MATHEMATICS IN THE BEHAVIORAL AND SOCIAL SCIENCES

JOHN W. BISHIR/DONALD W. DREWES

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North Carolina State University

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PART I. Paul Klee. *Disturbed Equilibrium.* 1922. Paul Klee Foundation. Museum of Fine Arts, Berne. (photo by Museum of Modern Art, New York)

PART II. Paul Klee. *Centrifugal Forces.* 1929. Paul Klee Foundation. Museum of Fine Arts, Berne. (photo by Museum of Modern Art, New York)

PART III. Paul Klee. *Inscription.* 1918. Felix Klee.

PART IV. Paul Klee. *Monument in the Orchard.* 1929. Paul Klee Foundation. Museum of Fine Arts, Berne.

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PREFACE

The utility of scientific knowledge is judged according to the degree to which it permits understanding of past events and prediction and control of future events. Description of events, being a historical record of past experience, cannot be used directly for future prediction and control. Rather, expectations of future events are generated within a logical system consisting of two or more abstract variables and the rules for specifying their interdependence. This logical system is called a *model.*

A logical system in which the variables are mathematical symbols and the structural relations between variables consist of a set of equations is called a *mathematical model.* Formulation of models in mathematical rather than literary or verbal form is often advantageous in that (1) mathematical language is often more concise and analytic than verbal language, (2) the relations between variables must be explicitly stated, thereby facilitating public scrutiny and evaluation of explanations, (3) the model builder has access to supportive mathematical theories, and (4) an explanatory structure can be readily generalized to the n-variable case.

This book introduces the mathematical methods most frequently used in the behavioral and social sciences. The mathematical content follows closely the recommendations made by the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America for students in the behavioral, management, and social sciences. Whenever possible, usage of mathematical content is illustrated by models drawn from the scientific literature. In other cases, hypothetical models are offered which attempt to capture the essence, if not the actuality, of the use of mathematical models.

The book is written for both those reasonably competent in mathematics who are seeking to relate mathematics to the social and behavioral sciences and those who have yet to develop their mathematical skills. Admittedly the task is easier for the former. To make it more feasible for the latter, the material is presented so as to facilitate an intuitive understanding of the mathematical concepts that govern useful techniques in the analysis of social and behavioral problems.

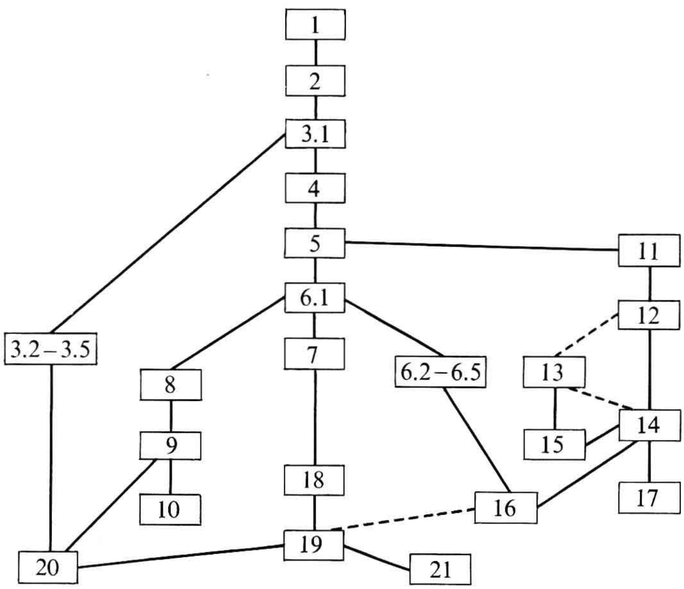
For those interested in pursuing applications in specific content areas literature references are footnoted within the textual material. Mathematical readings are listed at the end of each chapter.

Topics in the book are arranged according to a meaningful order of mathematical material rather than a hierarchy of models. Part I considers the finite mathematics relevant to the concept of system as a group of related entities. Matrices and linear algebra are discussed in Part II and applied to the analysis of systems containing n variables. Part III introduces differential and integral calculus and includes a parallel discussion of difference equations and their use in studying the dynamics of system behavior. Part IV deals with probability theory and its application to the development of random models in the social and behavioral sciences.

For each mathematical topic, we have included examples and problems from each discipline in proportion to usage. For instance, there are relatively more examples and problems drawn from classical economics in Part III (Calculus), whereas Part IV (Probability) contains proportionately more material drawn from psychology and management science.

The book contains more than 1500 problems, some within the main textual discussion to allow immediate verification of understanding. In addition, most sections contain problems which call for "proofs." Although these problems do not contain results required for an understanding of later material, it is our feeling that such problems facilitate the development of mathematical skills.

