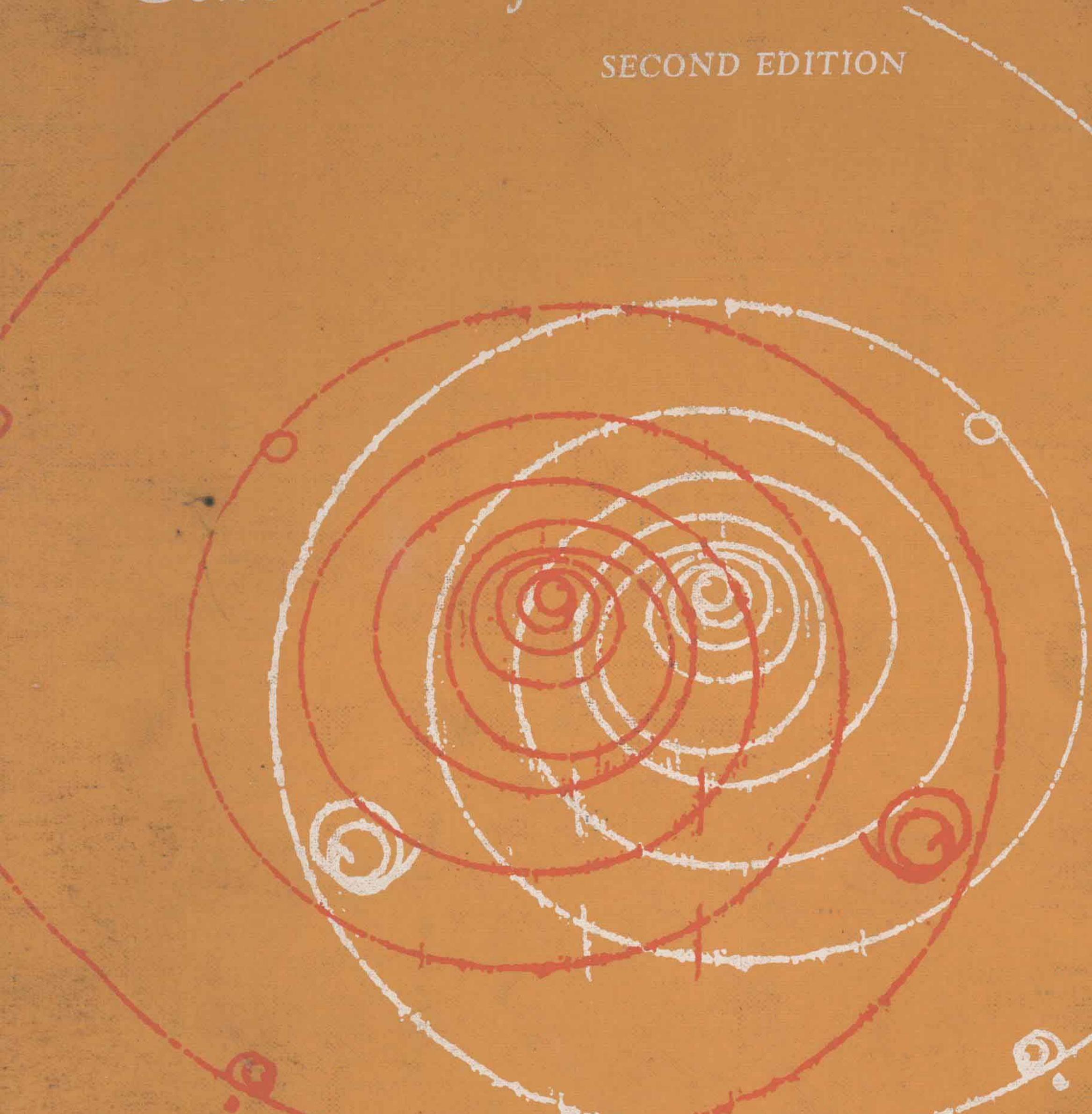
ROBERT T. SEELEY

Calculus of One Variable

SECOND EDITION



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In the editorial series of

I. M. SINGER

Massachusetts Institute of Technology

To Harold, Marguerite, and Rita

Preface

This second edition, like the first, aims for significant applications of calculus as early as possible. I was glad of the opportunity to improve the exposition and organization toward this end, and have practically rewritten the entire text. The organization is explained in the Introduction, but a few points deserve emphasis here.

