Numerical Techniques in Finance

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Preface

The aim of this book is to present some important models in finance and to show how they can be solved numerically and/or simulated. In this sense, this is a finance "cookbook." Like any cookbook, it gives recipes with a list of ingredients and instructions for making and baking. A recipe, however, is just a starting point; having followed it a number of times, you can think of variations of your own and make the results suit your own tastes and needs.

The minimum background you will need for this book is a good introductory course in finance. For some of the more advanced applications, it will help to have some knowledge of portfolio theory and/or speculative markets.

In addition to this theoretical background, it is necessary to know enough Lotus 1-2-3 to be able to set up a simple spreadsheet. Advanced Lotus techniques, such as Lotus functions, macros, and the use of data tables, are explained in the text. The models in the book are solved, and the programs written, in Lotus spreadsheets; where necessary, the Lotus Version 2 macro language is used.

The first four of the book's five parts address various areas of finance. The parts are largely independent, though chapters within the same part are likely to draw heavily on one another. The last part of the book focuses on technical topics. Some of these topics are mathematical (the Gauss-Seidel and Newton-Raphson methods, matrices, and random-number generators), and some deal with advanced aspects of Lotus. This part is meant to be used as a reference when you encounter an unfamiliar concept or a Lotus technique in one of the finance chapters.

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Contents

I CORPORATE FINANCE

Overview

Chapters 1–5 present some models of corporate finance that require extensive *numerical computation.*

The first is a model of financial-statement prediction. Such models are extremely important in that they allow the simulation of financial performance over time with the standard accounting constraints. This does not mean, however, that the models are merely technical. Indeed, the accounting constraints are broad enough to incorporate a variety of modes of corporate behavior. Because a corporation's balance sheet and its income statement must be solved simultaneously, this kind of model is ideal for a spreadsheet that allows circular arguments (as Lotus's does).

The second chapter uses the financial-statement-prediction model in valuing mergers and acquisitions. Valuation of this type requires that discount rates be related to the leverage of the company. The appendix of the chapter discusses the current state of the finance theory that relates the calculation of rates of return and leverage.

Chapter 3 shows how to derive debt capacity from riskless cash flows. This intrinsically important topic has applications in real-estate finance and in leasing.

Leasing is the topic of chapters 4 and 5. Chapter 4 discusses lease analysis using the equivalent-loan method. Chapter 5 discusses the lessor's analysis of leveraged leases and compares the internal-rate-of-return method of analyzing these leases with the accounting profession's multiple-phases method.

1 Simulating Financial Statements

1.1 Introduction

The usefulness of financial-statement projections for corporate financial management is undisputed. By examining *pro forma* financial statements, we can predict how much financing a firm will need in future years. We can play the usual "what if" games of simulation models, and we can ask what strains on the firm may be caused by changes in financial and sales parameters.

The use of spreadsheet models for the prediction of balance sheets is usually limited, however, to models in which all the items in the financial statement follow in a straightforward manner from sales or from other financial-statement items. To use a spreadsheet in this way—worthwhile though it is—is to fail to consider two other major types of financial-statement-prediction models that have been widely discussed in the literature:

- *Models in which it is assumed that the firm wishes to maintain a given ratio of debt to equity in its balance sheet* (see, for example, Warren and Shelton 1971). As will be shown further on, this assumption means that certain balance-sheet relations are determined from the simultaneous solution of several linear equations. In the Warren-Shelton model the firm solves a problem that involves some twenty simultaneous equations in as many unknowns.

- *Models in which the firm maximizes its value subject to a set of financing constraints* (see Myers and Pogue 1974 and further references in Brealey and Myers 1984).[1]

The latter models are theoretically preferable to the former, but the Warren-Shelton conception of financial planning is much more widespread in practice.

The Warren-Shelton model proceeds on two assumptions: that most balance-sheet items are directly or indirectly related to sales, and that the firm's primary financing concern is to maintain an appropriate balance between the value of debt and the value of equity in the balance sheet. Given these two assumptions, the firm predicts its sales for the coming year (or years) and predicts the interest rates it will have to pay on its long-term debt over the planning period. The model is solved by finding the solution to a set of simultaneous linear equations predicting both the balance sheets and the income statements for the coming years.

The introduction of simultaneity of financial relationships is usually thought to be beyond standard spreadsheet programs, and thus the Warren-Shelton-type financial-planning models are generally solved on larger computer systems. In fact, despite the simultaneity, these types of models can easily be set up and solved using commonly available microcomputer spreadsheet programs.

The next section presents a simple example that embodies all the essential features of the Warren-Shelton-type financial-planning model; the following section shows how this example may be solved using Lotus 1-2-3.

