

FOUNDATIONS OF CASUALTY ACTUARIAL SCIENCE



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Second Edition

Casualty Actuarial Society
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Textbook Steering Committee
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PREFACE

This landmark book is the first published, complete text containing the fundamentals of casualty actuarial science as practiced in North America. It is intended as an introduction to casualty actuarial concepts and practices. Its target audiences are members and students of the Casualty Actuarial Society, university and college students, plus insurance and general business professionals with a need for basic knowledge on these topics.

In designing the textbook, the Casualty Actuarial Society concluded that the readership would be best served by having each chapter written by an expert in the topic covered by the chapter. Therefore, each chapter is individually authored and the styles and organization vary somewhat. The chapters reflect the views of the individual authors and the content should not be considered as the official opinion of the Casualty Actuarial Society.

The book would not have been possible, were it not for the dedication of the chapter authors, Steven P. D'Arcy, Robert J. Finger, Charles C. Hewitt, Jr., Charles L. McClenahan, Gary S. Patrik, Matthew Rodermund, Margaret W. Tiller, Gary G. Venter, and Ronald F. Wiser, who produced actual text and met the challenge of deadlines.

The development of this text is not solely attributable to the authors' efforts, however. To the many who have helped along the way, on behalf of the Textbook Steering Committee, I offer sincere thanks. At the risk of overlooking someone's contribution, I would like to thank those who have helped this project to its conclusion.

The genesis of this book was in 1969 with L.H. Longley-Cook, at the time he joined the faculty of Georgia State University. The end product of the effort is this text, *Foundations of Casualty Actuarial Science*. Along the way, the project was furthered through the successive leadership efforts of Charles C. Hewitt, Jr., Richard L. Johe, W. James MacGinnittie, C.K. Khury, and Charles L. McClenahan. And, of course, there are those authors, especially Bernard L. Webb, whose efforts produced the predecessor draft chapters, known collectively as *Casualty Contingencies*.

Thanks also go to the members of the Textbook Steering Committee, whose organization, planning, leadership, and follow-through brought this

book to its publication: Donald T. Bashline, Lisa G. Chanzit, William R. Gillam, Richard A. Lino, and Edward P. Lotkowski. All of the work of the Textbook Steering Committee was done under the aegis of the Vice President—Development Charles A. Bryan to whom the committee owes much for his continued involvement and encouragement.

Each of the chapters was reviewed by an ad hoc panel of experts. The task of each panelist was to read a draft chapter, to note areas requiring editing, and to meet with the author at a group session to discuss the findings. Those unable to attend the meetings spent time on the telephone with authors or corresponded with them. Thanks go to: Nolan E. Asch, Steven D. Basson, Robert A. Bear, Paul J. Braithwaite, Charles A. Bryan, Jerome A. Degerness, James A. Faber, Patricia A. Furst, Robert A. Giambo, Owen M. Gleeson, Anthony J. Grippa, David N. Hafling, Philip E. Heckman, Charles C. Hewitt Jr., John J. Kollar, Howard C. Mahler, John S. McGuinness, Michael A. McMurray, Glenn G. Meyers, Michael J. Miller, Robert A. Miller III, Deborah M. Rosenberg, Alan I. Schwartz, Jane C. Taylor, Michael A. Walters, and Richard G. Woll.

Thanks also go to all those who offered their comments to the authors through special review sessions at meetings of the CAS. Each of the chapters was printed in draft form in the Forum and each author held an individual session at a CAS meeting to receive comments from the membership at large. I am unable to name these individuals, but I thank them heartily for their efforts nonetheless.

The penultimate effort of the book was the technical editing done by Alan Kennedy of the American Academy of Actuaries. His skilled hand may appear invisible to the reader, but its presence is much appreciated. And, finally, the actual printing of the text was accomplished through the efforts of the Editorial Committee, chaired by Eugene C. Connell, with Robert F. Lowe serving as editor of Special Publications.

It is the sincere hope of the Textbook Steering Committee that this textbook will serve its purpose well in the years to come. This is the first edition of what must be a living document. Undoubtedly, there will be continuous review, resulting in deletions, additions, and updates. I hope that this edition has started a process which will continue successfully over the years. It is my personal privilege to have been part of this process, at such an auspicious time.

PREFACE TO SECOND EDITION

The Second Edition of the Casualty Actuarial Society's textbook *Foundations of Casualty Actuarial Science* has been printed in response to the increasing demand for the book. This need has given the Editorial Committee the opportunity to incorporate the short set of errata to the First Edition and to develop a revised and expanded Index which should greatly assist the reader in using the text. An appendix to chapter four, titled "An Actuarial Model of Loss Development" has also been added. These enhancements would not have been possible without the efforts of many individuals, most notably, Tom Carpenter and Steven Basson, who incorporated the errata; and Abbe Bensimon, Ruy Cardoso, Daniel Crifo, James Golz, Rebecca Moody, Stephen Philbrick, Debbie Schwab and Patrick Woods, who revised and expanded the Index. This revision could not have occurred without their efforts.

