

DEVELOPMENTAL PSYCHOLOGY SERIES

# DEVELOPMENTAL MODELS OF THINKING

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## Preface

This volume presents detailed views of the state of the art in the field of developmental models of thinking. These views, generally divisible into mathematical and psychometric models, information processing models, and Piaget-centered models are provided by outstanding researchers in the field.

The problem of how intellectual abilities evolve during human ontogenesis is central to psychological research. Because of the relevance of this question for diagnostic purposes and for the improvement of instruction, further progress is strongly demanded from psychologists in the field. The present state of theory construction in developmental psychology is considered to be insufficient, despite the extensive work of Piaget and his group, which dominates the area and leads to a steadily growing number of studies. However, there do exist some new formalized developmental theories of thinking. Many of these theories show a high degree of precision and testability. Furthermore, they capture some of the essentials of Piaget's theory and, more importantly, overcome some of its insufficiencies. Moreover, there are experimental approaches that are explicitly guided by theoretical models of cognitive development. These theories focus more on the empirical examination of derived hypotheses and the refinement of models, and are less guided by Piaget's theory.

These new lines of research were the basis for a conference entitled "Developmental Models of Thinking," held at the University of Kiel (Germany) from September 11–14, 1977. The Kiel conference enabled for the first time a fruitful and critical comparison of all three major approaches—Piagetian, measurement, and information processing. The so-called measurement approach is based on mathematical and psychometric models of intellectual development. The information processing approach is best described by the work of the Carnegie-Mellon group.



However, as this volume shows, research not relying on computer simulation is also included.

The selection of participants was made primarily with respect to our goal to invite psychologists who contribute through their work to the construction of models in the field of cognitive development. Theoretical models were to be discussed, as well as the corresponding empirical results. These results provided a concrete basis for the comparison of different approaches because, in some cases, the empirical studies were based on the same problems. The contributions for this volume are based on some of the presentations during the conference. They have been considerably revised and rewritten for this volume in order to fit into an integrated, edited treatise.

The volume contains 12 chapters which form three subsets. Chapters 1–5 address the description of cognitive development through the use of mathematical and psychometric models. One characteristic of these contributions is the explicit consideration of interindividual differences. Chapters 6–10 represent variants of the information processing approach. And Chapters 11–12, provide an evaluation of the Genevan approach and of some recent directions taken by this type of research on cognitive development.

The book provides an extensive and integrative overview of the variety of ongoing theoretical and empirical research aiming at developmental models of thinking. This makes it of general interest to psychologists in all fields. It is of particular interest to workers in the fields of developmental, cognitive, and instructional psychology and to educators.

We wish to thank all of the authors for their readiness to follow our requests for revisions and modifications. Without this cooperation, this comprehensive volume would not have been possible. Furthermore, we are grateful to all the participants of the Kiel Conference in September 1977 for their contributions. This conference was sponsored by the German Stiftung Volkswagenwerk (Az. 34 314). The support of Urs Baumann and Hermann Wegener, faculty members of the Department of Psychology at Kiel University, enabled us to hold the conference. Mrs. Sinn provided invaluable assistance in taking care of all administrative problems.

We want to thank the Institute for Science Education at the University of Kiel and its managing director, Karl Frey.<sup>1</sup> The Institute was especially helpful in organizing the conference. We also wish to acknowledge the assistance of Aida Starke, Ute Kühl, Petra Möller, and Monika Wenhart in preparing this volume.

<sup>1</sup> The IPN is a research institute financed by Schleswig-Holstein and by the German federal government. The institute's function is a national one. Through its research work, it aims to further develop and promote science education.

# Overview

The contributions to this volume address the question of how to model cognitive development. The authors who have been brought together here are proponents of different research approaches. As a consequence, the answers vary considerably. A common feature, however, is the explicit search for theory-guided hypotheses about cognitive development and the goal of empirical studies in order to improve the theory.

Spada and Kluwe (Chapter 1) point out that the construction of formal models should be a central goal for developmental psychology. They review studies of two different models of thinking and intellectual development. These models are based on hypotheses about cognitive operations used by children of different ages to solve certain problems. They are also based on psychometric assumptions, and allow the specification and testing of hypotheses about quantitative and structural cognitive developmental changes in problem-solving processes. One model is based on assumptions of the deterministic model of structural learning (Scandura 1973) and corresponds to the findings of Inhelder and Piaget (1958). The second model is a generalization of the probabilistic latent trait model of Rasch (1966). Both models are compared on the basis of data that have been collected under different experimental conditions.

Lüer (Chapter 2), in his evaluation of mathematical models as a possible means for describing cognitive development, addresses the work done by Spada and Kluwe. His criticism focuses on restrictions and assumptions connected with some mathematical models. Favoring the information processing approach himself, Lüer claims that some mathematical models simplify the understanding of cognitive development. However, he shows connections between the mathematical approach and the information processing approach that probably make the distinction an arbitrary one.

