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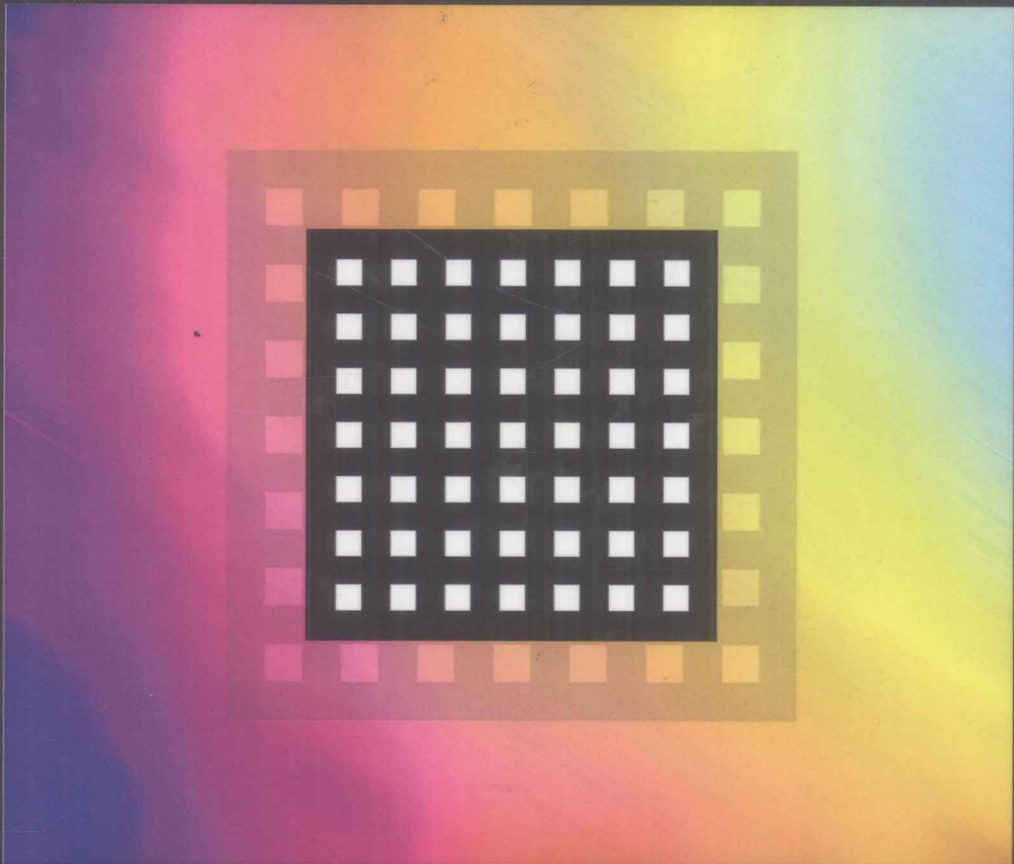

General Theory of Superconductivity

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Preface

The research field of the high-T_c superconductivity will approach its twenty-first birthday. The first publication in this field appeared in September, 1986. In the past twenty years a wealth of experience and knowledge in both experiment and theory on the high-T_c and low-T_c superconductivity was accumulated. It is the time to make a complete summary and upgrade the understanding of the superconductivity and the pseudogap. "General Theory of Superconductivity" is born in this background.

Many books about theory of superconductivity are available now, which may be called special theory of superconductivity. There are some main differences between the general and the special theory of superconductivity. First, the special theory only studies the conventional superconductor, but the general theory studies both the conventional and unconventional superconductor. High-T_c cuprates are one kind of unconventional superconductors. The essential difference between the conventional superconductor and the high-T_c cuprates is the coherence length of the Cooper pair ξ_o. For the conventional superconductor $\xi_o \sim 10^3 - 10^4$ $\AA$, but for the high-T_c cuprates $\xi_o \sim 20 - 30$ $\AA$. Second, the special theory only studies the superconducting state and only gives the equation to determine the pairing temperature T^*, at which the gap begins to appear, but the general theory studies both the superconducting and the pseudogap state and gives different equations to determine T^* and superconducting transition temperature T_c. Third, the special theory mainly discusses the phonon-mediated superconductivity, but the general theory discusses both the phonon-mediated and the two local spin-mediated superconductivity. The latter is the mechanism of the superconductivity and pseudogap in the high-T_c cuprates, and can quantitatively explain many experimental facts. Fourth, the special theory only studies macroscopic superconductivity, but the general theory studies both macroscopic and microscopic superconductivity. Fifth, the general theory introduces some new terms and gives many predictions. In addition, the general theory can give some further discussions on mechanism of superfluidity in ^{4}He and ^{3}He.

