STATISTICS

SECOND EDITION

NORMA GILBERT

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NORMA GILBERT
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Drew University

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PREFACE

This book is intended for students who need to understand how statistical decisions are made but who have little mathematical background (a year or two of high school algebra is sufficient). So many statistics texts aimed at this group have appeared that some explanation—other than the pleasure I've had in writing it—is needed of why another appears. Here are several reasons:

First, many formulas appear in any statistics book. I have tried to explain where a formula comes from, why it should be used, and when it should not be used. Explanations are often intuitive and informal, but a student should get a grasp of why a decision is reached rather than just learn what is reached.

Second, in some texts the notation gets pretty involved. In this one it doesn't. (The exception is Chapter 15; by then a student is well accustomed to statistics. Some instructors will prefer to omit this chapter.)

Third, the book is suited to self-paced ("Keller Plan") study as well as to the usual lecture approach: An introductory section at the beginning of each chapter explains its scope and where emphasis should be put. About 250 examples are worked out completely. Answers are given in detail to about 600 of the exercises in the belief that it does no good to know that the probability of an event, for example, is .40 if the student doesn't know how .40 is derived.

Last, some texts avoid probability, but an understanding of statistics without it is impossible. Others dive into probability immediately. Many students have trouble with this topic, however, so I have begun with descriptive statistics and delayed probability until Chapter 5; it is then used in every succeeding chapter.

Chapters 1 through 8, and parts of 9 and 10 should ordinarily be taken in sequence; material from the remainder of Chapters 9 and 10 and Chapters 11 through 17 can be selected according to the interests and speed of the students and the length of the course.

The second edition differs from the first in that (1) hundreds of new exercises have been added; many of these are statistical problems from real life; (2) the concepts behind confidence intervals and hypothesis tests are emphasized again and again; (3) notation is simplified; and (4) the errors in arithmetic have been corrected.

I am especially indebted to Dr. Peter Nemenyi who suggested many of the new and interesting problems. Special thanks are due to others who read and criticized the first edition: Katharine J. Kharas, Karen Rappaport, Donald L. Evans, Stephen B. Vardeman, Neil W. Henry, Martin D. Fraser, Michael Jacobs, and Fred Morgan.

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1 WHAT MATHEMATICAL BACKGROUND IS NEEDED?

Most of this chapter should be familiar stuff. Summation notation (Section 1.3, pages 3–4) may be new to you. You will use the remaining parts of this chapter so frequently, however, that you will be foolish indeed if you let anything slip by without understanding it. Even students who are good at mathematics are sometimes confused about whether multiplication or addition is carried out first [$4 + 2(8) = 4 + 16$ or $6(8)$?] and how parentheses are used to change the usual order.

1.1 INTRODUCTION

Many students approach statistics with apprehension, having resolved after junior year in high school never to take another mathematics course. But be of good cheer for two reasons. The background in mathematics which you need is remarkably slight. And in your high school courses you may have learned how to solve quadratic equations, for example, without understanding **why** you were learning to do this. Now those of you who are concerned about using mathematics are studying statistics because a knowledge of this material is necessary background for your study of psychology, economics, zoology, or sociology. You will find an almost immediate application of the mathematics you learn to a subject that interests you, and this delightful situation—in contrast to your past experience—will give most of you a new and splendid ability to cope with mathematics.

Try always to gain some perception of the general arguments that produce results rather than accept them blindly. An idiot can be trained (or a computer can be programmed) to plug numbers into formulas; you must learn when and why a particular formula is used. Occasionally an explanation, in order to be mathematically proper, would require that you know calculus; in these cases the explanation is purely intuitive or is omitted. Fair warning will be given when such occasions arise.

A special notation will be used to warn you of traps that have been set for you: an unusual twist, a problem that cannot be solved, or some special reason to be wary.

1.2 THE ORDER OF OPERATIONS

The word "operations" here refers to squaring, multiplication or division, and addition or subtraction. The rules agreed on by mathematicians are that these are carried out in the order given below when more than one operation is to be performed:

> (a) First square.
> (b) Then multiply or divide.
> (c) Then add or subtract.

