Living Mathematics: A Survey

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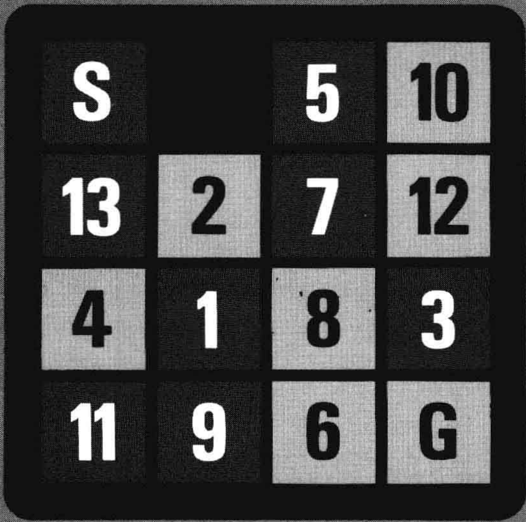

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Preface

Living Mathematics is intended for college students with varied mathematical backgrounds. The mixture of traditional and nontraditional topics represented in the text allows for flexibility in order to fit the needs of a particular course or instructor. The author's objectives in writing this text were as follows:

1. To instill college students with an interest in and appreciation for mathematics as an important influence in their lives, especially in the area of decision making;
2. To improve the average student's ability to attack and solve mathematical problems;
3. To eliminate some of the anxiety and outright dislike which many students feel toward mathematics by helping them achieve success in a college-level mathematics class.

With over ten years of classroom experience in liberal arts mathematics, the author explains each idea in the clearest possible terms with little left to the student's imagination. This approach relies heavily upon *intuition*, proceeding at all times from a concept for which the student has a good intuitive feel to the mathematical representation of that concept.

The arithmetic skills needed to understand a particular topic are presented at the beginning of the pertinent chapter in a section called Necessary Arithmetic rather than in a separate chapter. This allows for immediate reinforcement of the arithmetic and eliminates the necessity for time spent in deadly dull arithmetic drill. In fact, these arithmetic sections are designed to be handled by students on their own and could be assigned for independent study prior to the introduction of the main chapter content. Most of this arithmetic is not new to students; usually they just need a short refresher followed by some practice.

There are many examples discussed in detail in each section (over 1000 in all), and the author returns to the same examples whenever possible in moving to a new idea. Many varied exercises are included at the end of each section (totalling over 2300). Short summaries have been placed at the conclusion of each chapter, followed by review problems chosen carefully to tie together the ideas presented in the chapter. Answers to odd-numbered problems are included at the end of the text along with a glossary of terms.

In order to maintain student interest, the text concentrates primarily upon ideas not encountered in previous mathematics courses, constantly relating such ideas to areas of everyday importance to the reader.

In the chapter on sets, for example, the emphasis is upon the application of sets to later material, so stress is placed upon set-builder notation (for algebra), cardinal numbers and Venn diagrams (for probability), and DeMorgan's Laws (for logic).

Similarly logic and statistics are related to their misuse in advertising and politics; probability is related to expected value. Both of these areas, together with the obviously relevant consumer mathematics, algebra, and practical geometry are geared toward preparing students for rational decision making.

The writing style is conversational, employing a vocabulary designed to be readable by a typical student in a course of this nature. Anticipated questions are answered where they are expected to arise. Throughout the text, the student is led through a gradual but thorough exposure to the mathematics which will be important throughout his or her life.

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Contents

Preface ix

1 Sets 1

Necessary arithmetic 1
1.1 Sets 3
1.2 Cardinal number 5
1.3 Set relations 6
1.4 Set operations 12
1.5 Venn diagrams 20
1.6 Summary 30
Review exercises 31

2 Logic 33

2.1 Statements 35
2.2 Symbols 40
2.3 Truth tables 45
2.4 Variations on the "if . . . then" statement 55
2.5 More truth tables 60
2.6 Tautologies 63
2.7 Logical arguments 64
2.8 Summary 71
Review exercises 72

3 Number Systems 74

3.1 The tools 74
3.2 The rules 78
3.3 Modular systems 91
3.4 Prime numbers 103
3.5 Summary 126
Review exercises 128

4 Probability 130

Necessary arithmetic 130
4.1 Simple probability 143
4.2 Odds 158
4.3 Probabilities as products 164
4.4 Probabilities as sum 172
4.5 Summary 178
Review exercises 178