All the elementary functions are introduced and differentiated in Chapter II; this is convenient for the rest of the text, for instructors, and for readers who might well be encountering these functions elsewhere at the same time.

Differential equations appear early and often. The equations governing constant acceleration, exponential growth, and periodic motion are discussed and solved in Chapter III, and others are solved as the necessary techniques become available.

Definite integrals are introduced first as areas, then as Riemann sums, and finally are evaluated by the Fundamental Theorem of Calculus. It is possible, perhaps, to short-cut Riemann sums, but it then becomes more difficult to explain the applications of integration, and the Fundamental Theorem.

The last three chapters develop the ideas of approximations, sequences, and series. If the appendix material on limits and continuity is to be covered in any detail, it should follow approximations and sequences; without this background, the definitions and proofs are, inevitably, hard to understand.

The level of maturity rises gradually in the text as a whole, in individual chapters, and even within some chapter sections; every instructor should know how far to go with his particular class.

The significance of many theorems is easily hidden even from the best students. I make a special point to show why we need theorems on maximum values, convergence of Riemann sums, and convergence of infinite series. These need not be proved, but they must be understood. Those who do follow the proofs will find the Least Upper Bound Axiom presented with similar care.

This edition owes much to the reviews and suggestions by readers of the original and revised manuscripts, and particularly by users of the first edition. I thank all these collaborators anonymously; the entire list is too long, and a partial list has no logical stopping place. I am grateful to the publishers for their help in collecting this advice, and their generous attitude toward last-minute changes. Special thanks are due to the editor for his hard work in arranging page breaks and figures, and in rooting out errors and cloudy passages. In spite of all this, doubtless some errors remain. The author and publishers will be grateful to readers who bring these to our attention, or who have other comments to make.

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Chapter VI *Techniques of Integration*

Chapter VII *Vectors and the Laws of Motion*

Chapter VIII *Complex Numbers*

Introduction

The road to understanding in calculus is a long one. Before setting out, we would like to sketch how calculus developed, and introduce briefly some of the men who contributed to it. With this background, we outline the plan of the book, and suggest several ways to use it.

A THUMBNAIL SKETCH OF THE HISTORY OF CALCULUS

Calculus rests on mathematical developments that go back as far as four thousand years to the Babylonians and the Egyptians, but we won't start there. The immediate contributions came at the end of the Renaissance.

Preliminary steps

First came the development of algebra. In the 1500's, the Italians achieved spectacular results in the solution of the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, and in the process they advanced the use of negative and complex numbers. In 1585, Simon Stevin of Bruges published *La Disme*, the first proposal of a systematic use of decimal expansions. And there were improvements in notation by many, including René Descartes, abandoning clumsy verbal communication in favor of symbols very much like ours today. (This step is more important than it seems. A good notation not only saves space on the paper; it saves space in the memory and effort in the mind, both of which are at a premium.)

Second came the development of analytic geometry, due in large part to Descartes' *La Géometrie* (1637), which made it obvious that the new efficiency in algebra could be well employed in geometry. Pierre Fermat, a contemporary of Descartes, also made important contributions in this direction, finding equations of straight lines and conic sections.

Third, there were various methods for determining tangents, areas, and volumes that led naturally to the ideas of differentiation and integration, the two main ideas of calculus. The best known of these methods are Fermat's way of drawing tangents (discovered about 1629) and B. Cavalieri's determination of areas and volumes, published in 1635. And Isaac Barrow showed an important connection between these two ideas, tangents and area, that was a direct forerunner of the fundamental theorem relating differentiation and integration.