The following diagram shows how the book may be used in one-, two-, or three-semester courses. A solid line indicates dependence on the preceding chapter; a dashed line indicates use of only a portion of the preceding

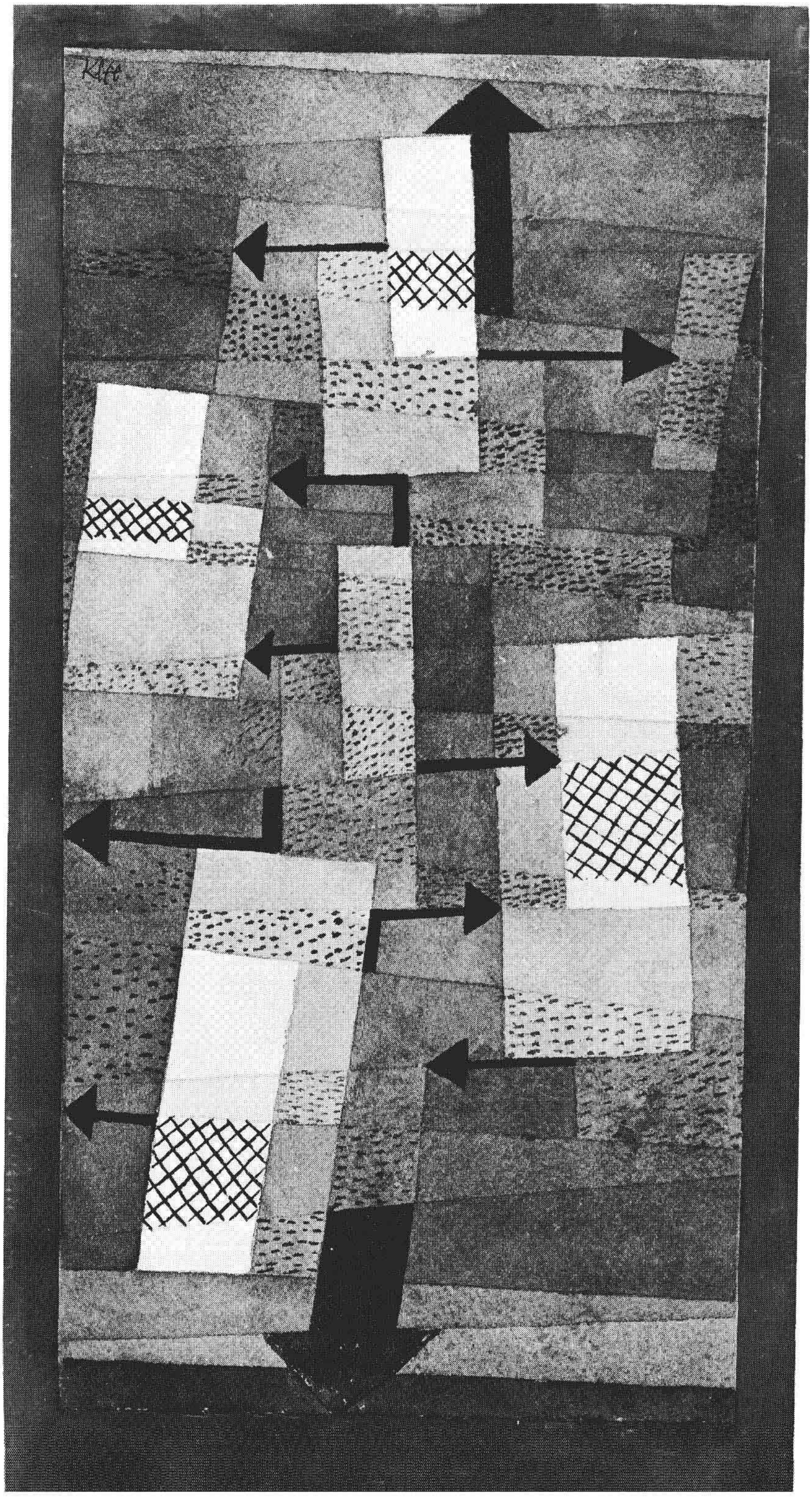


material. For example, a one-semester course emphasizing matrix algebra and its applications would survey the necessary parts of Chapters 1 through 6 before concentrating on Chapters 8 through 10. A one-semester course emphasizing basic discrete probability can be taught using parts of Chapters 1 through 7, together with Chapters 18 and 19. (In each of these cases, some selection is necessary from Chapters 1 through 7. In our experience, a typical class will be able to cover 200 text pages when meeting three times per week for one semester.)

