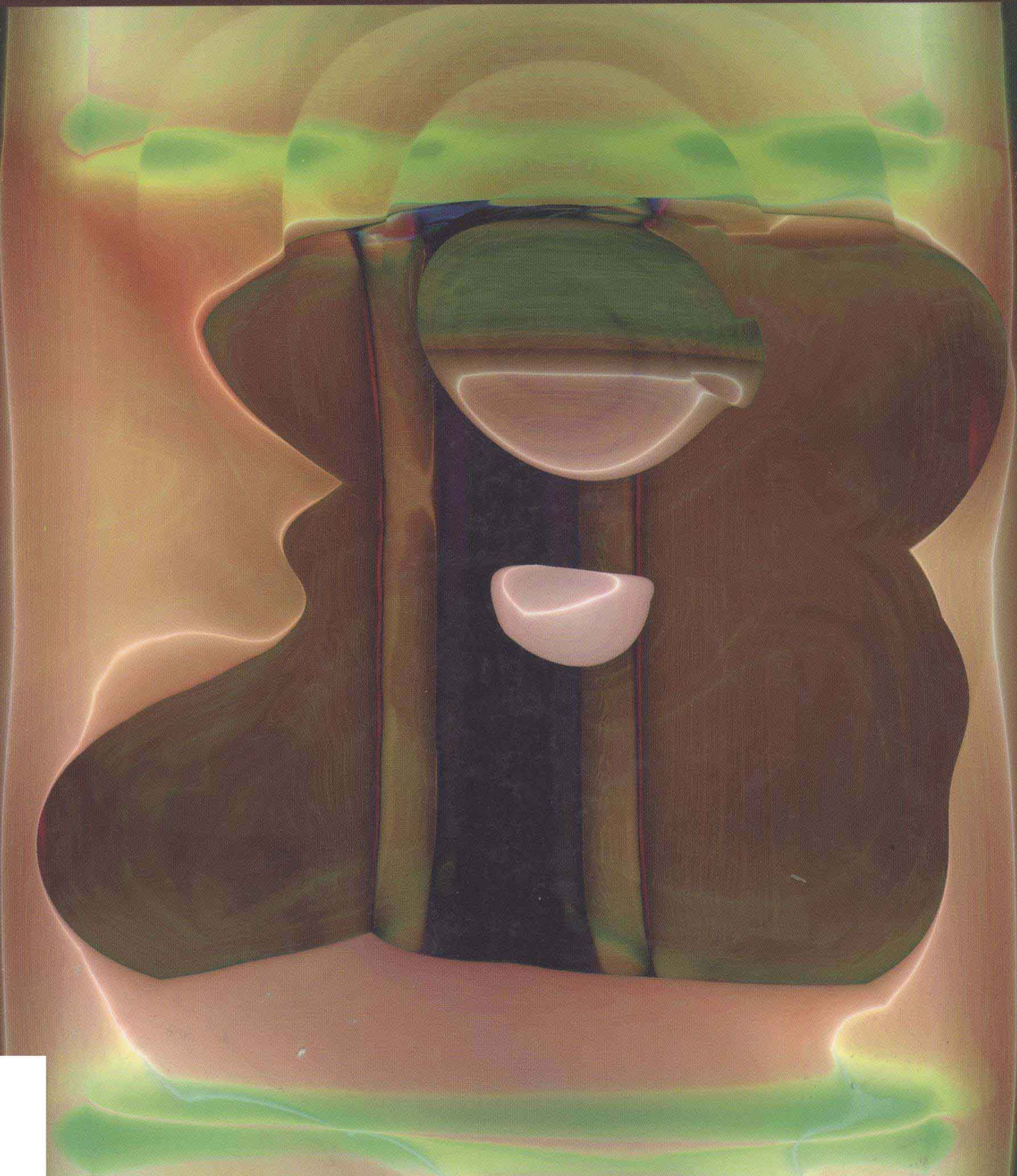
Study Guide
for Stewart/Redlin/Watson's

COLLEGE ALGEBRA 4th



John Banks

Study Guide
for
Stewart, Redlin, and Watson's
College Algebra
Fourth Edition

John Banks

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Preface to Fourth Edition

To master college algebra and move on to further studies, you need to practice solving problems. The more problems you do, the better your problem solving skills will become. You also need exposure to many different types of problems so you can develop strategies for solving each type.

