Vibrational Mechanics

Nonlinear Dynamic Effects,
General Approach, Applications

Dedicated to the memory of Nina

Preface to the Russian Edition

This book has two aims. The first aim is to describe a number of wonderful phenomena caused by the action of vibration on nonlinear mechanical systems. Here is a short list of such phenomena: vibrational displacement — the directed on the average "slow" motion or the change in the state of the system affected by "fast" undirected on the average mechanical actions (vibration); the change in the physical and mechanical properties of a body under vibration with regard to slow actions; the transformation of equilibrium states, in particular — their stabilization and destabilization under the action of vibration; the change in free oscillation frequencies of the system, caused by vibration; vibrational maintenance of rotation and self-synchronization of unbalanced rotors having an unexpected analogue in the peculiarities of the motion of celestial bodies; the seeming change in both magnitude and direction of the gravitational force. That enumeration is far from being complete. Many of those phenomena are widely used in technology and engineering. Still more effective use of such phenomena is expected in the nearest future. The author hopes this book will contribute to it.

The second, and perhaps the main aim of the book is to propose a general mechanical and mathematical approach to the description and investigation of this class of phenomena — the approach which may be called *vibrational mechanics* . Vibrational mechanics is a mechanics for the observer interested only in the "slow" motions of the system. Such motions appear together with the fast motions in a non-linear system under vibration and are, as a rule, of the main practical interest. It has been found that this observer, whom we call the observer $\mathbf{V}$ to distinguish him from the "ordinary" observer whom we will call the observer $\mathbf{O}$, in order to make a correct description of the behaviour of the system must add to all the slow forces, acting upon the system, certain supplementary slow forces, called *vibrational forces* . These forces are calculated according to certain rules. Thus, the vibrational mechanics is in some sense analogous to the mechanics of a relative motion.

All the above-mentioned phenomena can be easily and naturally explained by the appearance of vibrational forces. On the other hand, the disregard of those forces leads to misunderstanding and errors, which happened in the past and are still happening now. We will discuss them in this book.

Vibrorheology is an important part of vibrational mechanics. It mainly studies the change caused by vibration in the rheological characteristics of bodies with regard to the slow actions, and it also studies the corresponding slow motions.

Vibrational mechanics and vibrorheology play an important role in the new section of the applied theory of oscillations which has been formed in recent years — the *theory of vibrational processes and vibrational devices* . This theory studies the regularities of excitation and effects of vibration in different mechanical systems. It also embraces the theory of the machines in which vibration is used for beneficial purposes.

A few more words about the positions of the observers **O** and **V** . - positions which are widely used in this book and in some illustrations are marked by symbols and . The letters **O** and **V** are the initial letters of the words "ordinary" and "vibrational". In the world, perceived by the observer O, the ordinary laws of mechanics are valid in their direct form. Making and solving differential equations of motion, based on those laws, that observer gives correct descriptions of the phenomena, considered in the book. But his descriptions are rather intricate, with unnecessary details, as a result of which the interpretation of the results is sometimes quite embarrassing.

The observer **V** , as was mentioned, deliberately "does not notice" any fast forces or fast motions, though he does not forget that they do exist in reality. Staying within the frames of the "ordinary" mechanics, to make his descriptions correct, he must add to all the slow forces also the vibrational forces. The world of the observer **V** is much simpler than that of the observer **O** . In particular, the multidimensional system may be seen by him as a system of much smaller dimensions, an essentially non-conservative system may seem to him to be conservative, the discontinuous system may be seen by him as continuous or "smooth" and so on. As was mentioned, for the observer **V** vibrational mechanics and vibrorheology are mechanics and rheology respectively . In connection with the mistakes, which are often made when analyzing such phenomena, we can also speak of the position of the observer **W** (the first letter of the word "wrong"). He fails to take into account certain rather important circumstances. This observer either does not notice vibration and the fast forces, acting in the system, or does not take into consideration any possible consequences of their presence. So when considering the slow motions, he does not take into account a possibility of the appearance of vibrational forces. The world of this observer is full of "miracles", riddles and paradoxes. Trying to explain them he sometimes begins to doubt the validity of the basic laws of mechanics — the law of conservation of energy, the law of action and reaction. He believes that under the action of vibration the weight of the body changes, that the velocity of the center of inertia of the system may change only at the expense of internal forces, etc.

