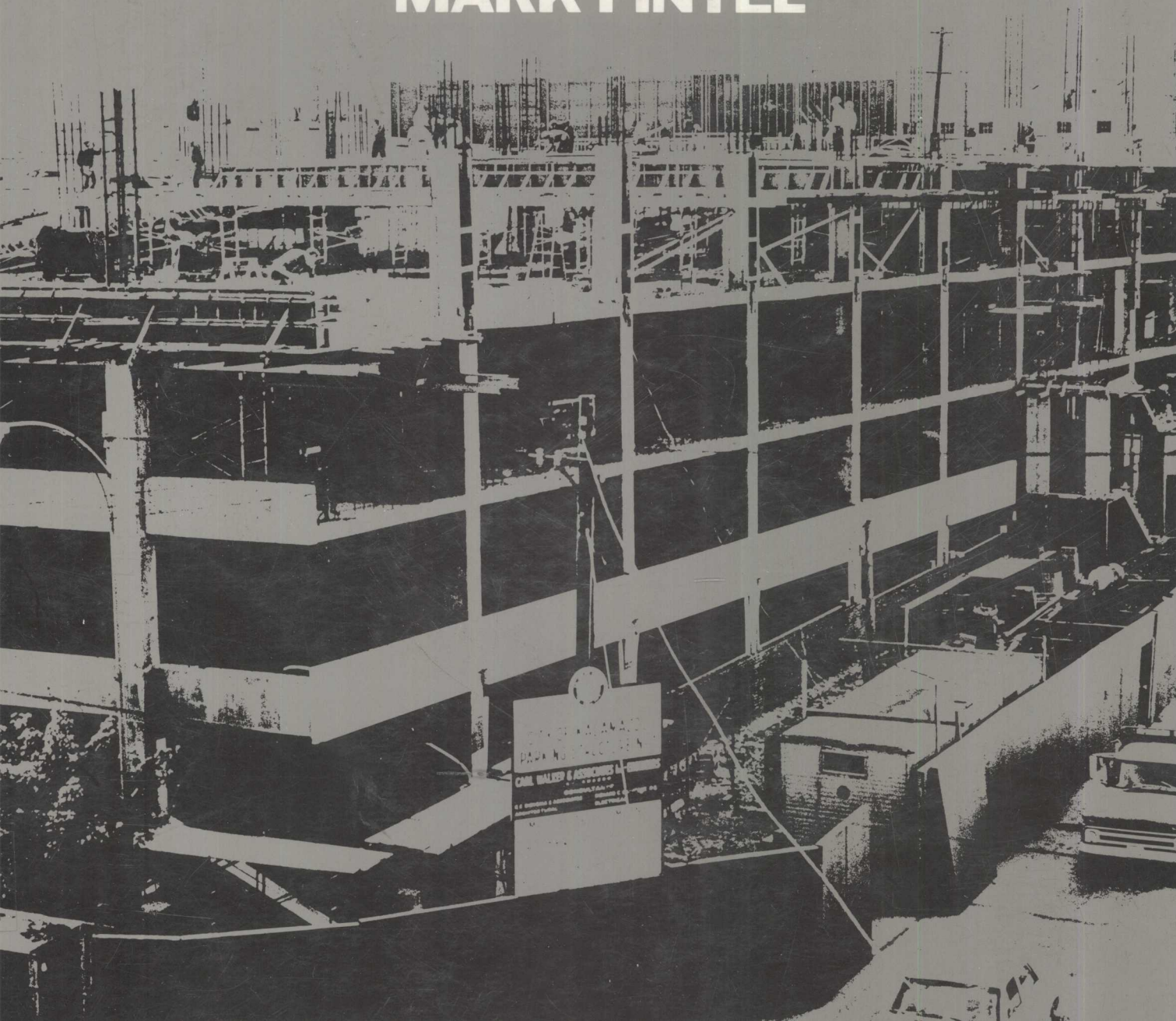


# HANDBOOK OF CONCRETE ENGINEERING

**Second Edition**

**Edited by**

**MARK FINTEL**



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# Handbook of **CONCRETE ENGINEERING**

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**Second Edition**

edited by **Mark Fintel**



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# Preface to Second Edition

The significant changes that have taken place in the state-of-the-art of concrete engineering and construction during the last ten years with the extreme success and popularity in the United States and abroad of the first edition of this handbook published in 1974 has made it imperative to issue this second edition. The following were the main contributors to the changes: (1) many innovations in design techniques have been introduced in the interim, particularly as a result of the wide use of computers in design offices thus allowing extreme sophistication of analysis; (2) the ACI code that serves as the basis for concrete design has undergone two significant revisions—in 1977, again in 1983. This latest edition of the handbook is based on the 1983 version of the ACI Code; (3) progress in construction equipment and methods; (4) higher strength materials.