The Casualty Actuarial Society

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Chapter 1

INTRODUCTION

by Matthew Rodermund

What It's All About

If it is agreed that an actuary is one who analyzes the current financial implications of future contingent events, then it might also be agreed that actuarial science concerns, first, the realistic perception of such contingent events and, second, the critical study of their current financial implications.

The foregoing definitions of the actuary and of actuarial science apply to all types of actuaries—life, health, pension, and casualty and property—but in different degrees and clothed in different perspectives. The future contingent events for life and pension actuaries involve, mostly, mortality, but life actuaries study the current financial implications of dying, and pension actuaries the financial implications of continued living. For health actuaries, the future contingent events are sickness and disability (with death as the extreme case), and they ponder the current financial implications of the need (by individuals and by the social order) for medical treatment and rehabilitation.

Surely, much can be written about the actuarial science of disciplines other than casualty and property insurance, but this book is about casualty actuarial science; to that discipline we will confine our remarks.

For casualty actuarial science (we will omit the word “property” for convenience, just as we do in the name of the Casualty Actuarial Society), the future contingent events are so widely varied that they cannot be described in a phrase. They may best be characterized by Murphy’s Law: If it can happen, it will. The current financial implications of such events defy precise measurement. And classical probability procedures haven’t helped much. That is why casualty actuaries must embrace *a priori*, or even intuitive, probabilities, in addition to experience indications, if they are to get on with their jobs.

The mention of probabilities reminds us to state the obvious, that probability theory (whether classical or Bayesian) forms the basis of actuarial science. If the actuaries hadn't had probability theory, they would have had to invent it. In "An Introduction to Credibility Theory," Laurence H. Longley-Cook (1962) quotes a statement by E. W. Phillips, from *Biometry of the Measurement of Mortality*, which is interesting because in 1935 it forecast as destiny for actuaries what was already rooted into their lives. It also foretold their future concerns:

The calculus of probability is a fascinating subject, and one which is destined to play a large part in actuarial science; and a day may come when it can truly be said of the actuary that he has fused together the theories of finance and probability.

The Beginnings

It all began with the advent of workmen's (now *workers*) compensation. That statement holds for casualty actuarial science, and it holds for the Casualty Actuarial and Statistical Society of America (CASSA), which later became the Casualty Actuarial Society. The first constitutionally accepted state workmen's compensation law, passed in Wisconsin in 1911, began to excite interest among scattered members of both the Actuarial Society of America (mostly in the East) and the American Institute of Actuaries (mostly in the Midwest). Even before the New York State Workmen's Compensation Act was passed, in 1914, the interested actuaries (plus many people whose interest was not actuarial but either statistical or social or both—including, among the latter, I. M. Rubinow, the founder and first president of CASSA) had realized the need to establish a technically sound basis for this new "social" insurance. Out of this interest came the professional society that we have inherited.

Considerable work in ratemaking for employers liability insurance had been done in the late 1890s; it depended largely on loss ratio comparisons, and these were studied for about eight industrial classifications in each of several regions of the country. In 1909, a conference on workmen's compensation was held (in Atlantic City) at which papers by future charter members of CASSA were among those presented. In his book, *Social Insurance*, published in 1913,

Rubinow included a section on industrial accidents. In 1914, Albert H. Mowbray, who was to be one of the charter members of CASSA, presented to the Actuarial Society of America a paper on the criteria for testing the adequacy of rates for workmen's compensation insurance (1914a). At the same meeting, Harwood E. Ryan, also to be a CASSA charter member, delivered "A Method of Determining Pure Premiums for Workmen's Compensation Insurance" (1914).

On November 7 that same year, CASSA was born.

The new society tackled the workmen's compensation problems directly. Among the first (and it has ever been thus) was the question of how to use relatively scanty experience to make justifiable rates. The second paper in Volume I of the *Proceedings*, by Mowbray (clearly one of the giants at that time), was "How Extensive a Payroll Exposure Is Necessary To Give a Dependable Pure Premium?" (1914b). That paper represented the first formal introduction to the concept of credibility, the concept that the volume of past experience of a risk or class of risks is a considerable factor in the weight, or "credibility," to be given such experience in using it for ratemaking.

It is the concept of credibility that has been the casualty actuaries' most important and most enduring contribution to casualty actuarial science. Any list of the great contributors to casualty actuarial science would also be a list of those who developed and implemented the theories of credibility: Albert H. Mowbray, Albert W. Whitney, G. F. Michelbacher, Winfield W. Greene, Francis S. Perryman, Paul Dorweiler, Thomas O. Carlson, Arthur L. Bailey, Laurence H. Longley-Cook, Robert A. Bailey (Arthur Bailey's son), LeRoy J. Simon, Frank Harwayne, Lester B. Dropkin, Allen L. Mayerson, Charles C. Hewitt Jr., Hans Bühlmann (a Swiss actuary). If we have omitted names of others who have made comparable contributions, we are sorry. The foregoing are the ones who stand out in our memory.

