BUSINESS ALGEBRA & MATHEMATICS

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PREFACE: TO THE INSTRUCTOR

This text has been designed for aspiring business and/or computer oriented students. A prime objective of *Business Algebra & Mathematics* is to acquaint students with a wide range of mathematical topics used in today's fast moving business world. Logic systems such as accounting, algebra, computers, mathematics, and statistics are essential tools for success by today's—and, tomorrow's—business people. More than at any time in the past, thorough mathematical skills are the key to students having the "edge" in the job market.

This *is* the computer age. It is our duty to prepare students for this environment. And this is an excellent beginning point at which to teach this process. Why both algebraic and business mathematics topics in one text? Businesses use both in every day operations at all levels. Also, quite simply, one intro-course/text can prepare students for accounting, programming, statistics, and more. One also might ask, since the use of computers is so widespread, who needs to learn math? *Everyone!* Before one can punch the correct keys, select computer menu options, etc., it is a must to *DO* the work *manually.*

This text should apply well for first-year college students, in whatever discipline; as well as, high school students wishing a good math and logic background for college entry. An over-riding concern in the development of the text content and presentation of topics was to effectively teach the student these necessary tools. A successful student is the focus of the entire text.

The text heavily uses clear, step-by-step, examples. Narrative explanations accompany these examples, but, the best way to assist the student in grasping a topic is to show him or her how it is done. Examples allow the student to practice and to see the steps and/or solutions. A good analogy is driving a car. To become proficient requires hands on experience doing the actual driving.

This text is organized and written based on actual teaching experiences. It can be used for a single semester, or broken into two—one for algebra and one for math. Additionally, be sure to emphasize to students, especially those weak in basic

arithmetic, to thoroughly review Appendix 1. Also, all students can improve their skills by review of the other Appendicies. Often students ignore supplemental information. Urge yours to read all topics included in the text—in chapters and appendicies.

Several features of this test make it both student oriented and instructor supportive:

1. A "Preface—To The Student" is designed, hopefully, to initiate the key element to learning—*motivation*!
2. As stated above, there are Appendicies to help weak students and/or to bolster advanced learning by all students.
3. Each chapter begins with a statement of *Learning Points* to key students in on primary chapter objectives early on, *and* ends with a *Glossary* of new terms not explained in detail in a particular Section's narrative and/or examples.
4. Boldface, italics, and other highlights are used to emphasize major points, new terms, formulas, basic concepts, and more.
5. Many topics include *RULES for → (the topic)*.
6. Many challenging problems are included in each Section to enrich the learning process. Their purpose is to allow the student to gain confidence by applying the concepts learned in each topic. Assign as many as time allows; and, encourage students to do others. Solutions to odd numbered problems have been included at the end of the text.
7. Relevant reference tables have been included for student use in problem solving.
8. The most significant feature of this text is the frequent use of examples. Students must be shown step-by-step procedures for problem-solving so that they can easily identify trouble spots when they occur.

Between my efforts and yours in the classroom, use of this text *will* start students in the correct direction to achieve success in the computerized, logic-oriented business world of the 1980's and beyond.

SPECIAL THANKS

This text and associated materials could not have been written, edited, or completed without my wife, Mary's total support and assistance. She edited the entire text, instructor's manual (and *did* all solutions!), as well as giving invaluable advice on text/solutions content.

Also, many DeVry Institute of Technology, Phoenix, students helped originally in solving problems. They inspired me to write this text, to better their successors' learning process.

Thomas F. George

PREFACE: TO THE STUDENT

Student: "Why don't I understand the material the first time I read it, Professor?"
Prof.: "You're *not* supposed to."

Often—*no*, next to always—it takes several readings of a topic to grasp its meaning. If you don't take the time to re-read until you understand, you will lose out. The loss will not just be a grade on the next test, or even in the course. The real loss will be *no job*.

Student: "Yeah, I suppose, but, the peer pressure makes it difficult to ask questions when I don't understand something."
Prof.: "Well, remember two things:"
"1) First, there is no dumb question."
"2) In class, the only opinion you should worry about is that of *your instructor*." In fact, *to learn is to ask*. Everyone, whether a child, adult, student, teacher, or professional must ask questions in order to learn.

