subject. Explicit expressions for the entropy of both a monatomic ideal gas and an ideal spin system are obtained. The entropy expressions lead to results for the other macroscopic properties for both the ideal gas and the ideal spin system. It is made clear that for ideal gases in the high-temperature, low-density limit, quantum effects may be neglected. The need to allow for the indistinguishable nature of identical particles in nonlocalized systems is emphasized. The expressions for the entropy and the chemical potential of an ideal gas are given in terms of the ratio of the quantum volume, which is introduced with use of the Heisenberg uncertainty principle, and the atomic volume or volume per particle. These forms for the entropy and chemical potential are easily remembered and provide a check on the validity of the classical approximation. In Chapter 6, the third law of thermodynamics is discussed with the use of expressions for the entropy and the temperature parameter obtained in Chapter 5.

After completing Section IB, the reader can proceed directly to the second half of the book. However, some reference to Chapter 7 is helpful to gain familiarity with the Helmholtz and Gibbs thermodynamic potentials that are used in later sections. The thermodynamic potentials are introduced briefly in Chapter 3, with the aid of the Legendre transform, which is discussed in Appendix D.

The final section in the first half of the book, Section IC, emphasizes the power of thermodynamics in the description of processes for both gases in Chapter 7 and condensed matter in Chapter 8. The Maxwell relations are obtained and used in a number of situations that involve adiabatic and isothermal processes. Chapter 9 concludes this section with a discussion of phase transitions and critical phenomena.

Chapter 10 in Section IIA gives a brief introduction to probability theory, mean values, and three statistical ensembles that are used in statistical physics. The partition function is defined as a sum over states, and the ideal localized spin system is used to illustrate the canonical ensemble approach. The grand canonical ensemble and the grand sum are discussed in Chapter 11. It is shown that for systems of large numbers of particles, for which fluctuations in energy and particle number are extremely small, the different ensembles are equivalent. Section IIB is concerned with quantum statistics. Chapter 12 reviews the quantum mechanical description of systems of identical particles and distinguishes fermions and bosons. Chapters 13 and 14 deal with the ideal Fermi gas and the ideal Bose gas,

respectively. Expressions for the heat capacity and magnetic susceptibility are obtained for the Fermi gas, whereas the Bose–Einstein condensation at low temperatures is discussed for the Bose gas. These chapters are illustrated with applications to a variety of systems. For example, Fermi–Dirac statistics is used to treat white dwarf stars and neutron stars. The radiation laws and the heat capacity of solids are discussed in Chapter 15, which deals with photons and phonons. The cosmic microwave background radiation is considered as an illustration of the Planck distribution.

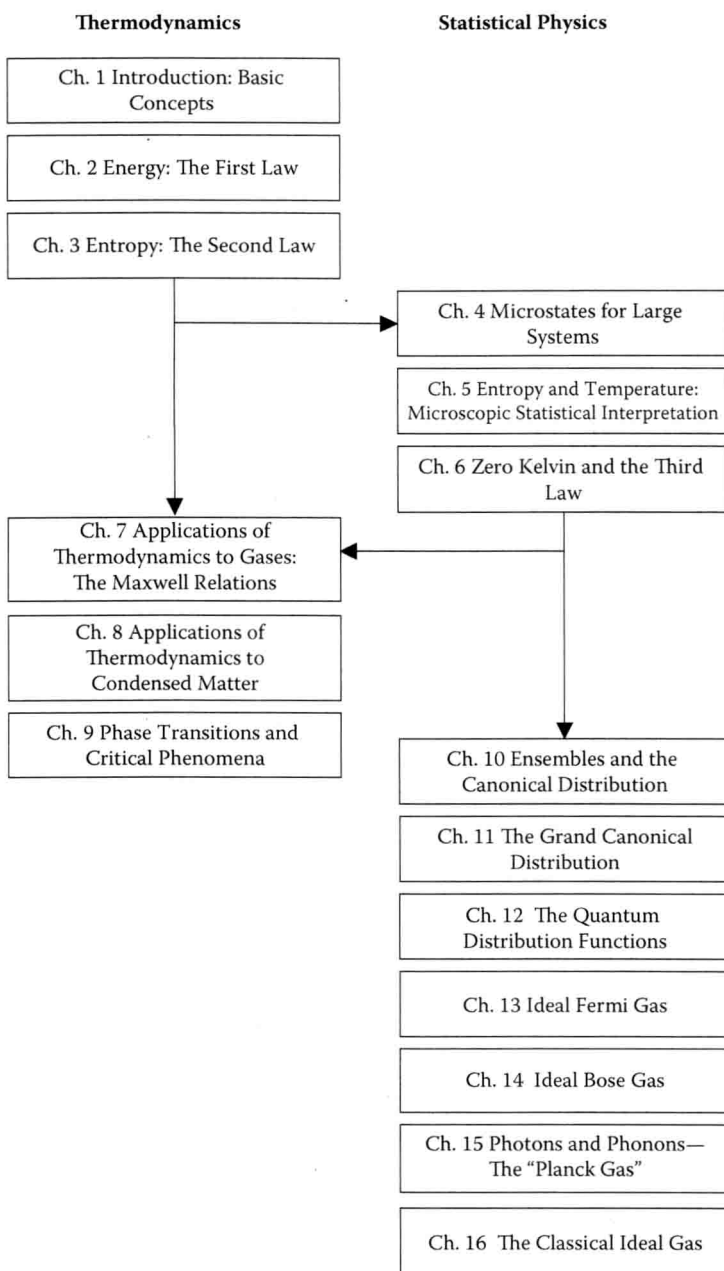
In Section IIC, Chapter 16 returns to the ideal gas treated in the classical limit of the quantum distributions, which automatically allows for the indistinguishable nature of identical nonlocalized particles. The internal energy of molecules is included in the partition function for the classical gas. The equipartition of energy theorem for classical systems is discussed in some detail. Nonideal systems are dealt with in Chapter 17 in terms of the cluster model for gases and the mean field approximation for spins. The Ising model for interacting spins is introduced and the one-dimensional solution of the Ising model is given for the zero applied field case. An introduction to Fermi liquid theory is followed by a discussion of the properties of liquid helium-3 at low temperatures. The chapter concludes with a phenomenological treatment of Bose liquids and the properties of liquid helium-4.

