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Dynamic Analysis of Non-Linear Structures by the Method of Statistical Quadratization

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ABSTRACT

Stochastic linearization is perhaps the most frequently used analytical method for analyzing the response of many nonlinear systems, as it provides reasonable estimates of the mean square response. However, the method is not, in general, well suited for estimating the power spectra of stationary responses of randomly excited nonlinear systems. In addition, for a Gaussian excitation, the linearized solution leads to a Gaussian probability distribution, whereas the true response is non-Gaussian. In this study, a higher order method termed equivalent stochastic "quadratization" is proposed to circumvent these shortcomings. The nonlinearity is replaced by a polynomial expansion up to a quadratic order. In this manner the Volterra series method can be used to approximate the response of the resulting nonlinear system. The system excitation is assumed to be Gaussian. However, the response is described by a non-Gaussian probability distribution. The method is developed for analyzing the stationary response of single and multi-degree-of-freedom systems; pertinent instructional examples are included. Further, a useful practical application of the proposed method is pursued for analyzing the stochastic response of compliant offshore platforms due to nonlinear drag forces. These are structures used to exploit oil resources in great water depths. The compliant nature of these platforms introduces nonlinear behavior which can not be neglected as in conventional offshore platforms. The method is applied for analyzing a specific three-degree-of-freedom model of a Tension Leg Platform (TLP) subject to wave and current forces. In addition to nonlinear drag forces, nonlinear potential forces significantly affect the TLP response. These forces are derived in the form of second order Volterra series. A stochastic response analysis of the TLP system due to combined nonlinear drag and nonlinear potential forces is performed to evaluate the relative significance of these forces.

The analytical results produced by the equivalent quadratization method for the instructional and practical problems considered, are found in good agreement with pertinent numerical data generated by Monte Carlo studies.

Clearly, the concept of quadratic, or even higher power, polynomial approximation of arbitrary nonlinearities and subsequent application of the Volterra series expansion for determining the random response of the derived equivalent nonlinear system, appears to be quite promising and meritorious. However, it is noted that the present study is strictly preliminary in nature, and reporting its findings in the present format conforms with the objective of the Lecture Notes in Engineering Series. Additional research is required to address versatility, reliability, and efficiency issues.

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CHAPTER 1
INTRODUCTION

1.1 Introduction

As offshore oil production moves into deeper water, compliant structural systems are becoming increasingly important. Examples of this type of structure are tension leg platforms (TLP's), guyed tower platforms, compliant tower platforms, and floating production systems. The common feature of these systems, which distinguishes them from conventional jacket platforms, is that dynamic amplification is minimized by designing the surge and sway natural frequencies to be lower than the predominant frequencies of the wave spectrum. Conventional jacket platforms, on the other hand, are designed to have high stiffness so that the natural frequencies are higher than the wave frequencies. At deeper water depths, however, it becomes uneconomical to build a platform with high enough stiffness. Thus, the switch is made to the other side of the wave spectrum. The low natural frequency of a compliant platform is achieved by designing systems which inherently have low stiffness. Consequently, the maximum horizontal excursions of these systems can be quite large.

The low natural frequency characteristic of compliant systems creates new analytical challenges for engineers. This is because geometric stiffness and hydrodynamic force nonlinearities can cause significant resonance responses in the surge and sway modes, even though the natural frequencies of these modes are outside the wave spectrum frequencies. High frequency resonance responses in other modes, such as the pitch mode of a TLP, are also possible.

One source of nonlinearity is the hydrodynamic drag force, which is due to flow separation around a submerged member. This force is frequently modeled mathematically by empirical equations such as the nonlinear Morison equation. For performing stochastic analyses, linearization methods such as described by Malhotra and Penzien(1970) are often utilized. However, responses at frequencies outside the wave spectrum frequencies are not predicted by linearization. Therefore, some response statistics may be significantly unconservative. In this study, a higher order method termed equivalent stochastic "quadratization" is proposed to circumvent this shortcoming of the equivalent stochastic linearization method.