To popularize the general theory, we are motivated to write this book as a textbook and expect that the research work will progress on the new level of the general theory. Easy to understand is one of our purposes. So this book interprets the general theory in simple language and graduate students can well know the methods to determine T^*, T_c, gap and pseudogap by using the program in chapter 5. This book concerns the physical pictures of theoretical results, the explanations of experiments and physical predictions besides pure theory. Therefore, the title of this book can be also called "General Physics of Superconductivity".

The whole book consists of seven chapters and one appendix. Chapter 1 introduces the basic phenomena and expounds the phenomenological theories. Chapters 2 and 3 give the theoretical explanations of concepts in chapter 1. Chapter 2 introduces BCS pairing theory of both conventional and unconventional superconductors. The gap and T^* equations are established in this chapter. Chapter 3 proposes a mechanism of the superconductivity and pseudogap in the high-T_c cuprates, and gives the T_c equation. Chapter 4 applies the microscopic theories in chapters 2 and 3 to explain many experiments in the high-T_c cuprates quantitatively. Since many experiments can be explained by the two local spin-mediated mechanism quantitatively in terms of unified values of related parameters, we believe that this mechanism is correct for the high-T_c cuprates. Some readers might be doubtful about the correctness of the new concepts in chapter 1. However, the theoretical grounds of these new concepts are given in chapters 2, 3, and 4. All the doubts about the new concepts will be removed after reading chapters 2, 3, and 4. Chapter 5 gives the numerical method to solve the non-linear integral equation used in chapter 4. Chapter 6 discusses many systems, in which the mechanism of superconductivity and pseudogap might be the same as that of the high-T_c cuprates. This chapter also discusses the mechanism of superfluidity in ^{3}He and ^{4}He. Chapter 7 expounds the extended Abrikosov pseudo-fermion representation of local spin operator, which is used in chapter 3. Since chapter 7 might be a little hard to understand, an appendix is attached to expound the basis of the pseudo-fermion representation in detail.

Authors
March 15, 2007.

Contents

Chapter 1

Introduction

1.1. Basic Phenomena and Concepts

1.1.1. Electric Properties (1911)

If direct current resistance $R = 0$, when $T \leq T_c$ [1] (1911), then the T_c is called superconducting transition temperature, or critical temperature. It is also called temperature of long-range phase coherence of Cooper pairs in general theory of superconductivity. The state at $T \leq T_c$ is called superconducting state. (For the exact statement on T_c of conventional superconductors see the sixth paragraph.) Bardeen, Cooper, and Schrieffer (BCS) proposed their epoch-making pairing theory of superconductivity [2], which forms the subject of chapter 2. In BCS theory, it was shown that even a weak net attractive interaction between electrons can cause an instability of the ordinary Fermi-sea ground state of the electron gas with respect to the formation of bound pairs of electrons occupying states with equal and opposite momentum. These so-called Cooper pairs have spatial extension or average size or coherence length ξ_o. One might well ask what really is the essential universal characteristic of the (macroscopic) superconducting state. The answer is that the existence of Cooper pairs is only a necessary condition. To have (macroscopic) superconductivity the Cooper pairs have to maintain long-range phase coherence over macroscopic distances. When the superconducting state is at rest, the long-range phase coherence contains two types. One is long-range unified phase of all Cooper pairs which comes from strong overlap of Cooper pairs. Another one is long-range phase difference locking which comes from separation of Cooper pairs. Note that when the superconducting state is at rest, the phase difference locked is actually zero (See section 1.1.5.). Microscopic superconductivity and supercurrent can occur without the long-range phase coherence. For the microscopic superconductivity and supercurrent see sections 2.6.2, 2.7.4 and 2.8. In this book "superconductivity" connotes "macroscopic superconductivity".

