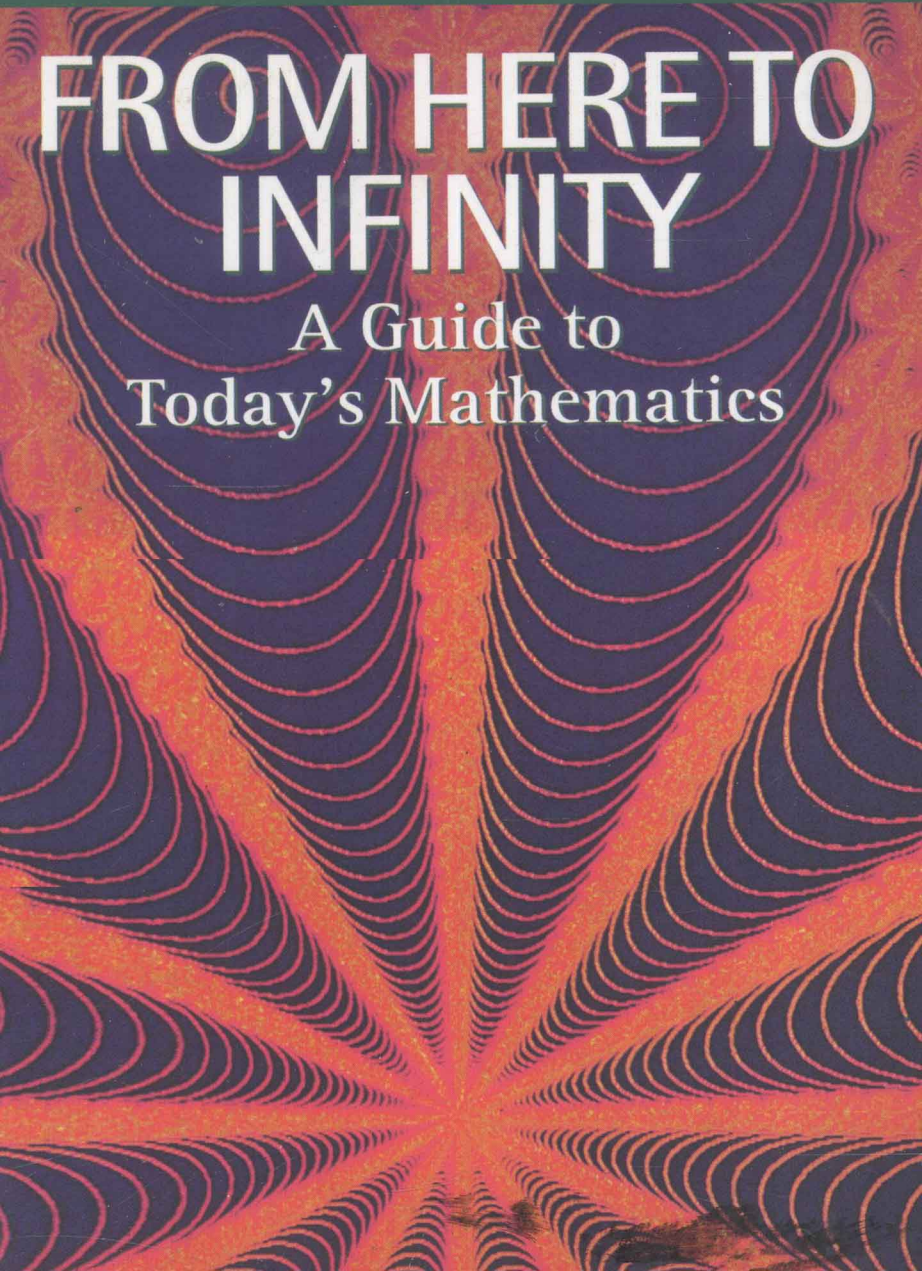


**IAN STEWART**

Author of *Does God Play Dice?*

# FROM HERE TO INFINITY

A Guide to  
Today's Mathematics



# From Here to Infinity

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Ian Stewart

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# From Here to Infinity

**Ian Stewart** is Professor of Mathematics at Warwick University. In 1995 he was awarded the Royal Society's Michael Faraday Medal for outstanding contributions to the public understanding of science. His many books include *Does God Play Dice?*, *Nature's Numbers*, *The Collapse of Chaos* (with Jack Cohen), *Game, Set and Math*, and *Fearful Symmetry: Is God a Geometer?* (with Martin Golubitsky). He contributes to a wide range of newspapers and magazines, and writes the 'Mathematical Recreations' column of *Scientific American*. He lives in Coventry.

## Foreword

Mathematics is not a book confined within a cover and bound between brazen clasps, whose contents it needs only patience to ransack; it is not a mine, whose treasures may take long to reduce into possession, but which fill only a limited number of veins and lodes; it is not a soil, whose fertility can be exhausted by the yield of successive harvests; it is not a continent or an ocean, whose area can be mapped out and its contour defined: it is as limitless as that space which it finds too narrow for its aspirations; its possibilities are as infinite as the worlds which are forever crowding in and multiplying upon the astronomer's gaze; it is as incapable of being restricted within assigned boundaries or being reduced to definitions of permanent validity, as the consciousness of life, which seems to slumber in each monad, in every atom of matter, in each leaf and bud cell, and is forever ready to burst forth into new forms of vegetable and animal existence.

JAMES JOSEPH SYLVESTER

## Preface

What is mathematics? What is it for? What are mathematicians doing nowadays? Wasn't it all finished long ago? How many new numbers can you invent anyway? Is today's mathematics just a matter of huge calculations, with the mathematician as a kind of zookeeper, making sure the precious computers are fed and watered? If it's not, what is it other than the incomprehensible outpourings of super-powered brainboxes with their heads in the clouds and their feet dangling from the lofty balconies of their ivory towers?