The contribution of Thomas (Chapter 3) represents a theory of develop-

mental growth that focuses on the functional relationship between means and standard deviations at various age levels.<sup>1</sup> Referring to early work of Thurstone, Thomas discusses the high sample correlation coefficients (e.g., for cognitive measures) computed on pairs of sample means and standard deviations obtained at each of the varying age points. He is able to show that analysis of variance as a model for developmental growth is readily falsified. He concludes that viewing growth phenomena within additive structures is inappropriate. Instead, Thomas proposes a multiplicative model in which the growth process is presented as a product of an age-specific growth parameter and random variables.

Bentler (Chapter 4) gives an extensive outline of the possibilities of studying cognitive development through causal modeling with qualitative data. Starting from a detailed discussion of the problems of description and explanation in developmental research, he presents a non-technical overview of latent attribute models (latent structure model, scalability model, latent trait model, factor analysis model), description models (dichotomous regression model, structural equation model), and multinomial response models (log-linear model, cross-classification with errors model).

Rudinger and Ruppel (Chapter 5) discuss the description of quantitative changes in cognitive development, taking into account the question of interindividual differences. Their main goal is to show the relationship between the quantitative approach and the qualitative-structural approach to cognitive development; they are considered to be complementary. Sternberg's componential analysis is discussed as a possible paradigm for combining the psychometric tradition and the qualitative-structural approach. However, the authors' evaluation of this paradigm lead them to a related model of mental abilities proposed earlier by Selz (1935). His model of intelligence and his method of training defined cognitive processes are considered to be promising with regard to the quantification of intellectual development.

Following this group of contributions, which deals predominantly with questions of the quantitative description and mathematical modeling of cognitive development, Klahr (Chapter 6) gives an outline of the information processing approach to research on cognitive development as it is followed at Carnegie-Mellon University. He focuses on computer simulation of different levels of cognitive competence as a method to elaborate a model of cognitive development. Basically Klahr aims at task-specific information processing models to account for learning and performance. Several empirical paradigms provide a good illustration of how a system

<sup>1</sup> This contribution was not originally included in the Kiel Conference since Professor Thomas was not able to accept an invitation due to previous commitments.

of production rules solves a central problem of information processing models, that is, what knowledge is available at different age levels. Klahr also discusses the advantages of self-modifying production systems.

Groner (Chapter 7), known for his work in mathematical learning theories, is an expert in mathematical psychology as well as in computer simulation. He and his coauthors, Keller and Menz, give a detailed criticism of the computer simulation approach to cognitive development.

The next three chapters by Joffe Falmagne, Greeno, and Resnick represent different examples of the information processing approach to cognitive development. The chapter by Joffe Falmagne (Chapter 8) aims at a theoretical framework for propositional reasoning and its development during childhood. The discussion of differences between her approach and the Piagetian approach leads to the tentative conclusion that young children do master some pattern of inductive inference at a fairly abstract level. Joffe Falmagne elaborates a theoretical framework that assumes that logical competence may develop in part as a result of a concept learning process: The structural concept being learned is a given rule of inference. A central part of the theoretical framework is obviously the changing availability of alternative modes of representation for verbally given information during the course of cognitive development.

The basic problem emphasized by Greeno (Chapter 9) is related to Joffe Falmagne's introductory discussion of the differences between the Genevan and the "propositional" approaches: the understanding of tasks. Greeno's chapter focuses on two forms of conceptual understanding: semantic interpretations of arithmetic, and arithmetic as a semantic model of a more abstract formal language. With respect to the first form, Greeno attempts a detailed description of how children generate a semantic interpretation of a given problem and of how they identify the appropriate relationship in the formal language of arithmetic. With respect to the second form of conceptual understanding, Greeno introduces an important distinction between implicit and explicit understanding. Unlike explicit understanding, an implicit conceptual understanding of arithmetic does not require knowledge of a formal language. However, the procedures that are used for reasoning in a given domain like arithmetic should follow the properties specified by a formal language. This distinction is important in light of the question of what children acquire during development. Referring to Piaget and his work on class inclusion, Greeno points out that Piaget did not assert that children acquire explicit understanding of a formal language of logic and set theory. As a consequence, Greeno claims a theory of procedures is necessary in order to analyze children's acquisition of general cognitive principles by studying their cognitive procedures.

Resnick's empirical and rational task analysis (Chapter 10) presents a way to analyze how children detect and solve problems. From her empirical and theoretical work, she concludes that stimulation of the development of task-oriented sequences of cognitive states will improve intellectual abilities. This development leads to an increasingly comprehensive understanding of the tasks and enables the children to reach the originally formulated objectives. It is of interest that careful experimental research on arithmetic and instruction led Resnick to study the role of invention in the process of the learning of thinking.

The last two chapters center on questions and problems connected with new research based on the theory of Piaget. Beilin (Chapter 11) ponders the alternative "revision or refinement of Piaget's theory," discussing the role of transformations and correspondences in cognition, the problem of decalage, and the interrelations between learning and development and between language and thought. Beilin, one of the American experts on Piaget-centered research, gives an integrative overview of the present state of the field.

Pascual-Leone (Chapter 12) critically examines Piaget's data base and methodology in order to illustrate the empirical invariances underlying the Piagetian notions of stage and equilibration. He also compares Piaget's data base and methodology with those of information processing approaches—in particular that at Carnegie-Mellon University. Of special interest is his discussion of the method of "metasubjective task analysis" and of the problem of "truly novel" performances.

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