There are two kinds of superconductors: conventional and unconventional superconductors. A great part of low-T_c superconductors belong to the conventional superconductors. The whole Josephson tunneling junction [3], the granular superconductor [4], and the high-T_c cuprates [5] belong to the unconventional superconductors. The basic difference between conventional and unconventional superconductors is as follows. In conventional superconductors the size of a Cooper pair is large (about 10^3 - 10^4 $\mathring{A}$). Therefore, the Cooper pairs are easy to strongly overlap. In unconventional superconductors there is, at least, one region in which there is no the Cooper pairs. In the high-T_c cuprates the size of a Cooper pair is small (about 25 Å). So, the Cooper pairs are separated into many single Cooper pairs (units), and there are no Cooper pairs in many regions in the high-T_c cuprates. For the whole Josephson tunneling junction the Cooper pairs are separated into two superconducting units disconnected with each other, i.e., two superconducting electrodes, by the middle insulating layer, in which there are no the Cooper pairs. For the granular superconductors the Cooper pairs are separated into many superconducting units disconnected with each other, i.e., superconducting grains. So, the granular superconductor also contains many regions, in which there are no Cooper pairs. Note that the Cooper pairs in the superconducting units separated are of the long-range phase coherence in the whole Josephson tunneling junction and the granular superconductors. The superconducting unit may be a superconducting film for a Josephson tunneling junction, or a superconducting grain for a granular superconductor. The unit of the high-T_c cuprates contains only a single Cooper pair. There is Josephson tunneling coupling between the two nearest-neighbor superconducting units in the whole Josephson tunneling junction and the granular superconductor, and between the two nearest-neighbor units in the high-T_c cuprates (See section 3.3.).

When the Cooper pairs begin to appear at T^* (pairing temperature) in the units, the high-T_c cuprates is still in normal state because the zero-point fluctuation of the phase difference between any two nearest-neighbor units, and the thermal disturbance can destroy the phase difference locking, and thus there is no long-range phase difference locking. Only at a lower temperature T_c ($< T^*$), when the Josephson tunneling coupling energy (See section 3.3.) can overcome both the thermal disturbance energy and the zero-point fluctuation energy, there is the long-range phase difference locking over macroscopic distances, and thus the superconductivity occurs in the high-T_c cuprates. Note that the long-range phase difference locking is also a type of the long-range phase coherence besides the unified phase of all Cooper pairs. The Cooper pairs at $T_c < T < T^*$ may be called non-supercurrent Cooper pairs or Cooper pairs without the long-range phase coherence. The Cooper pairs at $T < T_c$ consist of two sorts (See section 3.3.). One sort is the supercurrent Cooper pairs or Cooper pairs with the long-range phase coherence, and the other one is non-supercurrent Cooper pairs or Cooper pairs without the long-range phase coherence.

When the superconductivity begins to appear at T_c' in the superconducting units of Josephson tunneling junction, the whole Josephson tunneling junction is still in normal state. Only at a lower temperature T_c ($< T_c$'), when the Josephson tunneling coupling can overcome both the thermal disturbance and the zero-point fluctuation of phase difference on the two sides of the insulating barrier layer, there is the phase difference locking between the two superconducting units on the two sides of the insulating barrier layer. The T_c is called superconducting transition temperature of the whole Josephson tunneling junction. The Josephson tunneling junction belongs to one sort of the weakly coupled systems. The analyses for other weakly coupled systems are similar to the whole Josephson tunneling junction.

When the superconductivity begins to appear at T_c' in the superconducting units of granular superconductor, the granular superconductor is still in normal state. Only at a lower temperature T_c ($< T_c$'), when the Josephson tunneling coupling can overcome both the thermal disturbance and the zero-point fluctuation of phase coherence, there is the phase difference locking between any two nearest-neighbor superconducting units , i.e., the long-range phase difference locking. The T_c is called superconducting transition temperature of granular superconductor.