5 More Probability 180

5.1 Expected value 180
5.2 Counting 191
5.3 Summary 218
Review exercises 218

6 Mathematics for the Consumer 220

Necessary arithmetic 220
6.1 Ratio and proportion 235
6.2 Mark-up and mark-down 242
6.3 Building a home 247
6.4 Summary 268
Review exercises 269

7 More Consumer Mathematics 271

7.1 Compound interest 271
7.2 Present value 287
7.3 Economic indicators 288
7.4 Life insurance 293
7.5 Summary 301
Review exercises 301

8 Statistics 303

8.1 Fudging with statistics 303
8.2 Measures of central tendency 308
8.3 Measures of position 318
Necessary arithmetic 318
8.4 Measures of dispersion 324
Necessary arithmetic 324
8.5 Frequency distributions and their graphs 333
8.6 Summary 346
Review exercises 347

9 Algebra 348

Necessary arithmetic 348
9.1 First degree equations and inequalities 360
9.2 More equations 374
9.3 More inequalities 382
9.4 Summary 390
Review exercises 390

10 Graphing 392

10.1 Linear equations in two variables 392
10.2 Graphing linear inequalities 411
10.3 Summary 420
Review exercises 420

11 Metrics 422

11.1 Ports in the storm 423
Necessary arithmetic 430
11.2 Converting among metric units 441
11.3 Converting between systems 449
11.4 Summary 458
Review exercises 458

12 Geometry 460

Necessary arithmetic 461
12.1 Linear measure (distance) 466
12.2 Square measure (area) 481
12.3 Cubic measure (volume) 495
12.4 Multiple dimensions 507
12.5 Summary 517
Review exercises 518

13 Systems of Numeration 520

13.1 The system of Roman numerals 521
13.2 A positional (place value) system 525
Necessary arithmetic 526
13.3 Other positional systems 533
13.4 Arithmetic in other base systems 543
13.5 Summary 557
Review exercises 558

Solutions to odd-numbered problems 559

Glossary 603

Index 611

1 Sets

In recent years much has been said about set theory in mathematics courses from elementary through graduate school. Most of you have been exposed to sets before now but may have been unconvinced that they have any real significance in your lives. In truth, sets can be applied to many different areas of mathematics and can help you answer lots of everyday questions.

How many people in a television survey watch "Dallas" but not "Knott's Landing"?

By how many different routes can you jog in your neighborhood without traveling the same road twice?

What is the probability of rolling five of a kind in a game of Yahtze?

In this chapter we shall discuss some fundamental set language and learn to count the number of items in a set so that we can see how an understanding of sets assists us in the areas of counting, probability, statistics, and algebra.

Necessary Arithmetic

In this chapter dealing with sets you will be required to use some simple ideas from arithmetic. Basically you must be able to add, subtract, and multiply whole numbers. Remember that the numbers 0, 1, 2, 3, . . . are called **whole numbers,** and they are used for counting. We assume that each reader can perform such operations as

$$3 + 5 = 8$$

$$6 + 4 - 3 = 7$$

$$4 \cdot 3 = 12$$

But you will also encounter symbols like 2^3 or 3^2 or 2^5 and we need to be sure that you recall how to evaluate these expressions. In the example 2^3, 2 is called the base and 3 is called the exponent and $2^3 = 2 \cdot 2 \cdot 2 = 8$. In general, a^n means to use the base a as a factor in a product n times; that is,

$$a^n = \underbrace{a \cdot a \cdot a \cdots a \cdot a}_{n \text{ factors.}}$$

So in our examples

$$3^2 = 3 \cdot 3 = 9$$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

Often, in mathematics, we wish to compare two numbers and we do so by using symbols of inequality. We frequently say Mary is older than Tom, meaning Mary's age is greater than Tom's age. If Mary is 26 and Tom is 19, then we could write: 26 is greater than 19, which is abbreviated $26 > 19$. Similarly, we can write: $19 < 26$, which is read "19 is less than 26."