This study guide provides additional problems for each section of the main text, *College Algebra, Fourth Edition*. Each problem includes a detailed solution located adjacent to it. Where appropriate, the solution includes commentary on what steps are taken and why those particular steps were taken. Some problems are solved using more than one approach, allowing you to compare the different methods.

Developing a strategy on how to solve a type of problem is essential to doing well in mathematics. Therefore, it is extremely important that you work out each problem before looking at the solution.

If you immediately look at the solution before attempting to work it out, you short change yourself. You skip the important steps of planning the strategy and deciding on the method to use to carry out this plan. These problem solving skills cannot be developed by looking at the answer first.

Another method you can use to help study is to use index cards. Put a problem that you find challenging on one side of the card and the solution and its location in the text on the other side of the card. Add new problems to your collection whenever you do your homework. Once a week test yourself with these cards. Be sure to shuffle the cards each time.

Finally, I would like to acknowledge the many useful comments and suggestions provided by Anna Fox, Lothar Redlin, and Saleem Watson.

I hope you find that this study guide helps in your further understanding the concepts of college algebra.

John A. Banks

Contents

Chapter 5 Exponential and Logarithmic Functions

Chapter 6 Systems of Equations and Inequalities

Chapter 7 Matrices and Determinants

Chapter 8 Conic Sections

Chapter 9 Sequences and Series

Chapter 10 Counting and Probability

Chapter P
Prerequisites

Section P.1 Modeling the Real World

Key Ideas

A. Modeling with functions.

B. Finding Models.

A. In algebra we use letters to stand for numbers. This allows us to describe patterns, which we express in a formula.

1. The amount of power which can be generated by wind is given by the formula $P = 0.65 \times s^3$, where P is the power in watts and s is the speed of the wind in meters/sec. How much power is generated by the following wind speeds?

(a) 2 meters/sec.

We substitute $s = 2$ into the model and solve.

$$\begin{aligned} P &= 0.65 \times s^3 \\ &= 0.65 \times (2)^3 \\ &= 5.2 \text{ watts} \end{aligned}$$

(b) 4 meters/sec.

We substitute $s = 4$ into the model and solve.

$$\begin{aligned} P &= 0.65 \times s^3 \\ &= 0.65 \times (4)^3 \\ &= 41.6 \text{ watts} \end{aligned}$$

(c) 6 meters/sec.

We substitute $s = 6$ into the model and solve.

$$\begin{aligned} P &= 0.65 \times s^3 \\ &= 0.65 \times (6)^3 \\ &= 140.4 \text{ watts} \end{aligned}$$

2. Use the distance formula $D = RT$.
A VCR plays 12.5 cm of tape per minute.

(a) How much tape is played in 8 minutes?

We substitute the known values and solve.

$D = RT$

$$D = \left(\frac{12.5 \text{ cm}}{\text{minute}}\right)(8 \text{ minutes}) = 100 \text{ cm}$$

(b) How much tape is played in T minutes?

$D = RT$

$$D = \left(\frac{12.5 \text{ cm}}{\text{minute}}\right)(T \text{ minutes}) = 12.5T \text{ cm}$$

(c) A movie is 2 hours 15 minutes long. How much video tape is needed to put the movie on tape?

2 hours 15 minutes = 135 minutes

$D = RT$

$$D = \left(\frac{12.5 \text{ cm}}{\text{minute}}\right)(135 \text{ minutes}) = 1687.5 \text{ cm}$$

B. Finding patterns is an important skill that is developed by practice. This leads to writing formulas that can then be used to solve other problems.

3. The water in a hot tub is drained and replaced with fresh water. When the hot tub is restarted the temperature of the water is 68°. One hour later the water temperature is 79°.

   (a) Find a formula that models the temperature of the water h hours after the hot tub is restarted.

   We use the model that the temperature of the water in the hot tub is

   $$\textit{temp.} = \begin{pmatrix}\textit{initial}\\ \textit{temp.}\end{pmatrix} + \begin{pmatrix}\textit{increase in}\\ \textit{temp. per hour}\end{pmatrix} \times \begin{pmatrix}\textit{number}\\ \textit{of hours}\end{pmatrix}.$$

   Let T be the temperature of the water in the hot tub h hours after it is restarted.
   The initial temperature is 68° and during the first hour the temperature increases $79° - 68° = 11°$.
   Thus we get the model $T = 68 + 11h$.

   (b) Use the model developed in part (a) to find when the temperature reach 101°.