You can learn and succeed. The key is in your attitude. If you say, "I can't," you won't learn. If you say "I haven't time," you'll never get ahead. No one is stupid. But you must be *motivated*. Often, this takes time. Also, you must read, solve problems, ask questions, and, when things look bleak, do it all again, and again, if necessary. No one said that studying would be easy, so, put yourself in gear and press on!

Don't sell this text when your course is finished. No matter what job you accept after school, mathematical concepts/topics will continue to require your attention and expertise. You will find, as some time passes between this course and a later math/accounting/other course or your job, that review of this text will assist you. So learn all you can between these covers. You should also take detailed class notes of your instructor's words, board/projector examples, and other learning aids.

When working problems, use a dark pencil with an eraser. My father-in-law was an aircraft maintenance man who once told me enviously, "My wrench has no eraser, thus I cannot make a mistake. You and your pencil can; and, nobody will know." So,

don't worry if you make a mistake. You can always erase. Also, try to memorize formulas and step-by-step procedures until you do enough problems where they become second nature to you.

This text should serve you as a very useful tool toward your future success. Now it is up to you to learn the subject matter. Just *try*, and I'm sure you will have success.

Thomas F. George
(A Teacher & a Friend!)

CONTENTS

1

BASIC MATHEMATICAL CONCEPTS

Learning Points

1. Understand mathematical symbols.
2. Understand real numbers and operations with them.
3. Understand the properties of real numbers.
4. Understand the use of fractions, decimals, and percents.

SECTION 1.0. INTRODUCTION

In the beginning one is not expected to know all. This chapter gives you a foundation for the remainder of the text. Its difficulty factor is relatively low—so proceed with the expectation to succeed.

Topics you will learn include:

a. The various symbols and notations used in the "language" of math.
b. Real numbers, and how to perform addition, subtraction, multiplication, and division with both positive and negative real numbers.
c. The different properties of numbers and the concept of subsets. These fundamentals are the basis for the entire algebraic system.
d. The use of fractions, decimals, and percents, as well as conversion from one to another.

You may find Chapter 1 material familiar and wish to skip it. DON'T! Review of these basics will give you the solid foundation you need. Also, *review the appendices*, especially if you are weak in math skills—you will be happy you did. The old maxim that "practice makes perfect" is most appropriate in mathematics. Work as many problems as possible and you will excel.

SECTION 1.1. MATHEMATICAL SYMBOLS

Algebra deals heavily with equalities, inequalities, and relationships between data elements. Let us review some of the basic symbols.

COMPARATIVE SYMBOLS

Comparative symbols indicate relationships between quantities.

Symbol	*Explanation*
$=$	Equals ($X = Y$) says that X and Y are the same numbers, or are equal.
$\neq$	Does *not* equal ($7 \neq 5$).
$>$	Greater than ($7 > 5$).
$<$	Less than ($5 < 7$). *Note:* The symbols $<$ and $>$ always point to the smaller data element.
$\geq$	Greater than or equal to ($7 \geq X$).
$\leq$	Less than or equal to ($3 \leq Y$). *Note:* $7 \geq X$ means that X can only have values of 7 or less. $3 \leq Y$ means that Y can only have values of 3 or more.

COMBINATION SYMBOLS

Often we want to show two or more data elements as a single entity. To combine, or group, elements we use the following **combination symbols**:

( ) Parentheses
[ ] Brackets
{ } Braces

These are used in this book in the order of brackets within parentheses, and braces within brackets. For example: $(5 + [X - 2 - \{X + 10\} - 6])$.

A pair of parentheses is sometimes called a parenthesis.

OPERATION SYMBOLS

Operation symbols indicate what mathematical work is to be done.

Symbol	*Narrative*
$+$	Addition (the sum of the elements).
$-$	Subtraction (the difference of the elements).
$\times$ (or $\cdot$)	Multiplication (the product of the elements).
$\div$	Division (the quotient of the elements).

Example 1.1

Expression in Symbols	*Expression in English*
$7 + 5 = 12$	The sum of 7 and 5 is 12.
$12 - 5 < 10$	The difference of 12 and 5 is less than 10 (12 minus 5 is 7).
$6 \times 3 > 15$	The product of 6 and 3 is more than 15 (6 times 3 is 18).
$4(7 - 5) = 8$	Four times the difference of 7 and 5 is 8. (*Note:* The absence of a symbol or sign between elements denotes multiplication).
$-2 + (3[2 + 8] + 4) = 32$	Negative 2 plus the quantity (group) of three times the sum of 2 and 8, plus 4 equals 32.