The charts and tables of the chapter on the proportioning of sections are based on the metric system, making it possible for designers in Canada and overseas to apply the metric version of the 1983 Code to their own designs in the metric system.

Several chapters in the first edition have been deleted and replaced by new ones on post-tensioned slab systems (which are widely used in buildings), parking structures (due to their popularity), and structural plain concrete (has now been introduced in the ACI Code). All other chapters (except that covering tubular structures) have been thoroughly revised to reflect the changes brought about by the 1977 and 1983 ACI Codes and to incorporate the changes in the state-of-the-art since publication of the first edition of the handbook ten years ago.

MARK FINTEL

# Preface to First Edition

This handbook contains up-to-date information on planning, design, analysis, and construction of engineered concrete structures. Its intention is to provide engineers, architects, contractors, and students of civil engineering and architecture with authoritative practical design information.

The tremendous progress and changes in all the areas of concrete engineering in the last two decades seemed to indicate a need for a new *Handbook of Concrete Engineering*. In addition, the many inquiries on subjects of concrete engineering received daily by the editor in the course of his professional activity, heightened his enthusiasm in accepting the proposal from Van Nostrand Reinhold to assemble this handbook.

Much of the information contained in this book has evolved during the last 15 to 20 years. The subjects of a number of chapters are so recent that the material has never before been published in book form.

The following traditional engineering subjects are covered in chapters on proportioning of members (ultimate strength design); deflections; flat plates and flat slabs; foundations; properties of materials for reinforced concrete (geared particularly to design engineers); joints in buildings; thin shells; prestressed concrete; chimneys; silos and bunkers; concrete masonry; sanitary structures; pipes, marine structures; paving; construction methods and equipment; and structural analysis by force, displacement and finite element methods.

A number of chapters are of particular interest because they represent areas of recent technological advances: fire resistance, multistory structures; tubular structures; ductility of reinforced concrete members; earthquake resistance; large panel structures; and finally, computer applications and computer software.

Because of space limitations, a careful selection of topics has been made. The 27 chapters have been written by 28 experts from industry and universities, recognized as outstanding authorities in their respective fields. Their wide experience has resulted in concise chapters geared toward practical application in planning, design and construction of engineered concrete structures. Each chapter contains the general philosophy, the basic concepts, and applications elaborated by design examples. While ready-to-use formulas and design approaches are presented in the chapters, theoretical development of formulas has been omitted, and references for additional source material have been appended to most chapters.

The editor gratefully acknowledges the efforts of the authors in preparing high quality manuscripts, and their cooperation and patience with the editor and the publishers in all the stages of producing the handbook. Thanks to their unstinting cooperation, this book has been produced in a relatively short number of years.

MARK FINTEL

September, 1974

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# Proportioning of Sections— Strength Design Method

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MURAT SAATCIOGLU, Ph.D.\*

## 1.1 INTRODUCTION

This chapter contains up-to-date design information on proportioning reinforced concrete sections. It is intended to provide practical design information and design aids to structural engineers and designers who are involved in various phases of the design process. Readers are expected to be familiar with the fundamentals of reinforced concrete design, although the basic concepts are reviewed in a systematic manner for bridging the gap between theory and practice. Each subject category covered in the chapter contains a brief explanation of the relevant design concepts, illustrative examples, and the commonly used design aids. The material contained in this chapter includes: (1) fundamentals of reinforced concrete behavior, (2) description of the strength design method, (3) design of members for flexure, (4) design for flexure and axial load, (5) proportioning sections for shear and torsion, and (6) properties and use of reinforcement in concrete.

## 1.2 SI UNITS IN STRUCTURAL DESIGN

The International System of units, abbreviated SI (from the French, *Système International d'Unités*), has been accepted throughout the world. In this chapter SI units are followed

exclusively. All other chapters of the *Handbook* are in conventional U.S. units. While the basic ACI 318-83 continues to be written in conventional U.S. units, a metric version of the code, ACI 318M-83, has also been issued for those users who make their designs in SI units.

The SI system is an absolute system of units based on the quantities of length, time, and mass. In structural engineering the primary concern is on the units of length and force, with mass involved only when gravitational forces are being computed.