   We use the model $T = 68 + 11h$ with $T = 10$ to find h.

   $$\begin{aligned} T &= 68 + 11h \\ 101 &= 68 + 11h \\ 33 &= 11h \\ 3 &= h \end{aligned}$$

   So it will take 3 hours for the temperature to rise to 101°.

Section P.2 Real Numbers

Key Ideas

A. Real numbers.

B. Converting a repeating decimal to a fraction.

C. Properties of real numbers.

D. Set notations and interval notation.

E. Absolute value.

A. There are many different types of numbers that make up the **real number** system. Some of these special sets are shown below.

Symbol	Name	Set
$\mathbb{N}$	Natural (counting)	$\{1, 2, 3, 4, \ldots\}$
$\mathbb{Z}$	Integers	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathbb{Q}$	Rational	$\left\{r = \frac{p}{q} \,\middle|\, p,\ q \text{ are integers, } q \neq 0\right\}$
$\mathbb{R}$	Reals	Numbers that can be represented by a point on a line.

Every natural number is an integer, and every integer is a rational number. For example: $3 = \frac{3}{1} = \frac{6}{2} = \cdots$. But not every rational number is an integer and not every integer is a natural number. Real numbers that cannot be expressed as a ratio of integers are called **irrational.** π and $\sqrt{2}$ are examples of irrational numbers. Every real number has a decimal representation. When the decimal representation of a number has a sequence of digits that repeats forever it is a rational number.

1. Classify each real number as a *natural, integer, rational,* or *irrational* number.

(a) 17.312

This decimal number terminates. So the repeating sequence of digits is '0' and thus this number is rational.

(b) $-9.1\overline{27}$

Since the digits 27 repeat, this number is rational.

(c) $-18.101001000100001\ldots$

Since there is not a pattern where a portion is repeated, this number is irrational.
Here the pattern is $0\cdots01$ *where the number of zeros grows, so no one sequence is repeated.*

(d) $1.5765780\ldots$

Since there is not a pattern in which a portion is repeated, this number is irrational.

B. When the repeating sequence is different from '0' you can convert a rational number from its decimal representation to a fraction representation by following these steps:

1.	Set $x =$ the repeating decimal.
2.	Multiply x and the decimal representation by enough powers of 10 to bring one repeating sequence to the left of the decimal point.
3.	Multiply x and the decimal representation by enough powers of 10 so that the first repeating sequence starts immediately after the decimal point.
4.	Subtract the results of Step 2 from the results of Step 1. This creates an equation of the form *integer* $\times x =$ *an integer*.
5.	Divide both sides by the coefficient and reduce.

2. Convert each repeating decimal to its fractional representation.

(a) $0.\overline{4578}$

Let $x = 0.\overline{4578}$.
The repeating sequence has 4 digits in it and no digits before the repeating sequence. Start by multiplying both sides by 10^4.

$$\begin{array}{rl} 10000x = & 4578.\overline{4578} \\ 1x = & 0.\overline{4578} \quad \textit{Subtract.} \\ \hline 9999x = & 4578 \\ x = & \frac{4578}{9999} = \frac{1526}{3333}. \quad \textit{Divide.} \end{array}$$

So the fraction representation of $0.\overline{4578}$ is $\frac{1526}{3333}$.

(b) $3.12\overline{3}$

Let $x = 3.12\overline{3}$.
The repeating sequence has 1 digit in it and there are 2 digits before the repeating sequence starts, so we multiply both sides by 10^3: $1000x = 3123.\overline{3}$.
Next we need to multiple both sides by 10^2 to bring the 2 non-repeating digits to the other side of the decimal point: $100x = 312.\overline{3}$.
Thus we get:

$$\begin{array}{rl} 1000x = & 3123.\overline{3} \\ 100x = & 312.\overline{3} \quad \textit{Subtract.} \\ \hline 900x = & 2811 \\ x = & \frac{2811}{900} = \frac{937}{300}. \quad \textit{Divide.} \end{array}$$

So the fraction representation of $3.12\overline{3}$ is $\frac{937}{300}$.