Another source of nonlinearity is in the wave induced potential forces. These forces result from potential pressure gradients due to waves. Most compliant platform analyses in the literature model these forces by numerical methods such as finite element methods or sink-source methods, or by analytical methods based on slender member theory. The numerical methods are good for modeling systems with complex geometries, but are computationally expensive and more suitable for final design analyses. The interest of this study focuses on analytical methods since they are efficient and provide more insight into the fundamental behavior of compliant systems, although some accuracy may be sacrificed. Methods based on slender member theory, however, are inadequate because they do not consider wave scattering effects. For vertical cylinders, analytical methods that include wave scattering effects have been published in the literature, but have only been applied to compliant platform analyses in a limited manner. In this study, a more complete accounting of the potential forces is made.

1.2 Aim of Study

The purpose of this study is twofold. First, it is to verify the usefulness of the equivalent stochastic quadratization method as a tool for obtaining the response statistics of a compliant offshore system subject to nonlinear drag forces. A TLP system is used to develop and exemplify the proposed method. The verification procedure is presented in a systematic manner. The method is first developed as a general tool for analyzing nonlinear single-degree-of-freedom(sdof) systems subject to simple force excitations. The applicability of the method is then extended to general nonlinear multi-degree-of-freedom(mdof) systems, before finally applying it to a TLP system with three degrees of freedom. The second aim of the study is to analyze the response of the TLP system to combined nonlinear drag and nonlinear potential forces to evaluate the relative significance of these forces. Some of the more recent methods for modeling nonlinear potential wave forces are derived in a form which is more suitable for stochastic analyses of compliant systems. The estimation of the nonlinear low frequency surge response of a TLP system is of particular interest to this study. The nonlinear high frequency pitch response and its effect on axial tendon tensions is also to be investigated. In addition, the non-gaussian nature of the responses is considered to be an integral part of the analysis. This author is aware of no other analytical study which is more comprehensive in its modeling of the nonlinear wave forces and consequent responses of a TLP.

The remainder of this chapter is a discussion of modeling TLP systems and the environmental loads which act on a TLP's structural members. This is followed by a literature review of TLP response studies. The section on environmental loads is a somewhat involved review of hydrodynamic wave force theories since the response analyses of TLP's can not be understood without consideration of the hydrodynamic forces. The last section of this chapter gives a general scope of this study.

1.3 TLP Model

A TLP has a floating hull which is tied in place by tensioned vertical tendons. The typical TLP hull shape consists of four cylindrical column members arranged in a rectangular grid and connected at the base by cylindrical pontoon members. A diagram of the idealized TLP that is used in this study is shown in Figure 1.1.

The stochastic response of the hull due to wave and current induced forces is the primary interest of this study. Jefferys and Patel(1981) have shown that the inertia and wave forces acting on the tendons have a negligible effect on the motion of the hull. Therefore, the tendons are treated as massless springs which in conjunction with the hull buoyancy provide the restoring forces on the hull. The geometric nonlinearities inherent in the restoring forces are neglected since they are less important than the wave force nonlinearities. This is common assumption used in the literature.

Typically, the hull is considered to be a rigid body with six degrees of freedom. However, in the presence of a unidirectional flow field, which is parallel to the surge axis of the TLP, the hull responds in only three degrees of freedom. That is, surge or horizontal translation, heave or vertical translation, and pitch or rotation. This simple flow condition can be used to highlight the salient features of TLP responses. Therefore, it is used in the present analysis for simplicity and clarity of the results. It is noted that the pitch and heave motion directly influence the force in the tendons, while the surge motion has only an indirect influence through coupling with the pitch motion. The surge motion is most important in the analysis and design of the riser system and foundation, which are not modeled in the present study.

The surge natural period of typical TLP's is on the order of 70 to 120 seconds. The pitch and heave natural periods are much less and are in the range of 2 to 4 seconds. These periods are away from the dominant wave periods which are 4 to 6 seconds in normal sea states and 12 to 20 seconds in severe sea states. It is noted that the surge and pitch degrees