1.2 How Financial Models Work: Theory and Examples

Almost all financial-statement models are sales-driven; that is, most financial-statement variables are assumed to be functions of the sales level of the firm. For example, accounts receivable may be taken to be a direct percentage of the sales of the firm. A slightly more complicated example would postulate that the net-fixed-assets account is a step function of the level of sales:

$$\text{Net fixed assets} = \begin{cases} a & \text{if sales} < A \\ b & \text{if sales } A \leqslant \text{sales} < B \\ \text{and so on.} & \end{cases}$$

In order to solve a financial-planning model, one must distinguish between those financial-statement items that are functional relationships of sales and perhaps of other financial-statement items and those items that involve policy decisions. The asset side of the balance sheet is usually assumed to be dependent only on functional relationships. The current liabilities may also be taken to involve functional relationships only, leaving the mix between long-term debt and equity as a policy decision.

A simple example of this is the following. We wish to predict the financial statements for a firm whose current (year 0) level of sales is 1,000. The firm expects its sales to grow at a rate of 10 percent per year. In addition, the firm anticipates the following financial-statement relations:

CA/sales (ratio of current assets to sales): 15%

CL/sales (ratio of current liabilities to sales): 8%

NFA/sales (ratio of net fixed assets to sales): 77%

Expenses, excluding interest and depreciation = 80% of sales.

Depreciation

Fixed assets and depreciation are troublesome to model. For the moment, we shall employ the following model of depreciation of fixed assets (at the end of the chapter, we shall return to this topic, and explore some alternative models).

- The firm's depreciation policy is to depreciate all fixed assets over a ten-year life span, using straight-line depreciation.
- New assets are purchased at the end of the year.
- The accumulated depreciation in any given year is calculated as follows:

Accumulated depr(t)

$$= \text{Accumulated depr}(t-1) + \frac{\text{Assets at cost } (t-1)}{\text{Average depreciable life}}.$$

Long-Term Debt

The firm has a long-term debt of 280 in year 0, on which it pays interest of 10.5 percent. The debt is to be repaid in annual equal installments over 5 years. The firm estimates that interest on any new long-term debt will be 9.5 percent. The common-stock account in the balance sheet shows an initial balance of 450, and the firm has retained earnings in year 0 of 110. The firm has a 47 percent tax rate.

Financial Statement

The firm's balance sheet and its profit-and-loss statement look as follows.

Balance sheet

Assets	*Liabilities*
CURRENT ASSETS	CURRENT LIABILITIES
FIXED ASSETS	LONG-TERM DEBT
at cost	STOCK
depreciation	RETAINED EARNINGS
net	
TOTAL ASSETS	TOTAL LIABILITIES

Profit-and-loss statement

Sales
– Expenses
– Interest payments
– Depreciation

Profit before tax
– Taxes

Profit after tax
– Dividend

Addition to Retained Earnings

The facts given thus far suffice to determine the asset side of the balance sheet as well as the current liabilities. Three variables remain to be determined: NEW STOCK (which the firm will sell), NEW LONG-TERM DEBT, and the ADDITIONAL RETAINED EARNINGS for the year. To determine these, we have to examine the firm's financial policies.

As a matter of policy, the firm wishes to reduce its current 50 percent ratio of the book value of debt to the book value of equity to 40 percent over the next 5 years. This decision imposes a constraint on the financing, which the model must solve.

1.3 Setting Up and Solving the Model using Lotus

A schematic of the financial statements for year 1 follows. (Usually the numbers refer to the year. Thus, RET1 is year 1's retained earnings. The dollar signs indicate, loosely, which items you will want to copy absolutely to the next years.)

```
CURRENT ASSETS              ($SALES*(1+$SALES GROWTH)^YEAR1*$CA/SALES)
FIXED ASSETS
 at cost                    +NET FIXED ASSETS1+ACCUMULATED DEPR1
 accumulated depreciation +FIXED ASSETS AT COSTØ*0.1+ACCUMULATED DEPRØ
 net                        +SALES1*$NFA/SALES
TOTAL ASSETS                (CURRENT ASSETS1+NET FIXED ASSETS1)
```

(We have assumed that all new assets are purchased at the end of year 1, so that these assets will first be depreciated in year 2.)

```
CURRENT LIABILITIES     ($SALES*(1+$SALES GROWTH)^YEAR1)*$CL/SALES
LONG TERM DEBT          +DEBT/EQUITY1*(STOCK1+RETAINED EARNINGS1)
STOCK                   (STOCKØ+NEW STOCK1)
RETAINED EARNINGS       (RETAINEDØ+RETENTION1)
TOTAL LIABILITIES       @SUM(CL1,LTD1,STOCK1,RETAINED1)


determination of unknowns
NEW STOCK               (TOTAL ASSETS1-CURRENT LIAB1-LTDEBT1-STOCKØ-RETAINED EARNINGS1)
NEW DEBT                +TOTALNEWDBT1-TOTALNEWDBTØ
TOTAL NEW DEBT          +LTDEBT-$LTDEBTØ*(1-0.2*YEAR1)
```

Long-term debt is determined by the desired debt/equity ratio for year 1. Since 20 percent of the long-term debt on the books in year 0 has to be repaid in year 1, total new debt is determined by the expression given above. In each year *t*,

NEW DEBT = TOTALNEWDEBT(t) − TOTALNEWDEBT(t1).