Let us discuss exactly the characteristic temperatures of the conventional superconductors, such as tin and lead. In our physical views, the conventional superconductors have three characteristic temperatures: T^*, $T_{c,1}$, and T_c. T^* is a temperature at which the Cooper pairs without the long-range phase coherence begin to appear. $T_{c,1}$ is a temperature at which the Cooper pairs with the long-range phase difference locking (one type of the long-range phase coherence) begin to appear. T_c is a temperature at which the Cooper pairs with the unified phase (another type of the long-range phase coherence) begin to appear. The reasons are as follows. In conventional superconductors the average size of a Cooper pair ξ_o is at the order of $10^3 \sim 10^4$ Å. Assume $\xi_o = 10^4$ Å and electron number density is $n = 10^{22}/cm^3$. (Note that the ξ_o in the high-T_c cuprates is only 20 - 30 Å.) At T^* the Cooper pairs begin to appear. Because the number of Cooper pairs is very small at $T \approx T^*$, the distance between any two centers of Cooper pairs is larger than ξ_o. When the temperature lowers, the number of Cooper pairs increases. At $T_{c,1}$ ($< T^*$) the number density of Cooper pairs is $2 \times 10^{12}/cm^3$, i.e., there are 2 Cooper pairs in $(10^{-4}\ cm)^3$ volume. In this case the Josephson coupling energy overcomes both the zero-point fluctuation energy of phase coherence of two nearest-neighbor Cooper pairs and thermal disturbance energy (See section 3.3.), and thus the long-range phase difference locking between Cooper pairs is established. At $T_{c,1}$ the supercurrent Cooper pairs begin to appear. When the temperature lowers from $T_{c,1}$, the number of Cooper pairs continues to increase. At T_c ($< T_{c,1}$) the number density of Cooper pairs is $10^{13}/cm^3$, i.e., there are 10 Cooper pairs in $(10^{-4}\ cm)^3$ volume. In this case the Cooper pairs begin to strongly overlap, and thus the unified phase of all Cooper pairs is formed. From the measurements of resistance and specific heat jump the T^* and $T_{c,1}$ are in a very narrow temperature interval 10^{-3} K. Therefore, in physics, we deduce that $T_{c,1}$ and T_c are in a very narrow temperature interval 10^{-3} K as well. Because $T^* \approx T_{c,1} \approx T_c$ we can roughly say that the specific heat jumps at T_c instead at T^* and $T_{c,1}$, and the zero resistances occur at T_c instead at $T_{c,1}$. At present, there is only conceptual sense instead of observation sense to distinguish three temperatures for the conventional superconductors.

We interpret the alternating current resistance Z in superconducting state by the following table.

$$
Z \begin{cases}
= 0, & \text{when} \quad T = 0 \quad \text{and} \quad \begin{cases} \hbar\omega \leq 3.5 k_B T_c & \text{for Sn;} \\ \hbar\omega \leq 7 k_B T_c & \text{for } YBa_2Cu_3O_{7-\delta}; \\ \hbar\omega \leq 4.1 k_B T_c & \text{for Pb;} \end{cases} \\
> 0, & \text{when} \quad T = 0 \quad \text{and} \quad \begin{cases} \hbar\omega > 3.5 k_B T_c & \text{for Sn;} \\ \hbar\omega > 7 k_B T_c & \text{for } YBa_2Cu_3O_{7-\delta}; \\ \hbar\omega > 4.1 k_B T_c & \text{for Pb;} \end{cases} \\
> 0, & \text{when} \quad 0 < T < T_c \quad \text{and} \quad \hbar\omega > 0; \\
\approx Z_n, & \text{when} \quad 0 < T < T_c \quad \text{and} \quad \hbar\omega \gg k_B T_c.
\end{cases}
$$

Here ω is the frequency of alternating current, and Z_n is the alternating current resistance in normal state. For the high-T_c cuprates the condition of $Z = 0$ at $T = 0$ is that they are in superconducting state. If they are in pseudogap state (See section 1.1.6.), then always $Z > 0$ at any temperature including $T = 0$. From the alternating current resistance at $T = 0$