A distinct feature of SI appears in the use of the term “weight.” In the SI system, “mass” is used to express the amount of material in kilograms (kg). It should not be confused with weight, since weight is a force applied by gravity on the mass. Kilogram is *not* a force unit and only indicates the amount of matter in an object. The kilogram is defined as the mass of a certain platinum-iridium cylinder that is kept at the International Bureau of Weights and Measures near Paris, France. An accurate copy of this cylinder is kept at the National Bureau of Standards in the United States. Mass of an object remains the same everywhere on earth, whereas weight differs going from sea level to the top of a mountain because of the change in gravitational acceleration. In application of SI units to structural engineering, the conventional term “unit weight” is replaced by “mass density.” For example, mass density of normal density concrete is  $2400 \text{ kg/m}^3$ . Again, mass density is *not* the same as what is conventionally known as weight of a unit volume of

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material. Therefore, in computing dead loads, mass density should be multiplied by the gravitational acceleration,  $g = 9.8 \text{ m/s}^2$ , to obtain force.

In SI units the unit of force is the newton (N). One newton is defined as the force required to give a one-kilogram mass an acceleration of one meter per second square. Or, simply:

$$F = ma$$

$$\text{newton} = \text{kilogram} \times \text{meter/second square}$$

Since the load is a force term, loads and forces are expressed in newtons, kilonewtons, or meganewtons for concentrated loads. Distributed loads are expressed as kilonewton per square meter or kilonewton per meter. It should be noted, however, that  $\text{kN/m}^2$  is a load unit and is not used for stress. For strength and stress, pascal, kilopascal, and megapascal can be used.

$$\text{pascal} = \text{newton/square meter}$$

Concrete strength and reinforcement yield stress are expressed in units of megapascal (MPa).

Length in structural engineering is generally expressed as millimeters and meters. The meter, originally defined as one

ten-millionth of the distance from the pole to the equator along the meridian through Paris, was later defined as the length of a certain platinum-iridium bar kept at the International Bureau of Weights and Measures. A more accurate definition of a meter is 1,650,763.73 wavelengths of a certain radiation of the krypton-86 atom.

Generally, in structural design, meter is used to express span lengths and column heights. For cross-sectional characteristics such as dimensions, areas, or moments of inertia, as well as the area of reinforcement, units are expressed in millimeters.

The other unit of interest to engineers is a unit to express moment. Moments are generally expressed in kilonewton meters ( $\text{kN} \cdot \text{m}$ ).

Table 1-1 provides conversion factors from the conventional U.S. units (British system of units) to the SI units for those quantities that are commonly referred to in a structural design process. The metric bar sizes (consistent with the SI units) are discussed in Section 1.8.

### 1.3 BEHAVIOR OF REINFORCED CONCRETE

In designing reinforced concrete members it is important to understand the basic material and member behavior. In this

TABLE 1-1 Metric Conversion Factors

<i>To Convert From</i>	<i>To</i>	<i>Multiply By</i>
<b>LENGTH</b>		
inch (in.)	millimeter (mm)	25.4
inch (in.)	meter (m)	0.0254
foot (ft)	meter (m)	0.3048
yard (yd)	meter (m)	0.9144
<b>AREA</b>		
square foot (sq ft)	square meter ( $\text{m}^2$ )	0.09290
square inch (sq in.)	square millimeter ( $\text{mm}^2$ )	645.2
square yard (sq yd)	square meter ( $\text{m}^2$ )	0.8361
<b>VOLUME</b>		
cubic inch (cu in.)	cubic millimeter ( $\text{mm}^3$ )	16387.1
cubic foot (cu ft)	cubic meter ( $\text{m}^3$ )	0.02832
cubic yard (cu yd)	cubic meter ( $\text{m}^3$ )	0.7646
gallon (gal) Canadian	liter	4.546
gallon (gal) Canadian	cubic meter ( $\text{m}^3$ )	0.004546
gallon (gal) U.S.	liter	3.785
<b>FORCE</b>		
kip	newton (N)	4448.2
kip	kilonewton (kN)	4.4482
pound (lb)	newton (N)	4.4482
<b>FORCE PER UNIT LENGTH</b>		
pound per linear foot (plf)	newton per meter (N/m)	14.5939
<b>FORCE PER UNIT AREA</b>		
pound per square foot (psf)	newton per square meter ( $\text{N/m}^2$ )	47.8803
<b>STRESS, MODULUS OF ELASTICITY</b>		
pound per square inch (psi)	kilopascal (kPa)	6.895
pound per square foot (psf)	pascal (Pa)	47.8803
kip per square inch (ksi)	megapascal (MPa)	6.895
<b>MOMENT</b>		
foot kip	kilonewton meter ( $\text{kN} \cdot \text{m}$ )	1.3558
<b>MASS, DENSITY</b>		
pound (lb)	kilogram (kg)	0.4536
pound per linear foot (plf)	kilogram per meter ( $\text{kg/m}$ )	1.488
pound per square foot (psf)	kilogram per square meter ( $\text{kg/m}^2$ )	4.882
pound per cubic foot (pcf)	kilogram per cubic meter ( $\text{kg/m}^3$ )	16.02