The variable TOTALNEWDEBTØ (which is, of course, zero) has been put in only for consistency in copying the formula.

```
income statement
SALES                   ($SALES*(1+$SALES GROWTH)^YEAR1)
EXPENSES                +SALES1*$EXPENSES/SALES
INTEREST PAYMENTS       +$LTDEBTØ*(1-0.2*YEAR1)*$CURRENT%+TOTALNEWDBT1*$NEW INTEREST
DEPRECIATION            +ACCUMULATED DEPR1-ACCUMULATED DEPRØ
PROFIT BEFORE TAX       +SALES1-EXPENSES1-INTEREST1-DEPR1
PROFIT AFTER TAX        +PBT1*(1-$TAX)
DIVIDENDS               (PAT1*$PAYOUT)
RETENTION               +PAT1-DIV1
```

The additional retained earnings are derived from the firm's income statement. The net profits after taxes depend on the firm's total debt, which depends on the new debt raised, which depends on the firm's desired debt/

equity ratio. In Lotus, none of these circularities is material. (As soon as Lotus detects a circularity in the argument, you will see a CIRC at the bottom of the screen. This does not prevent a solution, however.)

One way to solve this model is to write out the equations for the various balance-sheet items explicitly and to solve them as a system of simultaneous linear equations. This is the approach taken by Warren and Shelton (1971). Another way to solve the model is to use recursion. A recursive solution looks like the schematic spreadsheet solution above: Each unknown in the model is written as a function of other unknowns, and the unknowns are substituted one into the other. At any stage, the current value of an unknown depends on the previous values of the other unknowns on which it is dependent.

As an example of recursion, take our equation for STOCK. Stock in any year is the sum of the STOCK in the previous year plus the NEW STOCK. NEW STOCK is defined in terms of other balance-sheet items (RETAINED EARNINGS for the current year, the LONG TERM DEBT for the current year, and so on). These items, in turn, are functions of balance-sheet and income-statement items. Ultimately, our model involves extensive circularity of argument: STOCK depends on NEW STOCK, which depends on RETAINED EARNINGS, which depends on RETENTION, which depends on PROFIT AFTER TAX, which depends on INTEREST PAYMENTS, which depends on LONG TERM DEBT, which depends on STOCK. This is only one example of the kinds of circularities involved.

Recursive methods involve taking an initial solution for the variables and successively "plugging this solution into" the model. Every time we do this, we get an imprecise approximation to a solution, since the current value of any variable depends on the previous values of the other variables. At some point the approximations will converge; that is, the current value of STOCK based on the previous values of all the other variables will be the same as the current value of these variables based on the previous value of STOCK.[2]

Because the solution involves a circularity of arguments, the model will not converge immediately to the correct solution. Several presses of the F9 ("calc") button usually suffice to cause the model to converge.[3]

The finished product looks like this (all numbers have been rounded off to the nearest whole number, so there may be some rounding errors):

INITIAL SALES	1000
SALES GROWTH	0.10
CA/SALES	0.15
CL/SALES	0.08
NFA/SALES	0.77
EXPENSES/SALES	0.80
INITIAL DEBT	280
CURRENT INTEREST	0.105
NEW INTEREST	0.095
INITIAL STOCK	450
INITIAL RETAINED	110
TAX RATE	0.47
DIVIDEND PAYOUT	0.70

	0	1	2	3	4	5
balance sheet						
CURRENT ASSETS	150	165	182	200	220	242
FIXED ASSETS						
at cost	1100	1287	1500	1744	2020	2335
depreciation	330	440	569	719	893	1095
net	770	847	932	1025	1127	1240
TOTAL ASSETS	920	1012	1113	1225	1347	1482
CURRENT LIABILITIES	80	88	97	106	117	129
LONG TERM DEBT	280	300	320	342	364	387
STOCK	450	502	561	628	704	791
RETAINED EARNINGS	110	123	136	149	162	175
TOTAL LIABILITIES	920	1012	1113	1225	1347	1482
determination of unknowns						
NEW STOCK		52	59	67	76	87
NEW DEBT		76	77	77	78	79
TOTAL NEW DEBT	0	76	152	230	308	387
RATIO OF DEBT/EQUITY		0.48	0.46	0.44	0.42	0.40
income statement						
SALES	1000	1100	1210	1331	1464	1611
EXPENSES	800	880	968	1065	1171	1288
INTEREST PAYMENTS	29	31	32	34	35	37
DEPRECIATION	110	110	129	150	174	202
PROFIT BEFORE TAX	61	79	81	83	83	83
PROFIT AFTER TAX	32	42	43	44	44	44
DIVIDENDS	22	29	30	31	31	31
RETENTION	10	13	13	13	13	13