Many students find the **number line** a useful device in understanding inequalities. To construct a number line, we arbitrarily choose a zero point and a length to represent one unit. Then all points at unit intervals to the right of zero are labeled with consecutive positive integers. See Fig. 1-1. (Those to the left correspond to negative integers for which we have very little need in this course.) The arrows at either end indicate that the numbering continues indefinitely; we should also mention that there are many numbers in between every two integers (fractions and decimals, such as $\frac{1}{2}$, 0.75, and 3.7). Before we lose our main point, however, recall that we were discussing inequalities. Very simply, we note that $26 > 19$ because 26 lies to the right of 19 on the number line. In general, if a and b are

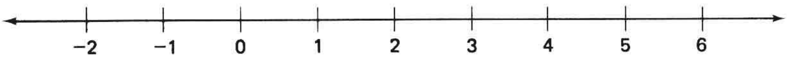

FIGURE 1-1 Number Line

numbers,

$a > b$	if	a lies to the right of b on the number line.
$a < b$	if	a lies to the left of b on the number line.
$a = b$	if	a and b occupy the same position on the number line.

1.1. Sets

For most of us the word *set* brings some specific example to mind: a set of encyclopedias, a set of dishes, a set of exam scores. We can say that a set is a collection of things. The items in a set are called **elements** or **members** of the set. Capital letters are usually used to name sets and the elements are enclosed in curly braces. For example, we write

$$A = \{\text{plate, cup, saucer, bowl}\}$$

$$B = \{\text{a, e, i, o, u}\}$$

$$C = \{1, 3, 5, 7, 9, \ldots, 97, 99\}$$

Each of these sets has been specified by **list** (or roster) and each set contains a finite number of elements (i.e., if you were to count the elements in these sets, you would stop at some specific number). Sets A, B, and C, then, are called **finite sets.** In set C notice that the dots are used to show that this set continues in the manner indicated; so you know that the next element is 11, then 13, and so on. Once the pattern is clearly established, we can use dots to represent elements that are too numerous to include.

Here is another example of a set specified by list.

$$N = \{1, 2, 3, \ldots\}$$

It should be clear that the next element is 4, then 5, and so on. Is this set finite? There seems to be no last element; so we could never stop counting. This is an example of an **infinite set.** In fact, this special set N is called the set of **natural numbers.** If we were to add the element 0, then we would create the set of **whole numbers** mentioned earlier.

$$W = \{0, 1, 2, 3, \ldots\}$$

The set W is also an infinite set.

Notice that all sets considered so far have been what we call **well-defined sets;** the reader can tell exactly what is in each set and what is *not* in each set. For example, cup is in set A, but spoon is not in set A; 15 is in set C, but 10 is not in set C; 83 is in set N, but $\frac{1}{2}$ is not in set N. Because mathematicians love shorthand methods of writing statements with as few words as possible, they invent symbols that can be used to replace words. To state that some element belongs to a set, mathematicians use the symbol $\in$. Thus to say "cup is in set A," we may write "cup $\in A$." Similarly, we write spoon $\notin A$, where the slash through the $\in$ means "does *not* belong to." Let's translate our other statements into this shorthand form.

$$15 \in C, \qquad 10 \notin C$$

$$83 \in N, \qquad \frac{1}{2} \notin N$$

In mathematics we are concerned only with well-defined sets, but you may wonder what a set that is not well defined looks like. The sets {all pretty girls} or {all tall men} are not well defined because interpreting the words "pretty" and "tall" differs from person to person. We shall avoid sets that are not well defined.

Sometimes it is not convenient to list all the elements of a set due to lack of space. In these cases, we can specify a set by **rule** (or description). It would be awkward (though possible) to list all the students in this class at this time, for instance, but we may specify the set as K = {all students in this class at this time}. Some other examples might be

P = {all U.S. Presidents}

A = {all letters of our alphabet}

R = {real numbers}

D = {outcomes in the toss of one die}

If necessary, each of these sets could be changed from rule form to list form because each is well defined. Which of these sets are finite? Infinite?

Another way of specifying a set is called **set-builder notation,** which is simply a shorthand way of describing the elements of a set. It is important that you learn to read set-builder notation so that it will make sense to you. It is not difficult if you practice a little.

For example, D = {x: x is outcome in toss of one die} is read, "Set D is the set of all elements x such that each x is an outcome in the toss of one die." What follows the colon (read "such that") merely describes the conditions for membership in the set. So here $D = \{1, 2, 3, 4, 5, 6\}$.

EXAMPLES

1. B = {x: x was a Beatle}
   B = {John Lennon, George Harrison, Ringo Starr, Paul McCartney}

2. $E = \{x\colon\ x \in N \text{ and } x < 7\}$. Recall that $N = \{1, 2, 3, \ldots\}$ and $x < 7$ means x is less than 7. So $E = \{1, 2, 3, 4, 5, 6\}$.