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However, the position of the observer **W** may sometimes also prove useful. In inventors' activities a temporal disregard of the laws of physics and mechanics, which hinder in attaining their aim, is an effective method of finding new technical solutions. We mean here the so called *fantastic analogy*, proposed by Gordon [176].

Now a few words about the terms *vibrational mechanics* and *vibrorheology*. Their introduction seems to be justified, first, by the peculiarity of the class of phenomena that are to be considered and, secondly, by the availability of a common methodical approach to the investigation of these effects. These terms, used by the author in his presentations and publications met no objections on the part of his colleagues [89, 92, 95, 100]. Equations, describing the slow motions, are called by us *the main equations of vibrational mechanics*. It is surprising and remarkable that for a comparatively wide class of systems these equations can be written in the form, typical of the potential system with dissipation, while the initial system is essentially non-potential. Systems of this type are named by us *potential on the average dynamic systems* . A number of important practical problems, in particular problems on synchronization (resonances) under the rotation of solid bodies, are reduced to studying those systems.

Vibrational mechanics is considered in this book as a special case of a more general concept which can be named *mechanics of systems with hidden motions*. Both in the process of mechanical-mathematical modeling real systems and in striving to simplify the models, one has a natural desire "not to see", "to partly ignore" certain components of motions of the system and even some degrees of freedom which seem to be of minor importance. The question arises: by which mechanics should the observer be guided lest he should come into conflict with reality? The answer is analogous to the previous one: certain additional forces should be introduced into the equations, describing the motions which are to be considered. Just like above, the mechanics of systems with partly ignored motions can be interpreted from the points of view of the three observers $\mathbf{O}'$, $\mathbf{V}'$ and $\mathbf{W}'$ similar to observers $\mathbf{O}$, $\mathbf{V}$ and $\mathbf{W}$. It is remarkable that vibrational mechanics is connected with both the classic works on mechanics and modern investigations. By the former we mean the works by Thomson, Tait, and Routh on mechanics of systems with cyclic coordinates, the works by Reynolds on the theory of turbulence, by Poincare and Lyapunov on celestial mechanics and theory of the stability of motion, by Krylov, Bogolyubov, Mitropolsky and Malkin on methods of averaging and the theory of periodic solutions of differential equations with a small parameter. By the latter we mean the works by Kapitsa and his followers on the behavior of a pendulum with a vibrating

axis of suspension, on the theory of vibrational processes and devices and also on the problem of resonances in the Solar system. That common character of ideas and their continuity has been systematically reflected in this book.

The first part of the book is devoted to the theoretical basis of vibrational mechanics, in particular to the description of the general approach, mentioned above, which is mainly based on the so called *method of direct separation of motions* . The main statements have been formulated here as theorems. It should be noted however that the elaboration of the mathematical apparatus of vibrational mechanics cannot as yet be considered completed. Other chapters of the book are devoted to numerous applications of vibrational mechanics.

The book is provided with a detailed table of contents and detailed captions to figures. It seems to us both of them will be helpful to the readers.

The book is meant first of all for the specialists in both theoretical and applied mechanics, in the theory of non-linear oscillations and vibrational technology. It may also prove to be useful for mathematicians involved in the theory of ordinary differential equations as a source of some new problems.