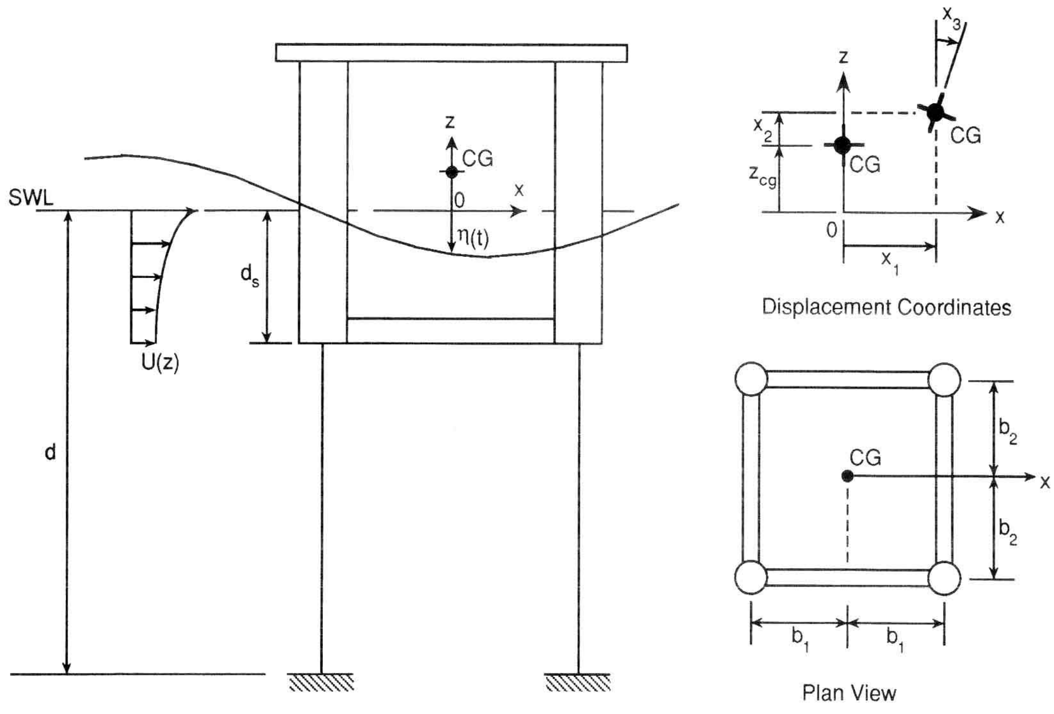

Figure 1.1 Diagram of Idealized TLP

of freedom are coupled through the added mass matrix and the hydrostatic stiffness matrix. The natural frequencies, however, are not very different than if the off diagonal terms in the mass and stiffness matrices are neglected. This indicates that the coupling is small. The heave degree of freedom is not coupled with the other degrees of freedom.

It is assumed that the total fluid force acting on the TLP hull can be obtained by summing the fluid forces acting on individual hull members as though other members are not present. In actuality, the presence of nearby members alters the flow field and, hence, the force acting on a member. If the members are spaced on the order of five diameters away from each other, however, the effect is small. This is a reasonable assumption for most TLP hulls.

1.4 Environmental Loads

The environmental loads acting on a TLP are due to waves, current, and wind. Only the response due to waves and current is considered in this study. Despite considering the TLP system to be linear, the response is still nonlinear because the wave and current induced forces are nonlinear. The linearity of the force depends on its relation to the wave elevation from linear wave theory. A linear force is linearly related to the wave elevation, a quadratic force is quadratically related to the wave elevation, and so on. In offshore systems, it is convenient to express the force and the resulting response as a Volterra series in which the wave elevation is the input function such as described by Yamanouchi(1974) and Vassilopoulos(1967). The series is usually truncated after second order. The linear force is called the first order force while the quadratic force is called the second order force or drift force. In the frequency domain, linear and quadratic transfer functions are needed to describe these forces.

It is well known that the surge response of a TLP subjected to wave and current loads consists of a wave frequency response, a mean response, and a low frequency or slowly varying response at the TLP's surge natural frequency. High frequency responses at the pitch natural frequency can also occur, although this has received less attention in the literature. The wave frequency response is due predominantly to linear potential forces acting on the hull and to a much lesser extent to viscous forces. The mean, low frequency, and high frequency responses are due to higher order wave forces, in particular, quadratic wave forces. These forces are due to both potential and viscous effects. In general, both effects contribute significantly to the total drift force. Further, the viscous forces induce a