section, discussions are devoted to behavior of reinforced concrete members subjected to flexural, axial, shear, and torsional stresses. Background information on the *ACI Code* design provisions is presented. Derivations of some of the *ACI* design equations are given.

### 1.3.1 Flexure

A simple reinforced concrete beam loaded by gravity loads exhibits the basic characteristics of flexural behavior. Depending on the magnitude of bending moments, the beam deforms either in the elastic range or in the inelastic range. If the bending moment is smaller than the cracking moment, then the beam is in the elastic range. Figure 1-1(a) illustrates strain and stress distributions at the midspan section prior to flexural cracking. At this stage, stresses in concrete are proportional to strains. The internal force couple formed by tension and compression in concrete provides the resistance to the externally applied moment. The reinforcement in concrete is basically inactive at this stage, mainly because it has not strained enough to develop sizable stresses. Principles of elastic theory can be employed for computing stresses in concrete. The presence of reinforcement in concrete makes the beam nonhomogeneous; therefore, the transformed area concept is used to account for the reinforcement by an equivalent concrete area. Moment of inertia of the transformed concrete section can then be used to

compute stresses in concrete. Extreme fiber tension in concrete is:

$$f_t = \frac{M(h - c)}{I_{tr}} \quad (1-1)$$

As the applied load is increased, the maximum tension in concrete approaches the modulus of rupture. At this point, tension cracks start forming in concrete. The “cracking moment” can be computed using eq. (1-1) with modulus of rupture,  $f_r$ , substituted for  $f_t$ :

$$M_{cr} = \frac{f_r I_{tr}}{h - c} \quad (1-2)$$

This equation is the same as eq. (9-8) of *ACI 318M-83* with the exception that  $I_{tr}$  is replaced by  $I_g$  for simplicity.

Once the cracks start forming, they propagate quickly toward the neutral axis under increasing loads. The neutral axis shifts upward with progressive cracking. The strain distribution assumes a new shape, resulting in a larger curvature. Since the concrete cannot resist tensile stresses, the reinforcing steel is called upon to resist the entire tension. At moderate loads, concrete stresses and strains continue to be proportional. Up to about 50% of  $f'_c$  the linear relationship between stresses and strains produces reasonably accurate results. Figure 1-1(b) shows strain and stress distributions