Example 1

$2(3) + 4 = ?$

$2^2(3) + 1 = ?$

$3 - \frac{12}{2^2} = ?$

$2(3) + 4 = 6 + 4 = 10.$

$2^2(3) + 1 = 4(3) + 1 = 12 + 1 = 13.$

$3 - \frac{12}{2^2} = 3 - \frac{12}{4} = 3 - 3 = 0.$

But suppose you don't want to do things in that order? What if, for example, 2 and 3 are first to be added and then the result squared? Parentheses ( ), square brackets [ ], or curly brackets { } are used, and a new rule is added:

> If there are parentheses or brackets, simplify what is inside them first before continuing.

Example 2

$6(4 + 1)^2 = ?$

$[2^2(3) - 2]^2(4 - 1) = ?$

$2(3^2 + 4 \div 2) - 3(2^2 - 4)^2 = ?$

$6(4 + 1)^2 = 6(5)^2 = 6(25) = 150.$

$[2^2(3) - 2]^2(4 - 1) = [4(3) - 2]^2(3) = [12 - 2]^2 3 = 10^2(3) = 100(3) = 300.$

$$\begin{aligned} 2(3^2 + 4 \div 2) - 3(2^2 - 4)^2 &= 2(9 + 4 \div 2) - 3(4 - 4)^2 \\ &= 2(9 + 2) - 3(0)^2 \\ &= 2(11) - 0 = 22. \end{aligned}$$

1.3 Σ NOTATION

You will soon need to learn seven Greek letters. The first of these is the Greek (capital) S, called **sigma,** and written Σ. We shall use Σ as an abbreviation for the word "sum" or for the phrase "the sum of ______."

Notation: Σ = Sum.

If X values are 1, 2, 3, 5, then $\Sigma X = 1 + 2 + 3 + 5 = 11$. Add all the X values.

If the weights W of five students are 110, 100, 160, 200, 190 pounds, then $\Sigma W = 110 + 100 + 160 + 200 + 190 = 760$ pounds.

Remember the rule for order of operations when Σ (standing for addition) is combined with exponents, multiplication, or parentheses.

Example 1 If Y values are 4, 3, 1, 6, 2, find (a) ΣY, (b) ΣY^2, (c) $(\Sigma Y)^2$.

(a) $\Sigma Y = 4 + 3 + 1 + 6 + 2 = 16$.

(b) $\Sigma Y^2 = 4^2 + 3^2 + 1^2 + 6^2 + 2^2 = 16 + 9 + 1 + 36 + 4 = 66$.

(c) $(\Sigma Y)^2 = 16^2 = 256$.

There are three rules for working with summations:

Rule 1. $\Sigma(X + Y) = \Sigma X + \Sigma Y$. (This rule only makes sense if the number of X values is the same as the number of Y values.)
Rule 2. If k is a constant, $\Sigma kX = k\Sigma X$.
Rule 3. If A is a constant, $\Sigma A = nA$, where n is the number of values that are being added.

Example 2 X values: 4, 1, 6, 3; corresponding Y values: 2, 3, 7, −1. (a) $\Sigma(X + Y) = ?$ (b) $\Sigma 2X = ?$ (c) $\Sigma 5 = ?$ (d) $\Sigma(X - Y) = ?$

(a) $\Sigma(X + Y) = \Sigma X + \Sigma Y = (4 + 1 + 6 + 3) + (2 + 3 + 7 - 1) = 14 + 11 = 25$.

(Rule 1)

(b) $\Sigma 2X = 8 + 2 + 12 + 6 = 28$, but also
$= 2\Sigma X = 2(4 + 1 + 6 + 3) = 2(14) = 28$.

(Rule 2, which says a constant can be factored out)

(c) $\Sigma 5 = 4(5) = 20$. (Rule 3) It is assumed that the number of values being added (here, 4) is known from the context of the problem. A more sophisticated summation notation, using subscripts, can be adopted, but we shall not need it.

(d) $\Sigma(X - Y) = \Sigma[X + (-1)Y]$
$= \Sigma X + \Sigma(-1)Y$ (Rule 1)
$= \Sigma X + (-1)\Sigma Y$ (Rule 2)
$= \Sigma X - \Sigma Y$
$= (4 + 1 + 6 + 3) - (2 + 3 + 7 - 1)$
$= 14 - 11$
$= 3.$