C. The basic properties used in combining real numbers are:

Commutative Laws	$a + b = b + a$	$ab = ba$
Associative Laws	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Distributive Laws	$a(b + c) = ab + ac$	$(b + c)a = ab + ac$

The number 0, called the **additive identity**, is special for addition because $a + 0 = a$ for any real number a. Every real number a has a negative, $-a$, that satisfies $a + (-a) = 0$. **Subtraction** is the operation that undoes addition and we define $a - b = a + (-b)$. We use the following properties to handle negatives:

$(-1)a = -a$	$-(-a) = a$	$(-a)b = a(-b) = -(ab)$
$(-a)(-b) = ab$	$-(a+b) = -a - b$	$-(a-b) = b - a$

The number 1, called the **multiplicative identity**, is special for multiplication because $a \cdot 1 = a$ for any real number a. Every nonzero real number a has an inverse, $1/a$, that satisfies $a \cdot (1/a) = 1$. **Division** is the operation that undoes multiplication and we define $a \div b = a \cdot \frac{1}{b}$. We use the following properties to deal with fractions:

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{ac}{bc} = \frac{a}{b}$	$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$
If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.		

3. Use the properties of real numbers to write the given expression without parentheses.

(a) $4(3 + m) - 2(4 + 3m)$

$12 + 4m - 8 - 6m$

$= 4 - 2m$

$$4(3+m) - 2(4+3m) = 12 + 4m - 8 - 6m$$
$$= 4 - 2m$$

Remember to properly distribute the -2 over $(4 + 3m)$.

(b) $(2a + 3)(5a - 2b + c)$

$$(2a+3)(5a-2b+c)$$
$$= (2a+3)5a + (2a+3)(-2b) + (2a+3)c$$
$$= 10a^2 + 15a - 4ab - 6b + 2ac + 3c$$

C. The real numbers can be represented by points on a line, with the positive direction towards the right. We choose an arbitrary point for the real number 0 and call it the **origin**. Each positive number x is represented by the point on the line that is a distance of x units to the right of the origin, and each negative number $-x$ is represented by the point x units to the left of the origin. This line is called the **real number line** or the **coordinate line**. An important property of real numbers is **order**. Order is used to compare two real numbers and determine their relative position.

Order	Symbol	Geometrically	Algebraically
a is less than b	$a < b$	a lies to the left of b	$b - a$ is positive
a is greater than b	$a > b$	a lies to the right of b	$a - b$ is positive
a is less than or equal b	$a \leq b$	a lies to the left of b or on b	$b - a$ is nonnegative
a is greater than or equal b	$a \geq b$	a lies to the right of b or on b	$a - b$ is nonnegative

4. State whether the given inequality is true or false.
   (a) $-3.1 > -3$

   False. $-3.1 - (-3) = -0.1$

   (b) $2 \le 2$

   True. $2 - 2 = 0$

   (c) $15.3 \ge -16.3$

   True. $15.3 - (-16.3) = 31.6$

5. Write the statement in terms of an inequality.
   (a) w is negative.

   $w < 0$. w is negative is the same as saying that w is less than 0. You can also express this as $0 > w$.

   (b) m is greater than -3.

   $m > -3$ or $-3 < m$

   (c) k is at least 6.

   $k \ge 6$ or $6 \le k$

   (d) x is at most 7 and greater than -2.

   $-2 < x \le 7$

D. A **set** is a collection of objects, or **elements**. A capital letter is usually used to denote sets and lower case letters to represent the elements of the set. There are two main ways to write a set. We can **list** all the elements of the set enclosed in { }, *brackets*, or list a few elements and then use ... to represent that the set continues in the same pattern. Or we can use **set builder notation**, "$\{x \mid x \text{ has property } P\}$." This is read as "*the set of x such that x has the property P.*" The two key binary operations for sets are called **union** and **intersection**. The *union* of two sets is the set that consists of the elements that are in *either* set. The *intersection* of two sets is the set that consists of the elements in *both* sets. The **empty set, $\emptyset$,** is a set that has no elements. **Intervals** are sets of real numbers that correspond geometrically to line segments. When the endpoint are included in the interval use the symbols [and]. When the endpoint are excluded in the interval use the symbols (and). Study the table on page 18 of the main text and notice the differences in how (,), [, and] are used.

6. Let $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{3, 6, 9, 12\}$, and $C = \{1, 3, 5, 7, 9, 11\}$.
   Find $A \cup B$, $A \cup C$, and $B \cup C$.

   $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$.

   $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

   $B \cup C = \{1, 3, 5, 6, 7, 9, 11, 12\}$.