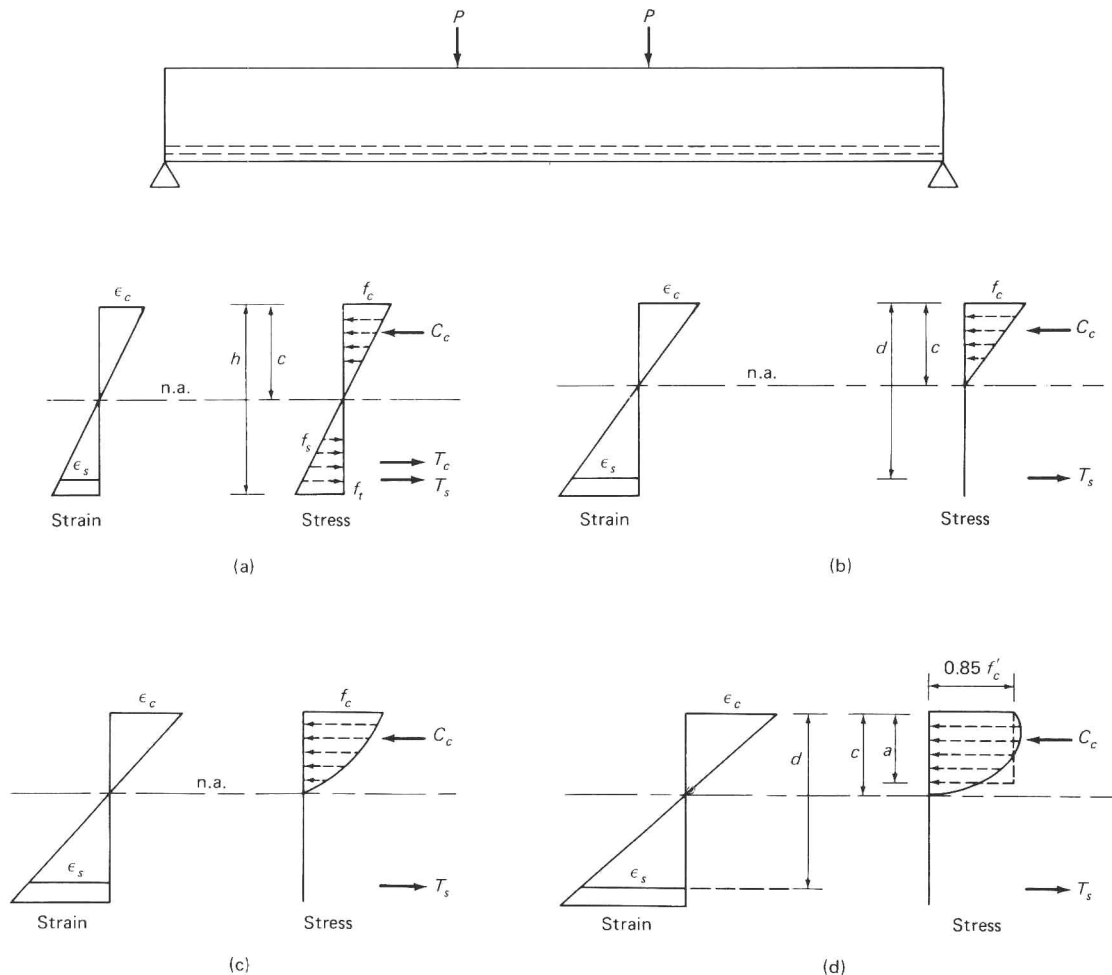


Fig. 1-1 Flexural behavior of a simple beam at different load stages.

tions at a moderate load level beyond cracking. The internal flexural resistance at the section is provided by a force couple consisting of concrete compression  $C_c$  and steel tension  $T_s$ . For a given neutral axis location  $c$ , the moment  $M$  can be found using eq. (1-3):

$$M = T_s \left( d - \frac{c}{3} \right) \quad (1-3)$$

where

$$T_s = A_s f_s.$$

To compute stresses in concrete, the device of the transformed section can still be used. Here the cracked concrete is assumed to make no contribution. The transformed section consists of compression concrete and the equivalent concrete area of the reinforcing steel.

When the applied load is further increased, stresses in concrete and steel increase correspondingly. At higher levels of stress, the proportionality of stresses and strains ceases, and the material nonlinearity is observed. Although a linear strain distribution can still be used to be consistent with the assumption that "plane sections before bending remain plane after bending," the stress distribution is of the same shape as the concrete stress-strain curve.

Figure 1-1(c) shows strain and stress distributions at higher loads, close to the ultimate load level. The internal resisting couple continues to be formed by the concrete compression and steel tension; the appropriate stress-strain relationships for concrete and steel can be used to find the coupling forces. A common practice is to utilize a bilinear idealization for the stress-strain relationship for steel. This idealization implies that the stresses and strains in steel are proportional, as before, up to the yield stress. Beyond this stress level, stresses in steel are taken equal to the yield stress, irrespective of the magnitude of strain. For concrete, one has to find the area under the stress distribution curve that has the same shape as the stress-strain relationship of concrete.

In properly proportioned members, the reinforcement does not yield under service loads. However, the concrete may crack in flexure even under low service loads. This is typical behavior for reinforced concrete and is not an undesirable occurrence. Reinforcement would not be needed if the concrete did not crack. Flexural cracks, in well-proportioned members, are very small hairline cracks. Their presence is hardly noticeable, and they do not present any undesirable appearance or corrosion problems.

The ultimate capacity of the beam is eventually reached as the applied load continues to increase. Failure is governed by one of two modes, depending on the amount of reinforcement. If relatively low percentage of steel is used in a section, the steel on the tension side starts yielding. This triggers widening of the cracks and continued shifting of the neutral axis upward. The cracks become visible with the increase in curvature. The deflection also increases to excessive levels. When this happens, the concrete is subjected to higher strains and stresses to maintain compatibility and equilibrium at the section. Eventually, concrete crushing occurs as the secondary mode of failure. Until the collapse mechanism forms, however, excessive deflections and wide cracks provide warning of an imminent failure. This type of ductile failure initiated by yielding of reinforcement is a desirable failure mode for flexural members.