Section IID deals with special topics that include the density matrix, chemical reactions, and an introduction to irreversible thermodynamics. Chapter 18 introduces the density matrix formulation with applications to spin systems and makes a connection to the classical phase space approach. Topics covered in Chapter 19 are the law of mass action, adsorption on surfaces, and carrier concentrations in semiconductors. Chapter 20 deals with irreversible processes in systems not far from equilibrium, such as thermosmosis and thermoelectric effects.

For a one-semester course, the important sections that should be covered are Sections IA, IB, IIA, and IIB. If students have had prior exposure to elementary thermodynamics, much of Section IA may be treated as a self-study topic. Problems given at the end of each chapter provide opportunities for students to test and develop their knowledge of the subject. Depending on the nature of the course and student interest, materials from Sections IC, IIC, and IID can be added.

A diagram that illustrates the structure and the interrelationships of the first 16 chapters of the book is given in the following figure.

Statistical and Thermal Physics Topics Covered in Chapters 1 to 16



Acknowledgments

My thanks go to numerous colleagues both in Johannesburg and in Tallahassee for helpful discussions on the concepts described in this book. In teaching the material I have learnt a great deal from the interactions I have had with many students. Their comments and responses to questions have often been enlightening.

Finally, I wish to thank my family for their continuing support during this project. In particular, I owe a great deal to my wife Renée, who in addition to preparing most of the figures, provided the necessary encouragement that helped me to complete the book.

Physical Constants

Avogadro number, N_A	$6.023 \times 10^{23} \text{ mol}^{-1}$
Bohr magneton, μ_B	$9.27 \times 10^{-24} \text{ J T}^{-1}$
Boltzmann constant, k_B	$1.38 \times 10^{-23} \text{ J K}^{-1}$
Electron charge, e	$1.60 \times 10^{-19} \text{ C}$
Electron mass, m_e	$9.11 \times 10^{-31} \text{ kg}$
Gas constant, R	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
\hbar	$1.055 \times 10^{-34} \text{ J s}$
Nuclear magneton, μ_N	$5.05 \times 10^{-27} \text{ J T}^{-1}$
Permeability constant, μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Permittivity constant, ϵ_0	$8.554 \times 10^{-12} \text{ F m}^{-1}$
Planck constant, h	$6.624 \times 10^{-34} \text{ J s}$
Proton mass, m_p	$1.67 \times 10^{-27} \text{ kg}$

CONVERSION FACTORS

1 atmosphere (atm)	$1.01 \times 10^5 \text{ Pa}$
1 electron volt (eV)	$1.60 \times 10^{-19} \text{ J}$
1 Joule (J)	10^7 erg
1 liter (L)	10^{-3} m^3
1 mass unit (u)	$1.66 \times 10^{-27} \text{ kg}$

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