Days To Remember

One of the memorable moments in the development of casualty actuarial science came in 1917, at a meeting of the Actuarial Section of the National Reference Committee on Workmen's Compensation

Insurance. The event is described in Albert Whitney's famous paper, "The Theory of Experience Rating," presented at the May 1918 CASSA meeting. According to Whitney, the committee—Winfield W. Greene, chairman; Albert H. Mowbray; Benedict D. Flynn; George D. Moore; and Joseph H. Woodward; all charter members and future presidents of the Society—was seeking to formulate a plan of experience rating of workmen's compensation risks.

The problem of experience rating, Whitney wrote, "arises out of the necessity of striking a balance between class experience on the one hand and risk experience on the other." Whitney's paper traced and analyzed verbally and mathematically the general line of reasoning pursued by the committee, which apparently had struggled at some length with the problem of the weight to be given risk experience, examining and rejecting many suggestions and assumptions. The committee used the term "credibility" and the notation Z to express this "weight," and sought to quantify it. Then Win Greene suggested that the relatively complicated second term of the denominator of an equation that the committee agreed summed up its thinking (No. 22 in Whitney's exposition) be taken as a constant. The development of his suggestion resulted in

$$Z = \frac{P}{P + K} .$$

Voila!

That formula (where P is exposure and K a constant), which underlies most of the credibility studies since then, has generally been attributed to Albert Whitney, because it first appeared and was analyzed in his paper (referred to above), but apparently it sprang out of the deliberations of the special actuarial committee on workmen's compensation, and, specifically, was one of Win Greene's suggestions.

Casualty actuarial science was born at that moment. The concept of credibility clearly has fascinated the casualty actuarial profession, and, later on, some of the life actuaries, who took it up mainly for group insurance.

In his 1918 paper on the theory of experience rating, Whitney explored the implications of the credibility concept contained in the

statement of the Z formula. He recognized, for instance, that reasonable values of K would have to be determined by judgment, depending on underlying factors.

Such judgment considerations were treated by Michelbacher in "The Practice of Experience Rating," presented at the same 1918 CASSA meeting as the Whitney paper. It was quite a day for actuarial science. Michelbacher's paper complemented Whitney's, setting forth the development of a practical plan from the theoretical principles discussed by Whitney. In the plan, greater credibility was given to a greater amount of observable data. Workmen's compensation loss experience was divided into two groups—death and permanent total disability losses in one, all other losses in the second. Credibility factors were calculated separately for each group. In later years the losses were divided into three groups—serious, non-serious, and medical.

Whitney had assumed that inherent hazards differed among classifications of risks, and he assumed a knowledge of the distribution of such hazards; but in his mathematical development he, in effect, reversed his assumptions and fell back on Bayes's Rule, which, prior to Laplace's generalization, declared that, a priori, all possible events were equally likely. Whitney's efforts were criticized, but he was aware that the casualty actuaries had practical problems of statistical estimation to attend to—specifically, reliable and marketable ratemaking where classical statistics didn't provide acceptable answers—and he pursued his own line of study.

Among the other practical problems confronting actuaries was finding a way to establish full credibility. The Z formula didn't allow full credibility, but there were many buyers of workmen's compensation insurance who insisted that they should be rated solely on their own experience. How this problem has been resolved over the years, in many lines of insurance, in many kinds of experience and retrospective rating, and in classification rating, is one of the great stories in casualty actuarial science, and is covered in the various chapters of this book.

Retrospective Rating

Albert Whitney, who developed the theory of experience rating, had also shown an early interest in retrospective rating, and he passed along his interest to Paul Dorweiler, his understudy in the National Workmen's Compensation Service Bureau. Retrospective rating, which was explored, described, and refined by Dorweiler in the 1920s, 1930s, and 1940s (Dorweiler 1927, 1933, 1936, 1941), and also by other well-known actuaries in the 1940s, was the next—after experience rating—important contribution to the methodologies of casualty actuarial science by members of the Society.

It was a rating scheme applied on top of experience rating, and it permitted workmen's compensation risks whose estimated premiums were greater than certain specified minimums to limit their final retrospective premiums, depending on losses, to amounts between preselected maximum and minimum percentages of the audited premiums. Obviously, risks whose loss records were better than average could save on their workmen's compensation costs, first prospectively through experience rating, then at policy expiration through retrospective rating. Insurance charges in the retro plan protected the insurance company against the probability that the risk, because of high losses, would exceed the preselected maximum premium; and there was a saving from the excess charge to recognize the probability that, because of low losses, the calculated retrospective premium would be less than the minimum.

Again the credibility concept, in which the measurement of risk is related to the volume of experience, came into play, because the charges and savings were higher for small premium accounts than for large. Originally there were several tabular retrospective rating plans whose maximum and minimum premium percentages were specified for varying sizes of risk, the range between the maximum and minimum percentages being less for small risks than for large. Another plan (Plan D, so-called) was created for risks that preferred to select their own maximums and minimums. Moreover, Plan D made it possible to combine large workmen's compensation and liability insurance risks in a single rating scheme, which applied also on an