Figure 1-1(d) shows the strain and stress distributions at the ultimate load condition when the failure is governed by steel yielding. This is the ultimate load condition considered

in design. Although the strain distribution is linear, concrete stress distribution shows a parabolic variation. In order to find the internal resisting couple at the section, one must integrate concrete stress distribution to determine the compression force. ACI 318M-83 permits the simplification of representing concrete stress distribution by an equivalent rectangular stress block. Accordingly, concrete stress of  $0.85 f'_c$  is assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section. The height of the rectangular stress block is defined as the neutral axis location  $c$  multiplied by the factor  $\beta_1$ . This factor is equal to 0.85 for concrete strengths up to and including 30 MPa. For strengths above 30 MPa,  $\beta_1$  reduces at a rate of 0.08 for each 10 MPa, provided that it is not below 0.65. The Code recommends the maximum usable strain of 0.003 at the extreme concrete compression fiber. This value represents a conservative maximum strain for concrete. The concrete compression,  $C_c$ , the reinforcement tension,  $T_s$ , and the internal resisting moment,  $M_n$ , can be found as shown below:

$$C_c = 0.85 f'_c ab \quad (1-4)$$

$$T_s = A_s f_y \quad (1-5)$$

Equilibrium requires  $C_c = T_s$  and hence:

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad (1-6)$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) \quad (1-7)$$

$$M_n = A_s f_y \left( d - \frac{f_y A_s}{1.7 f'_c b} \right) \quad (1-8)$$

In heavily reinforced sections, relatively high tension capacity is provided, and the capacity of the concrete in compression is exhausted prior to yielding of the reinforcement. Concrete crushing occurs in a sudden and brittle manner. Because of the nonductile behavior of concrete crushing, this type of proportioning should be avoided.

If concrete crushing and steel yielding occur simultaneously, the section is said to be "balanced." A section reaches its balanced point when the maximum compression fiber strain is 0.003, and the steel strain is equal to its yield strain. Equations (1-4) through (1-8) are equally applicable to a balanced section, since this condition is a special case of an ultimate moment condition at which the limits of the two materials are reached simultaneously. In Fig. 1-1(d), if  $\epsilon_y$  is substituted for  $\epsilon_s$ , the linear strain condition gives:

$$\begin{aligned} \frac{c}{d} &= \frac{\epsilon_c}{\epsilon_c + \epsilon_y} = \frac{0.003}{0.003 + f_y/200,000} \\ &= \frac{600}{600 + f_y} \end{aligned} \quad (1-9)$$

From force equilibrium:

$$C_c = T_s$$

$$0.85 f'_c \beta_1 cb = A_s f_y$$

Substituting eq. (1-9) for  $\frac{c}{d}$  and  $\rho_b = \frac{A_s}{bd}$ :

$$\rho_b = \frac{0.85 f'_c \beta_1}{f_y} \times \frac{600}{600 + f_y} \quad (1-10)$$



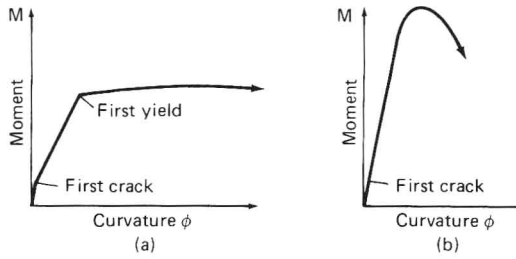


Fig. 1-2 Moment-curvature relationships.

Equation (1-10) gives the balanced section reinforcement ratio. Reinforcement ratios lower than this value produce ductile failure with reinforcement yielding occurring before crushing of concrete.

The load stages discussed above can best be summarized by a moment-curvature relationship. Each of the above load stages represents a characteristic point on the moment-curvature curve. Curvatures at different load stages can be determined from the corresponding strain diagrams as:

$$\phi = \frac{\epsilon_c + \epsilon_s}{d} \quad (1-11)$$

Figure 1-2(a) shows a typical  $M-\phi$  curve for a singly reinforced section failing in tension. The first characteristic point on the curve is the point at which concrete cracks. Up to this point, the section is elastic. At cracking, there is a slight increase in curvature under constant moment. The curve then continues with a lower slope (reduced stiffness due to cracking) until the first yielding in steel takes place. At this point there is a substantial reduction in the slope of the curve. The curvature increases with very little increase in moment until concrete crushing occurs. The moment-curvature relationship for a heavily reinforced section failing in compression is shown in Fig. 1-2(b). Comparison of the two curves indicates that the beam failing in tension is capable of developing larger inelastic curvatures and hence shows a ductile behavior. Moment-curvature curves for a singly reinforced rectangular section corresponding to three different areas of steel are shown in Fig. 1-3.