A two-semester course emphasizing the general ideas of "finite mathematics" including matrix algebra and Markov chains would use Chapters 1 through 9 and 18 through 20. (The material need not be covered in order.) A two-semester course emphasizing techniques of calculus consists of Chapters 1 through 6, and 11 through 17, with options of including none, some, or all of Chapters 13, 15, 16, and 17. Finally, the entire book may be covered in three semesters. In this case, the large number of problems and the presence of Chapters 10, 17, and 21 allow considerable latitude for the individual preferences of the instructor.

It is a pleasure to thank the many people who have contributed so much to the completion of this book. Special thanks go to Mrs. Carol Little, Mrs. Debra Currin, and Mrs. Sherry Ford for their many hours of careful typing; to Miss Jane Woodbridge of Harcourt, Brace and World for a superb job of editing; to Dr. Nicholas Rose, Chairman of the Department of Mathematics at North Carolina State University, for his assistance and encouragement; to Professor David Rosen of Swarthmore College for a careful review which led to substantial improvement of Part III on calculus; and especially to our wives for their encouragement and understanding through the seemingly endless job of writing.

John W. Bishir
Donald W. Drewes


Klee

CONTENTS

PART I

Finite Mathematics

SET THEORY 1

1.1 SET MEMBERSHIP

The language and techniques of set theory provide the basic tools for many branches of mathematics of significance to the behavioral and social scientist. Sets also appear in mathematical models developed for the behavioral sciences. Thus we begin our mathematical discussion with a look at sets.

A set is any collection of objects. We may speak, for example, of the set of closing quotations on the New York Stock Exchange for September 23, or the set of possible dominance relations among a group of ten people, or the set of all solutions of the equation $x^2 - 1 = 0$, or the set of all purple cows. Other terms, such as *class* or *collection*, are sometimes used as synonyms for set.

The objects in a set are called *elements*, or *members*, of the set. We say that elements *belong to* the set. The notation $x \in A$ is used to indicate that the object x is a member of the set A. If, for example, P is the set of all psychotics, we might indicate that Mr. X is psychotic by writing $X \in P$.

One way of describing a set is to enclose, in braces, letters or numbers separated by commas to represent the members of the set. Thus, the set S of all solutions of the equation $x^2 - 1 = 0$ could be written as $S = \{1, -1\}$. If R and L denote, respectively, "the rat turns right" and "the rat turns left," then the set of possible choices made in a single run through a T-maze is $C = \{R, L\}$. Similarly, {Adams, Jefferson, Monroe} denotes the set of U.S. Presidents who died on July 4.

PROBLEMS

1. Write the following sets.

(a) The set of countries lying in both the Eastern and Western hemispheres.

(b) The set of living former Presidents of the United States.

(c) The set of States having only one representative in the House of Representatives.

(d) The set of possible combinations of coins which amount to 41¢.

A set may also be described by stating a criterion which members of the set must satisfy. For instance, the set $S = \{1, -1\}$ may be denoted in the alternative form $S = \{x: x^2 - 1 = 0\}$ read "S is the set of all objects x having the property that $x^2 - 1 = 0$." In general, the notation $A = \{x: p(x)\}$ means that A is the set of all objects x about which the proposition $p(x)$ is true. It is common to use this method of representing a set when the members are not known exactly or are too numerous to list. Thus we represent the set of all millionaires by $M = \{x: x \text{ is a millionaire}\}$ and the set of all real numbers larger than 4 by $G = \{y: y \text{ is a real number and } y > 4\}$. We have, for example, Onassis $\in M$ and $9 \in G$.

PROBLEMS

2. Denote the following sets.

(a) The set of citizens of Canada.

(b) The set of stocks listed on the American Stock Exchange.

(c) The set of positive, even integers.

Some sets, such as the set of all English words that begin with the letters *qa*, contain no elements. Such sets are called *empty sets*, or are referred to as the *null set*, and are denoted by the symbol $\emptyset$. Other empty sets are the set of U.S. Presidents who have lived to be 100 years old and the set of all audible tones greater than 20,000 cps.

PROBLEMS

3. Which of the following sets are empty?

(a) $\{0\}$

(b) The set of integer solutions of the equation $x^2 - 3 = 0$.

(c) $\{\emptyset\}$

The members of a set may themselves be sets. A social club represents a certain set of people, and it may be of interest to speak of the set of social clubs in a given town. However, we never allow a set to be a member of itself. The following example, due to Bertrand Russell, shows why.

Example 1 *Russell's Paradox* Suppose we wish to divide the collection of all sets into two smaller collections one of which, M, is to contain all those sets which *are* members of themselves while the other, N, is to contain those sets which *are not* members of themselves. Obviously, every set is assigned