1.4 EXERCISES

1. $4.38 + .12 + .02 =$

2. $6.21(.01) =$

3. $5(0) =$

4. $\dfrac{27(75)}{25(9)} =$

5. $\dfrac{25 + 75}{25 + 50} =$

6. Add: $-10, 5, -3, 2, 3, -5, 2, 10$.

7. Add: $-10, -2, 5, 3, 4, 12, -8, -6$.

8. $3(5^2) + 5 =$

9. $3(5^2 + 5) =$

10. $4 - \dfrac{10^2}{20} =$

11. $3(1 - 2)^2 - 4^2(2^2 - 3) =$

12. $2^2(2^2 + 1)^2 + 3(1 + 3)^2 =$

13. X values: 2, 5, 3. Find (a) ΣX, (b) ΣX^2, (c) $(\Sigma X)^2$, (d) $\Sigma 3X$, (e) $\Sigma 2X^2$, (f) $\Sigma 4$.

14. Y values: 4, 1, 1, 3. Find (a) ΣY, (b) $\Sigma 4Y$, (c) $(\Sigma Y)^2$, (d) ΣY^2, (e) $\Sigma 3$.

15. Z values: 2, -1, 0, 6, 3. Find (a) ΣZ, (b) ΣZ^2, (c) $\Sigma 5$, (d) $(\Sigma Z)^2$, (e) $\Sigma(-Z)^2$.

16. Find (a) ΣfX, (b) ΣfX^2, (c) $(\Sigma fX)^2$.

f	X
3	2
4	5
2	3

17. X: 2, -1, 0, 3. Find $\Sigma X^2 - \dfrac{(\Sigma X)^2}{4}$.

18. Find (a) ΣfY, (b) $(\Sigma fY)^2$, (c) ΣfY^2.

f	Y
2	0
9	1
3	4
1	8

19. X values are 3, 5, 8, 2, 0, 3, 0; corresponding Y values are 2, 3, 1, −1, 0, 2, 4. Find (a) ΣX, (b) ΣY, (c) $\Sigma(X + Y)$, (e) $(\Sigma X)^2$.

ANSWERS

1. 4.52   2. .0621   3. 0

4. $3(3) = 9$ ("Cancel" first; not $\frac{2025}{225} = 9$.)

5. $\frac{100}{75} = \frac{4}{3}$ (Don't try to cancel the initial 25's.)

6. Cancel −10 and 10, −5 and 5, −3 and 3. $2 + 2 = 4$.

7. Cancel −10, −2, and 12; 5, 3, and −8. $4 - 6 = -2$; OR add the positive numbers and negative numbers separately: $24 - 26 = -2$.

8. $3(25) + 5 = 75 + 5 = 80$.

9. $3(25 + 5) = 3(30) = 90$.

10. $4 - \frac{100}{20} = 4 - 5 = -1$.

11. $3(-1)^2 - 16(4 - 3) = 3(1) - 16(1) = -13$.

12. $4(4 + 1)^2 + 3(4)^2 = 4(5)^2 + 3(16) = 4(25) + 48 = 100 + 48 = 148$.

13. (a) 10, (b) $4 + 25 + 9 = 38$, (c) $10^2 = 100$, (d) $3(10) = 30$, (e) $2(38) = 76$, (f) $3(4) = 12$.

14. (a) 9, (b) $4(9) = 36$, (c) $9^2 = 81$, (d) $16 + 1 + 1 + 9 = 27$, (e) $4(3) = 12$.

15. (a) 10, (b) $4 + 1 + 0 + 36 + 9 = 50$, (c) $5(5) = 25$, (d) $10^2 = 100$, (e) 50.

16. (a) (Add a ΣfX column) $6 + 20 + 6 = 32$, (b) (Add a ΣfX^2 column) $12 + 100 + 18 = 130$, (c) $32^2 = 1024$.

17. $14 - \frac{4^2}{4} = 10$.

18. (a) $0 + 9 + 12 + 8 = 29$, (b) $29^2 = 841$, (c) $0 + 9 + 48 + 64 = 121$.

19. (a) 21, (b) 11, (c) $21 + 11 = 32$, (d) $21 - 11 = 10$, (e) $21^2 = 441$.

1.5 FORMULAS

A number of formulas will be developed in this course; they simply give shorthand directions for carrying out operations. Study the following examples, and learn how to translate from English into mathematical language and vice versa.

Example 1 Translate into English: $\Sigma(X - 4)$.
Subtract 4 from each X value, then add the new set.

Example 2 Write a formula that gives directions to subtract 4 from each X value, square each number in the new set, and then add the squares.

$$\Sigma(X - 4)^2$$