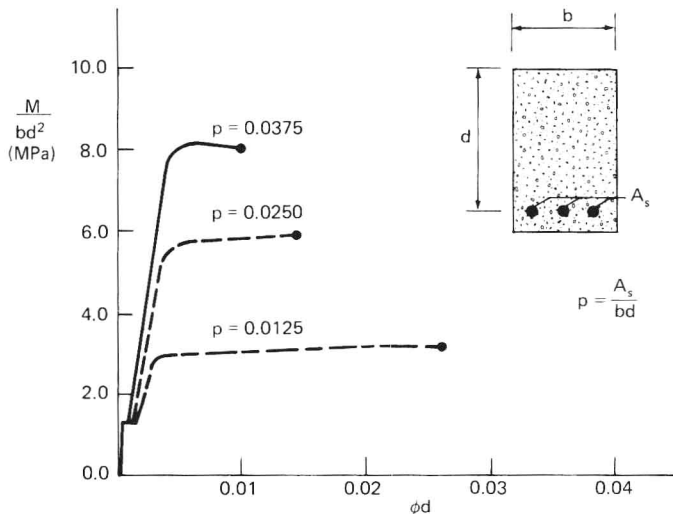


Fig. 1-3 Moment-curvature curves as affected by percentage reinforcement.

### 1.3.2 Flexure and Axial Load

Generally, concrete members subjected to axial loads are simultaneously subjected to bending moments. Concentrically compressed members rarely occur in practice. Even if a member appears to be resisting a concentric load, there is always some eccentricity due to construction imperfections. Furthermore, bending moments due to continuity are always present in monolithic structures. Lateral loads also impose bending moments on columns. Consequently, it is safe to assume that some eccentricity always exists in compression members. The provisions of ACI 318M-83 allow for some moment capacity in a section even if the results of analysis indicate a concentric load. This is an indirect procedure to assure a minimum design moment capacity. Earlier editions of the *Code* explicitly spelled out the minimum eccentricity requirement for compression members.

Reinforced concrete sections under combined axial force and bending moment behave in much the same manner as flexural members. The only difference between the two is the presence of additional uniform compression. A good example is a column, which behaves elastically under a relatively moderate eccentric load, the same as a beam. Figure 1-4(a) shows stress and strain distributions at a column section. By using elastic theory and the transformed area concept, we can find stresses in the concrete. The maximum and the minimum fiber stresses are computed as shown below:

$$f_{\max} = \frac{P}{A_{tr}} \pm \frac{Mh}{2I_{tr}} \quad (1-12)$$

Axial force and moment resistance of a column section at this load stage can be computed by applying the principles of static equilibrium.

When a column is subjected to higher eccentricity, tensile stresses can exceed the modulus of rupture, and concrete cracking occurs. Figure 1-4(b) illustrates this stress condition. If the maximum compression is less than about 50% of  $f'_c$ , elastic theory is still applicable with the appropriate transformed section. However, as the applied load is increased, material nonlinearity becomes more pronounced. Concrete stress distribution forms a parabolic profile following the concrete stress-strain relationship. Steel yielding may take place with increasing load. The ultimate load capacity is reached either by crushing of the concrete or yielding of the steel. This load stage is of interest to designers. Figure 1-4(c) shows a rectangular column section eccentrically compressed by a force  $P_n$ . If a rectangular stress block is used to express concrete stress distribution at ultimate, the nominal strength for axial force and flexure can be computed using the equilibrium equations:

$$P_n = 0.85 f'_c ab + A'_s f'_s - A_s f_s \quad (1-13)$$

$$M_n = P_n e = 0.85 f'_c ab \left( \frac{h}{2} - \frac{a}{2} \right) + A'_s f'_s \left( \frac{h}{2} - d' \right) + A_s f_s \left( d - \frac{h}{2} \right) \quad (1-14)$$

Columns with large eccentricities fail because of the yielding of tension reinforcement. Concrete crushing takes place shortly after the steel yields. At failure,  $\epsilon_{cc} = 0.003$ ,  $f_s = f_y$  and  $f'_s$  is usually equal to  $f_y$ . Conversely, in columns with small eccentricities, concrete crushing occurs prior to tension steel yielding. In this case,  $\epsilon_{cc} = 0.003$ ,  $f'_s$  is usually