(very low temperature) in the above table we know that there is a threshold energy for absorption of electromagnetic wave. The existence of the threshold energy means that there is an energy gap in the single particle excitation spectrum. BCS pairing theory gives that the threshold energy is equal to doubled energy gap of superconductor (See chapter 2.), and the energy gap leads to formation of Cooper pairs. The gap or Cooper pairs alone can not lead to superconductivity because the Cooper pairs may be separated into two sorts: supercurrent and non-supercurrent Cooper pairs, i.e., Cooper pairs with and without the long-range phase coherence.

1.1.2. Diamagnetism (Meissner effect, 1933)

Inside superconductors in superconducting state the magnetic induction $B = 0$, and the $B = 0$ is history-independent. This phenomenon is called Meissner effect [6]. Meissner effect is a perfect diamagnetism. This perfect diamagnetism occurs at magnetic field smaller than the thermodynamic critical field H_c for type-I superconductor (See Fig. 1.5.), and at magnetic field smaller than the lower critical magnetic field H_{c1} for type-II superconductor (See Fig. 1.5.). From Meissner effect we realize that superconductors in superconducting state are different from the perfect conductors since the latter tends to trap magnetic flux in, and leads to $B = 0$ history-dependent. The origin of Meissner effect is as follows. The magnetic field parallel to the surface of superconductor decays exponentially, with a penetration depth λ_L, in the interior. $\lambda_L(T = 0\text{ K})$ ranges from 500 to 10 000 $\mathring{A}$, depending on the material. From Maxwell fourth equation, we know that accompanying this parallel field is a surface current density. This surface current density screens the magnetic field from the interior of superconductor.

1.1.3. Electronic Specific Heat

In the conventional superconductors, such as tin and lead, when applied magnetic field is zero, the electronic specific heat, C_e, as a function of temperature is shown in Fig. 1.1. Usually people say that the specific heat jump occurs at T_c. But in the view of general theory, for the conventional superconductors the specific heat jump occurs at T^* and $T_{c,1}$. At T^* the Cooper pairs without the long-range phase coherence begin to condense into the chemical potential. This is a second order phase transition. At $T_{c,1}$ the Cooper pairs with the long-range phase difference locking, i.e., the supercurrent Cooper pairs, begin to appear. This is also a second order phase transition. At T_c there is no new phase transition. That the transition width of the jump is only 10^{-3} K [7,8] means that T^* and $T_{c,1}$ are in a very narrow temperature interval. So $T_c \approx T_{c,1} \approx T^*$ and we can roughly say that the specific heat jumps at T_c for conventional superconductors.

The measurement showed that the electronic specific heat well below T_c was dominated by an exponential dependence so that [9,10]

$$C_{es} = \gamma T_c d e^{-bT_c/T}, \tag{1.1}$$

where the normal state electronic specific heat is $C_{en} = \gamma T$, and d and b are numerical constants. $b = 1.5$. Such an exponential dependence implies a minimum excitation energy per quasiparticle of $1.5k_B T_c$ for some materials, i.e., the existence of gap for quasiparticle

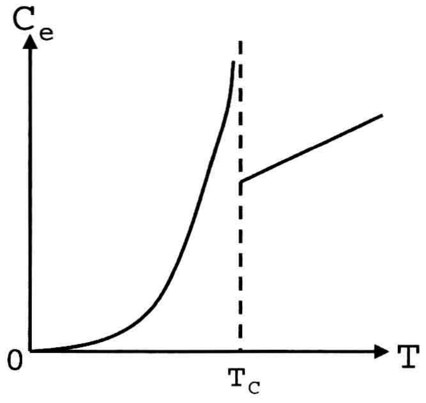

Figure 1.1. The usual schematic plot of C_e vs. T for conventional superconductors, such as tin. The transition width at T_c is only 10^{-3} K. At $T > T_c$ $C_e = C_{en}$. At $T \leq T_c$ $C_e = C_{es}$.

excitations. Note that the existence of gap does not mean superconductivity, in general. The appearance of superconductivity needs the long-range phase coherence of the Cooper pairs.