We can consider mathematical symbols similar to English punctuation. Consider:

"Allen," said Bill, "is funny."

Allen said, "Bill is funny."

Punctuation helps to explain the meaning of each sentence. *Mathematical symbols* do the same. Consider a similar situation in mathematical symbols:

Example 1.2

$$6 \times 3 + 5$$

Solution A: $18 + 5 = 23$

Solution B: $3 + 5 = 8 \times 6 = 48$

Note that we have two answers to a single problem. If we multiply and then add, we get 23. However, if we add and then multiply, we get 48. To clear up such problems, we must establish some **rules of operations.**

RULES OF OPERATIONS

Let us agree to certain established rules, or priorities, in working with expressions.

Rules of Operations

When an expression is written horizontally, the following priority order MUST be used:

First Priority:	1. Combine quantities in parenthesis.
↓	2. Raise quantities to a power.
	3. Multiply/Divide from left to right.
Last Priority:	4. Add/Subtract from left to right.

Note. Sometimes you cannot combine quantities, such as where you have a number and a variable in a parenthesis. Also, there is no priority between multiplication and division or between addition and subtraction. The choice is yours. We can put the rules listed above into an acronym: PPMA ("Paul Picked My Apple")—Parenthesis, Powers, Multiply (Divide), and Add (Subtract).

Example 1.3

$$6 \times 4 \div 3 + 2^3$$

Step 1 $= 6 \times 4 \div 3 + 8$

Step 2a $= 24 \div 3 + 8$ } Do 2a then 2b *or* 2b

Step 2b $= 8 + 8$ } then 2a.

Step 3 $= 16$

Example 1.4

$$(7 - 12)^3 - 12^2$$

Step 1 $= (-5)^3 - 12^2$

Step 2 $= -125 - 144$

Step 3 $= -269$

Note. In $(-5)^3$ we are cubing the quantity -5 (or $-5 \times -5 \times -5$), where $-5 \times -5 = +25$, and $+25 \times -5 = -125$. In -12^2 we are squaring the quantity 12 (or 12×12). *Watch your signs*!

Example 1.5

$$
\begin{aligned}
& 18 \div 3 + (9 - 7)^4 \times 3 - (1 - 4)^3 \\
\text{Step 1} \quad & = 18 \div 3 + 2^4 \times 3 - (-3)^3 \\
\text{Step 2} \quad & = 18 \div 3 + 16 \times 3 - (-27) \\
\text{Step 3} \quad & = 6 + 48 - (-27) \\
\text{Step 4} \quad & = 54 + 27 \\
\text{Step 5} \quad & = 81
\end{aligned}
$$

("−" times "−" is a "+")

Individual steps can be combined. However, be very careful. When in doubt show all steps, at least until you have developed a firm understanding of the procedure. Let us look at one further example with "multiple" or "layered" data groupings.

Example 1.6

$$
\begin{aligned}
& 4[2 - (a + 3b) + (6 + b)] \\
\text{Step 1} \quad & = 4[2 - a - 3b + 6 + b] \\
\text{Step 2} \quad & = 4[8 - a - 2b] \\
\text{Step 3} \quad & = 32 - 4a - 8b
\end{aligned}
$$

Note. Since the two parentheses are totally separate, both are removed in Step 1. Note also that the first one is $1(a + 3b)$. In effect we have -1 times $(a + 3b)$. One must multiply the quantity outside into *all* quantities inside the parenthesis. Step 2 combines "like" quantities—numbers and in this case the "b" variables.

PROBLEMS

Write in words an equivalent statement for the following. Where possible, use words such as *sum*, *difference*, *product* or *times*, and *quotient* or *divide*.

1. $42 > 17$
2. $15 > 9$
3. $x + 2 = 7$
4. $p + 6 = 7$
5. $9 - 3 = 6$
6. $3 - 9 = -6$
7. $5 + 3 \neq 10$
8. $4 + 5 \neq 12$
9. $4y - 2 = 10$
10. $3t - 4 = 12$
11. $3(t + 2) = 30$
12. $4(a + 3) = 40$
13. $x - 2 \leq 7$
14. $y - 4 \leq 10$
15. $t < r$
16. $x < y$