7. Let $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{3, 6, 9, 12\}$, and $C = \{1, 3, 5, 7, 9, 11\}$.
Find $A \cap B$, $A \cap C$, and $B \cap C$.

$A \cap B = \{6, 12\}$.

$A \cap C = \emptyset$.

$B \cap C = \{3, 9\}$.

8. Let $A = \{x \mid x < 8\}$, $B = \{x \mid 3 \leq x < 7\}$, and $C = \{x \mid 5 < x\}$.
Find $A \cup B$, $A \cup C$, and $B \cup C$.

$A \cup B = \{x \mid x < 8\}$.

$A \cup C = \{x \mid -\infty < x < \infty\} = \mathbb{R}$.

$B \cup C = \{x \mid 3 \leq x\}$.

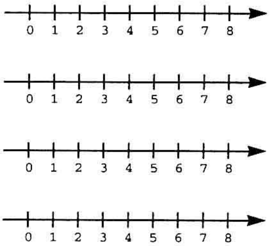
9. Let $A = \{x \mid x < 8\}$, $B = \{x \mid 3 \leq x < 7\}$, and $C = \{x \mid 5 < x\}$.
Find $A \cap B$, $A \cap C$, and $B \cap C$.

$A \cap B = \{x \mid 3 \leq x < 7\}$.

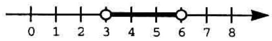
$A \cap C = \{x \mid 5 < x < 8\}$.

$B \cap C = \{x \mid 5 < x < 7\}$.

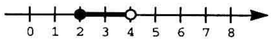
10. Graph the intervals $T = (3, 6)$, $V = [2, 4)$. Find $T \cup V$ and $T \cap V$ in interval notation, then graph.



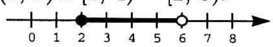
$T = (3, 6)$



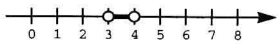
$V = [2, 4)$



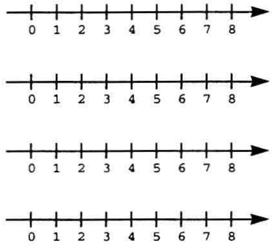
Unions are elements in *either* set.
$T \cup V = (3, 6) \cup [2, 4) = [2, 6)$.



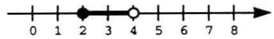
Intersections are elements in *both* sets.
$T \cap V = (3, 6) \cap [2, 4) = (3, 4)$.



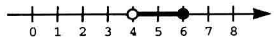
11. Graph the intervals $V = [2, 4)$, $W = (4, 6]$. Find $V \cup W$ and $V \cap W$ in interval notation, then graph.



$V = [2, 4)$

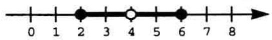


$W = (4, 6]$



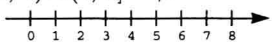
Unions are elements in *either* set.

$V \cup W = [2, 4) \cup (4, 6]$. *Note we cannot express the union in simpler notation.*



Intersections are elements in *both* sets.

$V \cap W = [2, 4) \cap (4, 6] = \emptyset$.



E. The **absolute value** of a number a, denoted by $|a|$, is the distance from the number a to 0 on the real number line. Remember absolute value is always positive or zero. It is also defined as $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$. If a and b are real numbers, then the distance between the points a and b on the real line is $d(a, b) = |b - a|$. By Property 6 of negatives $|b - a| = |a - b|$ (*since the distance between the points a and b is the same as the distance between the points b and a*).

12. Find each absolute value.

(a) $|-7|$

Since $-7 < 0$ we use the definition $|a| = -a$, so $|-7| = -(-7) = 7$.

(b) $|8|$

Since $8 \geq 0$ we use the definition $|a| = a$, so $|8| = 8$.

(c) $|\pi - 5|$

$|\pi - 5| = -(\pi - 5) = 5 - \pi$, since $\pi - 5 < 0$.

(d) $\left|\sqrt{10} - 3\right|$

$\left|\sqrt{10} - 3\right| = \sqrt{10} - 3$.
Since $\sqrt{10} > \sqrt{9} = 3$, so $\sqrt{10} - 3 > 0$.

13. Find the distance between each pair of numbers.

(a) 4 and 10

$|4 - 10| = |-6| = 6$.
Remember to do the work on the inside of the absolute value bars *first*!

(b) 5 and -3.

$|5 - (-3)| = |5 + 3| = |8| = 8$.

(c) -2 and -9

$|-2 - (-9)| = |-2 + 9| = |7| = 7$.