Let us interpret the second order phase transition in the high-T_c cuprates, which are one sort of the unconventional superconductors, in detail in this paragraph. At $T = T^*$ ($> T_c$) the Cooper pairs begin to appear. Although the Cooper pairs at $T_c < T < T^*$ have no long-range phase coherence, they condense into the chemical potential, which is the Bose-Einstein condensation. Therefore, the high-T_c cuprates have a second order phase transition at $T = T^*$. The magnitude of the specific heat jump at $T = T^*$ is given in section 2.5.4. The order parameter of the second order phase transition at $T \leq T^*$ is the amplitude of the wavefunction of Cooper pairs. The Cooper pairs at $T_c < T < T^*$ have no long-range phase coherence, and thus they are not the supercurrent Cooper pairs. The section 3.3 of this book demonstrates that at $T = T_c$ ($< T^*$) the supercurrent Cooper pairs begin to appear. This is also a second order phase transition. The order parameter of this second order phase transition is related to the number density of supercurrent Cooper pairs. The number density of supercurrent Cooper pairs is a complex function of the wavefunction of Cooper pair (See section 3.3.). At $T < T_c$ the high-T_c cuprates contain two sorts of Cooper pairs: supercurrent and non-supercurrent (See section 3.3.). Note that for the high-T_c cuprates the widths of specific heat jumps at T^* and T_c are wide because the gap has node and is anisotropic (See section 3.2.2.).

1.1.4. Isotope Effect (1950)

The phenomenon is that for element superconductors, $T_c \propto M^{-\alpha}$, where M is the atom mass and isotope exponent $\alpha \simeq 0.5$ [11]. The larger the M is, the lower the T_c. From this isotope effect people infer the importance of electron-phonon interaction for the superconductivity in these element superconductors. The isotope effect was found in some superconducting alloys and compounds as well. Actually, the electron-phonon interaction is important for a great part of superconductors. Note that the superconductivity caused by another mechanism other than the electron-phonon interaction can also have the isotope effect, which is shown for the T_c cuprates in section 4.6.

1.1.5. Josephson Effect (1962)

Josephson made from microscopic theory a prediction that a zero voltage supercurrent

$$I_s = I_c \sin(\phi_1 - \phi_2) \tag{1.2}$$

should flow between two superconducting electrodes in superconducting states, separated by a thin insulating barrier layer [3]. Here, $(\phi_1 - \phi_2)$ is the difference in the phases of the Cooper pairs, responsible for superconductivity, in the two electrodes, and I_c is the maximum supercurrent that the junction can support. The structure consisting of one thin insulating barrier layer and two superconducting electrodes on the two sides of the barrier layer is called Josephson tunneling junction or S-I-S junction. The supercurrent Cooper pair tunneling costs no energy when the two superconducting electrodes have the same chemical potential ($\approx$ Fermi level). When the dc supercurrent I_s is zero, i.e., the superconducting state of Josephson junction is at rest, the phase difference $(\phi_1 - \phi_2)$ is zero as well. When we increase the supercurrent, the phase difference $(\phi_1 - \phi_2)$ between the two superconducting electrodes adjusts to make the Josephson supercurrent $I_c \sin(\phi_1 - \phi_2)$ just the experimental value I_s. The increase of phase difference continues for increasing I_s until the point is reached where $I_s = I_c$, or $(\phi_1 - \phi_2) = \pi/2$. Then the current jumps from zero-voltage line to the curve of single-particle on the coordinate system of current vs. voltage.

Josephson further predicted that the I_s oscillates as a function of time if there is a voltage drop, V, on the S-I-S junction,

$$I_s = I_c \sin\left((\phi_1 - \phi_2)_\circ + \frac{2|e|V}{\hbar}t\right), \tag{1.3}$$

where e is the charge of free electron, and $(\phi_1 - \phi_2)_\circ$ is the phase difference at $V = 0$.