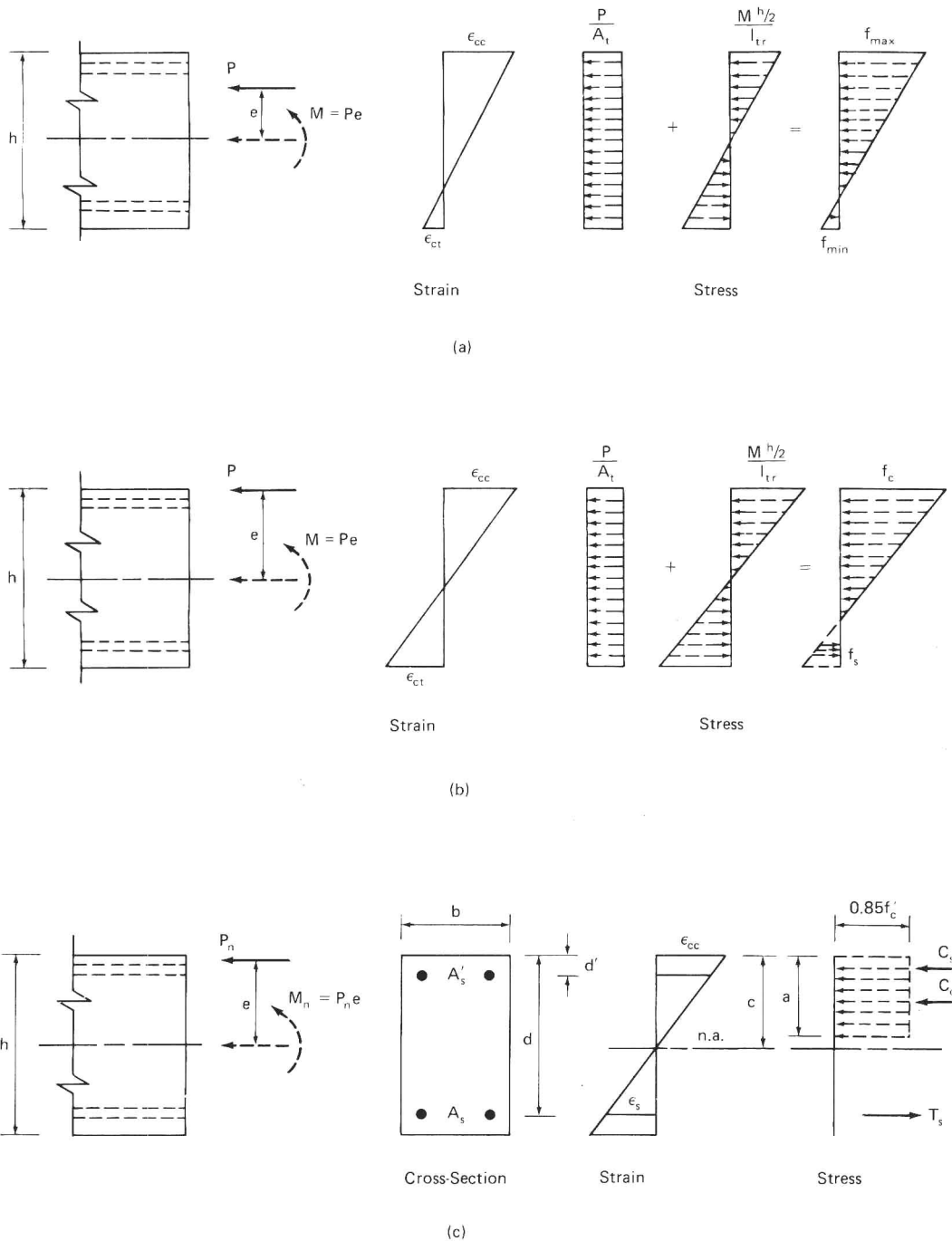


Fig. 1-4 Reinforced concrete under combined axial force and flexure.

equal to  $f_y$ , and  $f_s$  must be determined from a strain-compatibility analysis. Strain-compatibility analysis involves an iterative procedure. The neutral axis location is first assumed, and the corresponding stress in steel is found. This value is then compared with the stress computed to satisfy the static equilibrium. The iteration continues until a reasonable agreement is achieved.

Equations (1-13) and (1-14) give the axial load and moment combination that produces failure in a given column section. Depending on the eccentricity,  $e$ , different  $P_n$  and  $M_n$  combinations may govern the design. These combina-

tions are expressed in terms of well-known moment-axial force interaction diagrams. These diagrams are extremely useful design tools. The details of the interaction diagrams are discussed in Section 1.6.

### 1.3.3 Shear

Shear behavior of reinforced concrete is still a subject of interest for researchers. Most structural members are subjected