While working at the concept and apparatus of vibrational mechanics I had a happy chance to discuss the main points with V.I.Babitsky, V.V.Beletsky, V.V.Koslov, M.Z. Kolovsky, P.S.Landa, L.G.Loytsyansky, Ja.G.Panovko, A.A.Pervosvansky, V.V.Rumyantsev, O.V.Savinov, G.Ju.Stepanov, and K.V.Frolov. My young colleagues E.B.Kremer, O.Z.Malakhova, A.V.Petchenev. A.Ya. Fidline and N.P.Yaroshevich both on my request and on their own initiative made a number of complicated researches the results of which have been reflected in the book. Besides, O.Z.Malakhova read the manuscript very attentively and helped to eliminate some drawbacks. While reviewing the book, important remarks were made by M.Z.Kolovsky. I express my deep gratitude to those scientists.

I appreciate the unfailing understanding and support of my colleagues and the management of the Research and Design Institute of Mechanical Treatment of Mineral Resources ("Mekhanobr", St-Petersburg) which made it possible for me to devote myself to this most interesting work.

I am greatly thankful to my wife Nina Granat who passed away when the work on the book was practically completed. A gifted researcher herself (the reader will find some results of her work in the book), Nina selflessly released me from any domestic care. To Nina's fond memory this book is dedicated.

November 29, 1991, St.Petersburg

I.Blekhman

Preface to the English Edition

I am thankful to the World Scientific Publishing Co. for the opportunity to present my book to the English readers.

Five years have passed since the publication of this book in Russia [125]. The ideas and results stated there have been essentially developed and supplemented during that time. In this English edition of the book the author tended to at least partially reflect those developments:

- item 3.3 has been added, which is devoted to the characteristics of the advantages and limitations of the approach and methods stated in the book;
- item 5.3 has been added, which gives a short review of the results referring to the stability of composite pendulums with a vibrating axis of suspension, including the follower-loaded ones;
- item 7.8 was supplemented by the general definition of synchronization;
- item 10.6, devoted to the vibrational pumps and coaches has been considerably enlarged;
- a new chapter had been introduced. This chapter (chapter 18) contains some ideas on the problem of controlling the properties of the nonlinear systems by means of vibration. Here the new approach to the creation of the materials with given dynamic properties — the so called dynamic materials — is discussed most thoroughly;
- the bibliography contains additional references to both the latest investigations and to those made before, which unfortunately escaped the attention of the author when the book was being prepared.

In 1996 the author delivered a PhD course of lectures on vibrational mechanics at the Department of Solid Mechanics, headed by Professor Pauli Pederson, at the Technical University of Denmark (DTU).

Then similar lectures were delivered at the Institute of the Problems of Mechanical Engineering of the Russian Academy of Sciences; at the St.Petersburg Technical University (the former Polytechnical Institute) and at the Marine Technical University.

Professors D. A. Indeitsev, M. Z. Kolovsky and S. V. Sorokin, who had invited me, and their disciples as well as their colleagues at the DTU made an attentive and interested audience. The contact with them proved to be

both pleasant and useful. Some of them, including Professors J. J. Thomsen, S. V. Sorokin and their pupils J. S. Jensen, D. M. Tchernyak and E. C. Miranda found it interesting to make their own investigations of that problem. References to their investigations and short reviews of them are given in the book.

The author's contacts with Professors A. L. Fradkov, K. A. Lurie, and L. Sperling were both very useful and very pleasant.

I am deeply grateful to all those scientists.

I want to thank Associate Professor Minna Perelman for her careful translation in the process of which some fragments of the text were improved.

I am also thankful to my colleague Larissa Titova for her valuable assistance in preparing the manuscript.

I am grateful to Professor Mikhail Levinshtein for his helpful advice.

July 30, 1999, St.Petersburg

I.Blekhman

Contents

Preface to the Russian Edition vii
Preface to the English Edition xi

I Fundamentals of Theory of Vibrational Mechanics **1**

1 Introduction. Subject–Matter of Vibrational Mechanics **3**
1.1 Oscillatory Processes in Nature and Engineering 3
1.2 Damaging Vibration and Useful Vibration. Vibrational Engineering 3
1.3 The Theory of Vibrational Processes and Devices — a New Section of the Applied Theory of Oscillations 5
1.4 On the Effects Caused by the Action of Vibration in Non-linear Oscillatory Systems 6
1.5 Vibrational Mechanics and Vibrational Rheology. Observer **O** and Observer **V** 8
1.6 Vibrational Mechanics as a Section of the Mechanics of Systems with Hidden Motions 13
1.7 Mistakes and Paradoxes, connected with the Interpretation of the Phenomena under Consideration. The Observer **W** 14