Mathematics is all of these, and none. Mostly, it's just different. It's not what you expect it to be. Even when it looks as if it *is* what you expect it to be, you turn your back for a moment and it's changed. It's certainly not just a fixed body of knowledge, its growth is not confined to inventing new numbers, and its hidden tendrils pervade every aspect of modern life.

Mathematics is changing very rapidly indeed. You can tell that just by following the career of this book, which first saw daylight in 1987 under the title *The Problems of Mathematics*. By 1992 several fundamental new discoveries had made a second edition imperative. The title remained the same, but the cover became more jazzy, with a fancy computer graphic in place of a design made from coloured pins. And now it has become necessary to produce a third edition, with a user-friendly title and a distinct shift of emphasis. The new title, by the way, is supposed to indicate that mathematics combines relevance to everyday life ('here') with sweeping intellectual invention ('infinity'). The new emphasis is more on the overall ebb and flow of mathematics, and less on particular problems.

A lot happened during those eight years. The most dramatic was Andrew Wiles's proof of Fermat's Last Theorem, announced in 1993 but not completed until 1995, and then only after some heart-stopping glitches. Chaos and fractals outgrew their origins and smeared themselves across the face of science like bread and jam in

the hands of a toddler. The intellectual equivalent of egg smeared itself over the faces of a few diehard scientists who had alleged that chaos was just computer error and didn't really exist. Oh, Chaos Theory was hyped, no doubt about it—but it was hyped because it was *important*. The classical understanding of knots took a totally unexpected step sideways, matured overnight like a field of mushrooms, and transferred itself into the biochemistry of DNA and Richard Feynman's diagrammatic representations of the collisions of quantum particles. These areas in turn fed new ideas back into knot theory, and the dance still continues, swirling and whirling across the mathematical landscape. The circle was squared, literally, by chopping it into  $10^{50}$  pieces and reassembling them into a square with not a single point out of place. Schrödinger's cat proved it was alive after all by letting itself out of the bag and revolutionizing cryptography. One mathematician found a way to construct billions upon billions of 'Carmichael numbers', and three others promptly trumped him by using the same method to prove the existence of infinitely many. You will find all these stories, and many others like them, within these pages.

Progress does not always move in the forward direction; mathematics must occasionally mount a 'strategic withdrawal'. A proof of the Poincaré conjecture, the holy grail of topology, was announced in the first edition of this book and deleted from the second edition when one key step fell to bits. It may yet resurface in the fourth edition, you can never tell in this subject. The second edition contained an entire new chapter on the solution to the Kepler Problem, which is to prove that the most efficient way to stack spheres is the one known to every greengrocer. You won't find it in this edition, because the proposed proof has become distinctly controversial. It has not so much collapsed as disappeared behind an impenetrable veil of fog. Even though mathematical truth is much more clear-cut than truth in any other area of science, controversy can still muddy its waters. One of the current long-running debates focuses around so-called 'experimental mathematics', which emphasizes the role of computers in suggesting new truths. This would not be terribly controversial, except that it has been interpreted by some people as a

downgrading of the traditional concept of proof. (Look, it *isn't*, OK?) So now we have a meta-controversy about whether the original controversy was or was not aimed at a straw man. Heady stuff, and it certainly makes you re-examine your prejudices. Some mathematicians don't like controversies, but they do serve to remind us that mathematics is created by human beings who *care* about it, and I think that's an important message.

One characteristic of science at the end of the twentieth century is that traditional subject boundaries are dissolving. The same is true in mathematics. No longer is it sensible to carve the subject up into algebra, calculus, geometry, and so on. Every area impinges on every other. Many areas of mathematical research are now enriched by direct and active contact with applied science. Often the most interesting areas are not those in which mathematics has traditionally been used, and the most interesting applications involve mathematics that has not normally been considered useful.

I've found this in small ways in my own research. First, I genuinely find it hard to answer the question 'what field do you do work in?' I work in several fields, or none, so I either mumble something vague about 'nonlinear dynamics' or give the enquirer a twenty-minute potted history. Second, my own creations keep turning round and biting me. A few years ago a physiologist friend and I did some very 'blue skies' work on how to use group theory to classify patterns of movement in animal locomotion. A few weeks ago I learned that engineers were using our ideas to build and control a walking robot. Four years ago an industrial engineer telephoned me with a query about chaos; today we have a joint patent application for a machine that performs quality control analysis in the springmaking industry.

This is my own personal microcosm of the ever-broadening sweep of science, and it has affected how I view mathematics and its role. Partly for that reason, you will find an idiosyncratic selection of topics here. This is not in any sense a comprehensive study of the whole of today's mathematical sciences; it's just my personal picture of some of the bits that have captured my interest. But I do believe that I can justify my choices in terms of their wider significance. We



are living in an increasingly mathematical world. Mathematics is sometimes considered an art, but I think that mathematical thought patterns lie much closer to science. Mathematics is one of the keystones of the scientific endeavour, and science and its technological legacy affect every one of us in more ways than we know, or can know. So it pays to understand the kinds of things that mathematicians do; and my selection, idiosyncratic though it may be, is certainly a valid sample.

My aim in *From Here to Infinity* is to give you as much insight as I can into the questions raised at the start of this preface—what mathematics is, what it's for, what mathematicians do when they create new mathematics, and why that task wasn't finished long ago. At the very least, I guarantee that you won't emerge from the book thinking that mathematical research consists of inventing new numbers.

Though *sometimes*, of course—

I.N.S.

Coventry 1995



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