These predictions, known as dc and ac Josephson effect, respectively, have been confirmed by a large number of experiments.

The physical mechanism of the dc Josephson effect in S-I-S junction is as follows. Assume that the values of the superconducting transition temperature T_c' of the two superconducting electrodes are the same. At $T \leq T_c'$ the two superconducting electrodes are in superconducting states separately, and thus the supercurrent can flow in them separately. The whole Josephson tunneling junction is still in normal state at $T \cong T_c'$ because the Cooper pairs in two superconducting electrodes have not any long-range phase coherence yet. As the temperature goes down and reaches T_c, at which the Josephson coupling energy begins to overcome the zero-point fluctuation energy and the thermal disturbance energy [12], there is the phase difference locking across the insulating barrier layer, and thus the whole Josephson tunneling junction is in superconducting state. From Eq. (1) we know that the long-range phase coherence of Cooper pairs in the two electrodes, i.e., for example, ϕ_1 and ϕ_2 are constants for conventional superconductor, and the phase difference locking across the insulating barrier layer, i.e., constant phase difference $(\phi_1 - \phi_2)$ does not be destroyed by the zero-point fluctuation energy and thermal disturbance energy, are the two necessary and sufficient conditions for the dc Josephson effect. Note that for the usable Josephson

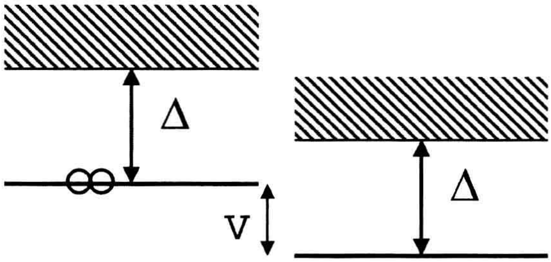

Figure 1.2. The schematic energy level diagram for the two superconducting electrodes. The shading denotes the energy levels occupied by quasiparticles. The two circles denote a Cooper pair. The real line denotes the chemical potential. Note that the Fermi energy level is nearly equal to the chemical potential.

tunneling junction the necessary value of Josephson coupling energy to overcome the zero-point fluctuation energy is actually very small (10^{-10} eV [12]). Therefore, T_c is nearly equal to T_c' for the usable Josephson tunneling junction.

Let us propose a mechanism of the ac Josephson effect. Assume that the two superconducting electrodes of the Josephson tunneling junction consist of the same superconductors. The schematic energy level diagram for the two superconducting electrodes is shown Fig. 1.2. All the Cooper pairs occupy the same energy level (Fermi energy level). The Fermi energy level can contain many Cooper pairs because the Pauli principle can not apply to Cooper pairs with spins 0 and 1. The oscillatory current in Eq. (1.3) comes from the tunneling of Cooper pairs between the Fermi energy level of one superconducting electrode and the Fermi energy level of the other [13]. For example, the Cooper pair on the left Fermi energy level tunnels to the right Fermi energy level. This tunneling of the Cooper electron pair does not conserve energy but takes an energy $2|e|V$ due to the voltage V. This Cooper pair must tunnel back, since it can not complete a transition where energy is not conserved. So long as the voltage V exists, the Cooper pair will tunnel to and from between the two superconducting electrodes [13]. Naturally, the frequency ω coming and going should satisfy the energy conservation,

$$\hbar\omega = 2|e|V. \tag{1.4}$$

From this relation, Eq. (1.4), we can understand Eq. (1.3).

Here we give an another similar mechanism for the ac Josephson effect. We assume that the Cooper electron pair on the left Fermi energy level tunnels to the right Fermi energy level. This tunneling of the Cooper pair does not conserve energy but takes an energy $2|e|V$. To keep the energy conservation the tunneling Cooper pair has no choice but to emit a photon with energy $\hbar\omega = 2|e|V$, and to return to the left Fermi energy level. The Cooper pairs move to and from between the left and the right, and thus the oscillatory current and the external radiation are formed in the S-I-S junction.

According to our mechanism, it is not necessary for Cooper pairs to have the long-range phase coherence and the phase difference locking across the barrier layer in the ac