Fourth, there were results on infinite series and products, primarily due to John Wallis, which appeared in 1655.

Finally, in physics there was a concerted study of motion, primarily the motion of falling bodies and of the planets. The most famous men in this work were Galileo, the astronomical observer Tycho Brahe, and Johann Kepler, who analyzed Tycho's observations and deduced three simple but profound empirical laws governing planetary motion.

The invention of calculus

Isaac Newton, a student of Barrow, gathered the mathematical developments together into one general theory, calculus, and applied it to solve the physical problems of the motion of falling bodies and of the planets. He showed that Kepler's laws imply the "inverse square" law of gravitation: Each planet is attracted to the sun by a force proportional to m/r^2, where m is the mass of the planet and r is the distance from planet to sun.

Newton did not shout *Eureka!* and run into the streets to announce his discovery. Perhaps his caution was due to an error in the commonly accepted distance to the moon, which was not corrected until 1679: because of the error, the force of gravity at the surface of the earth (as found from falling bodies) did not agree well enough with the force that Newton derived to explain the motion of the moon. Whatever the cause of the delay, by the time his invention of calculus was finally made known, much of the same theory had been found independently by Gottfried Leibniz, his contemporary. Thus Newton and Leibniz are both considered the inventors of calculus.

Neither Newton nor Leibniz succeeded in making the logic of their methods understood. Their reasoning was so mysterious that George Berkeley, an Irish bishop, published in 1734 the famous pamphlet *The Analyst* in which he defended his own faith by pointing out that Newton and his followers treated objects no more substantial than "ghosts of departed quantities," and that the foundations of religion were every bit as secure as those of Newton's analysis.

Development and application

In spite of the logical difficulties, both Newton and Leibniz had strong evidence that their methods contained some essential truth. Newton could explain the motion of the planets. And Leibniz had expressed his discoveries in a notation so apt that, although nobody understood exactly why, it led automatically to results that were seen to be correct.

From the end of the seventeenth century to the beginning of the nineteenth, calculus developed in the notation and outlook of Leibniz, but continued to find its inspiration and application in the project of explaining the physical world by mathematics, so successfully begun by Newton. The greatest mathematicians of this period were Leonhard Euler (1707–1783) and Joseph Louis Lagrange (1736–1813). Euler wrote the first widely read texts on calculus and others equally popular on algebra and trigonometry. He made advances in all fields of mathematics, and in dynamics, in the study of "least action" and energy, in the three-body problem of astronomy (the effect of mutual attractions between the earth, moon, and sun), in hydraulics, and in optics. Lagrange pursued these same questions, achieving greater unity and generality. His greatest work is the monumental *Mécanique analytique* (1788), which brought the science of mechanics close to its present form.

The great wealth of mathematical results, consistent with itself and with physical observations, proved beyond a doubt that calculus had abstracted certain essential features of the universe in which we live. But the logical foundations were still poorly understood, and even Euler was occasionally led by his formal manipulations to results that can hardly be considered correct. (One of these aberrations serves as a bad example in Chapter XI below. Unfortunately, most of Euler's voluminous and outstanding work is beyond the scope of this book, so we are not able to balance the bad impression created by this one example.)

The first great mathematician of the nineteenth century was Carl Friedrich Gauss (1777–1855), who made important contributions in the theory of the integers, use of infinite series, theory of surfaces, complex numbers, difficult numerical computations, astronomy, electricity and magnetism, surveying, and development of the telegraph.

Securing the foundations

A further contribution of Gauss was to the underlying logic of calculus, overcoming the valid objections to the work of the founders. This development continued with Augustin Cauchy's book *Cours d'analyse* (1821) and culminated in the work of Karl Weierstrass (1815–1897) and Richard Dedekind (1831–1916). Dedekind's contribution was a penetrating analysis of the nature of the real numbers; Weierstrass pointed out subtle logical oversights in the work of his predecessors, and in his own work he adopted the standards of rigor and logic that still apply today.