3. $K = \{x\colon x$ subscribes to *Newsweek*$\}$. It would be impractical to try to list the elements of this set, but it is clear what those elements are.

One particular set deserves special mention: the set containing no elements. For obvious reasons, it is called the **empty set** (or null set) and is written $\{\ \}$ or $\emptyset$.

Some examples of sets that are empty are

1. {green-haired Presidents}
2. $\{x\colon x \in N \text{ and } x < 1\}$
3. {outcomes greater than 7 on the toss of one die}

Each of these sets contains no elements and is therefore an empty set. Caution: you may write the empty set as $\{\ \}$ or $\emptyset$ but never as $\{\emptyset\}$, for that set is *not* empty! It contains one element—namely, the symbol $\emptyset$.

1.2. Cardinal Number

Corresponding to every well-defined finite set there is a whole number called the **cardinal number** of the set. To determine the cardinal number of a set, we merely count the elements in that set. The number of elements in a set A is called the cardinal number of set A and is denoted by $n(A)$. In this course we are not concerned with the cardinal number of an infinite set.

EXAMPLES

1. $A = \{\text{plate, cup, saucer, bowl}\}$
   $n(A) = 4$
2. $B = \{\text{a, e, i, o, u}\}$
   $n(B) = 5$
3. $D = \{\text{outcomes in toss of one die}\} = \{1, 2, 3, 4, 5, 6\}$
   $n(D) = 6$
4. $E = \{x\colon x \in N \text{ and } x < 7\} = \{1, 2, 3, 4, 5, 6\}$
   $n(E) = 6$
5. $F = \{x\colon x \in N \text{ and } x < 1\} = \{\ \} = \emptyset$
   $n(F) = n(\emptyset) = 0$
6. $G = \{x\colon x \text{ is a Beatle}\}$
   $n(G) = 4$
7. $L = \{\text{letters in the word OCCASIONS}\}$
   $= \{\text{O, C, A, S, I, N}\}$
   $n(L) = 6$

Notice that each different element is listed just once within a set. Notice also that the cardinal number of the empty set is 0 (example 5).

EXERCISE 1.1

1. Tell which sets are well defined
   (a) A = {baseball teams in the National League}
   (b) S = {U.S. Presidents who also served as Vice-Presidents}
   (c) T = {worthwhile TV shows}
   (d) C = {players on UCLA's basketball team}
   (e) D = {tasty ice cream flavors}
2. Write the following sets in list (roster) form and state the cardinal number of each.
   (a) C = {classes you are taking this semester}
   (b) M = {members of your immediate family}
   (c) F = {numerals on the face of a clock}
   (d) L = {letters in the word "mathematics"}
   (e) T = {natural numbers less than 12}
3. Write the following sets in rule (description) form and state the cardinal number of each.
   (a) V = {a, e, i, o, u}
   (b) P = {Nixon, Ford, Carter}
   (c) $A = \{1, 3, 5, 7, 9\}$
   (d) B = {penny, nickel, dime, quarter, half-dollar, silver dollar}
   (e) D = {2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, ace}
4. Write the following sets in list (roster) form and state the cardinal number of each.
   (a) $E = \{x\colon\ x$ is a New England state$\}$
   (b) $A = \{x\colon\ x \in N \text{ and } x \leqslant 3\}$
   (c) $D = \{x\colon\ x$ is digit in your Social Security number$\}$
   (d) $G = \{x\colon\ x$ is a female U.S. President$\}$
   (e) $K = \{x\colon\ x$ is a legal holiday in the United States$\}$
5. Write the following sets in set-builder notation.
   (a) $B = \{0, 1, 2, \ldots, 49\}$
   (b) C = {cheddar, mozzarella, Swiss, parmesan, . . .}
   (c) $P = \{5, 6, 7, \ldots\}$
   (d) S = {spades, clubs, diamonds, hearts}
   (e) T = {heads, tails}

1.3. Set Relations

In arithmetic you learned that numbers can be related by equality or inequality. In set theory there are three ways in which sets can be related: equality, equivalence, and subsetness.

Equality

Recall two of the sets dealt with in the last section.

$$D = \{\text{outcomes in the toss of one die}\}$$

$$E = \{x\colon\ x \in N \text{ and } x < 7\}$$