Section P.3 Integer Exponents

Key Ideas

A. Key exponent definitions and rules.

B. Scientific notation.

A. The key exponent definitions are:

$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors of } a}$	$a^0 = 1,\ a \neq 0$	$a^{-n} = \dfrac{1}{a^n},\ a \neq 0$

In addition to the exponent definitions, the following key exponent laws should be mastered.

$a^m a^n = a^{m+n}$	To multiply two powers of the same number, add the exponents.
$\dfrac{a^m}{a^n} = a^{m-n},\ a \neq 0$	To divide two power of the same number, subtract the exponents.
$(a^m)^n = a^{mn}$	To raise a power to a new power, multiply the exponents.
$(ab)^n = a^n b^n$	To raise a product to a power, raise each factor to the power.
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n},\ b \neq 0$	To raise a quotient to a power, raise both numerator and denominator to the power.
$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$	To raise a fraction to a negative power, invert the fraction and change the sign of the exponent.
$\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$	To move a number raised to a power from numerator to denominator or from the denominator to numerator, change the sign of the exponent.

1. Simplify.

(a) x^3x^5

$x^3x^5 = x^{3+5} = x^8$

(b) $w^{12}w^{-8}$

$w^{12}w^{-8} = w^{12+(-8)} = w^4$

(c) $\dfrac{y^8}{y^{15}}$

$\dfrac{y^8}{y^{15}} = y^{8-15} = y^{-7} = \dfrac{1}{y^7}$ or

$\dfrac{y^8}{y^{15}} = \dfrac{1}{y^{15-8}} = \dfrac{1}{y^7}$

(d) $(5x)^3$

$(5x)^3 = 5^3x^3 = 125x^3$

(e) $(m^3)^7$

$(m^3)^7 = m^{3\cdot7} = m^{21}$

(f) $\frac{2^9}{2^7}$

$\frac{2^9}{2^7} = 2^{9-7} = 2^2 = 4$ or

$\frac{2^9}{2^7} = \frac{1}{2^{7-9}} = \frac{1}{2^{-2}} = 2^2 = 4$

A. Shortcut: In problems like 1c and 1f above, compare the exponents in the numerator and denominator. If the exponent in the numerator is larger, we simplify the expression by bringing the factor up into the numerator. If the exponent in the denominator is larger, we simplify the expression by bringing the factor down into the denominator.

2. Simplify.

(a) $\frac{27^3}{9^5}$

Since both 27 and 9 are powers of 3, first express each number as a power of 3.

$$\frac{27^3}{9^5} = \frac{(3^3)^3}{(3^2)^5} = \frac{3^{3\cdot3}}{3^{2\cdot5}} = \frac{3^9}{3^{10}} = \frac{1}{3^{10-9}} = \frac{1}{3}$$

(b) $(2x^7y^5)(3x^2y^3)^4$

$$\begin{aligned}(2x^7y^5)(3x^2y^3)^4 &= (2x^7y^5)[(3)^4(x^2)^4(y^3)^4] \\ &= (2x^7y^5)(3^4x^8y^{12}) \\ &= (2)(81)(x^7x^8)(y^5y^{12}) \\ &= 162x^{15}y^{17}\end{aligned}$$

(c) $\left(\frac{x^4y}{z^6}\right)^3\left(\frac{xz^3}{y^4}\right)^5$

$$\begin{aligned}\left(\frac{x^4y}{z^6}\right)^3\left(\frac{xz^3}{y^4}\right)^5 &= \frac{(x^4)^3y^3}{(z^6)^3}\frac{x^5(z^3)^5}{(y^4)^5} \\ &= \frac{x^{12}y^3}{z^{18}}\frac{x^5z^{15}}{y^{20}} \\ &= (x^{12}x^5)\left(\frac{y^3}{y^{20}}\right)\left(\frac{z^{15}}{z^{18}}\right) \\ &= \frac{x^{17}}{y^{17}z^3}\end{aligned}$$

3. Eliminate negative exponents and simplify.

$\frac{3a^3b^{-4}}{2a^{-2}b^{-1}}$

$$\begin{aligned}\frac{3a^3b^{-4}}{2a^{-2}b^{-1}} &= \frac{3}{2}a^{3-(-2)}b^{-4-(-1)} \\ &= \frac{3}{2}a^5b^{-3} = \frac{3a^5}{2b^3}\end{aligned}$$