2 On the Mechanics of Systems with Hidden Motions **21**
2.1 General Statements and Main Equations; Theorem 1 . . . 21
2.2 Special Cases, Examples 26
2.2.1 Mechanics of the Relative Motion 26
2.2.2 A Solid Body with the Internal (Hidden) Degrees of Freedom 27
2.2.3 Systems with Cyclic Coordinates 31
2.2.4 Motion of a Body in a Viscous Incompressible Fluid 34
2.2.5 Vibrational Mechanics — Systems with Hidden Fast Motions 36

3 Basic Statements and Mathematical Apparatus of Vibrational Mechanics **37**
3.1 Vibrational Mechanics as the Mechanics of Systems with Hidden Motions 37

3.2 Method of a Direct Separation of Motions as an Effective General Method of Solving Problems of Vibrational Mechanics 38
3.2.1 Preliminary Remarks 38
3.2.2 The Initial Equation and Its Reduction to a System of Integro-differential Equations for the Explicit and Hidden Components of Motions 38
3.2.3 The Case when a Separate Equation for the Explicit Component is Obtained: Theorem 2 41
3.2.4 The Main Assumption of Vibrational Mechanics, Its Formalization and Conditions of its Fulfillment; Theorem 3 and 4 43
3.2.5 The Main Equation of Vibrational Mechanics. Vibrational Forces, Observers **O** and **V** 46
3.2.6 Method of an Approximate Derivation of the Expression of Vibrational Forces and of Composing the Main Equation of Vibrational Mechanics, its Grounding; Theorem 5 47
3.2.7 An Important Special Case; Theorem 6 50
3.2.8 On the Simplifications of Solving Equations for the Fast Component of Motion. Purely Inertial Approximation 53
3.2.9 Additional Remarks, Certain Generalizations 55
3.2.10 Summary of the Main Relations and Results 57
3.2.11 On the Procedure of the Practical Use of the Method 61
3.2.12 Short Bibliographic Review 62
3.3 On the Main Peculiarities and Advantages of the Approaches of Vibrational Mechanics and of the Method of Direct Separation of Motions as Compared to Other Similar Methods 63
3.3.1 Peculiarities and Limitations 63
3.3.2 Advantages 63
3.3.3 Final Remarks 65
3.4 On Other Methods of Obtaining Expressions for the Vibrational Forces and the Main Equations of Vibrational Mechanics 66

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4 Potential on the Average Dynamic Systems and Extremal Signs of Stability of Certain Motions **69**

4.1 On Potentiality on the Average and on the Extremal Signs of Stability 69

4.2 Systems with Synchronized Objects. Integral Signs of Stability (Extremal Properties) of Synchronous Motions 75

4.2.1 Systems with Almost Uniform Rotations, Synchronization of Vibroexciters; Theorems 7 and 8 75

4.2.2 Systems of Quasiconservative Objects, Canonical Systems; Theorem 9 85

4.2.3 Systems with Quasicyclic Coordinates; Theorem 10 . . 89

4.3 Systems with Kinematic Excitation of Vibration (the Minimax Sign of Stability); Theorem 11 91

4.4 Systems with Dynamic Excitation of Vibration; Theorem 12 96

II Vibrational Mechanics of Machines, Mechanisms and Pendulum Devices **99**

5 Devices of Pendulum Type **101**

5.1 Pendulum with a Vibrating Axis of Suspension 101

5.1.1 Brief Bibliographic Review 101

5.1.2 Equation of Motion 102

5.1.3 Solving the Problem by the Method of Direct Separation of Motions 103

5.1.4 The Use of the Mini-Max Criterion of Stability . . . 109

5.1.5 Behavior of the Pendulum Depending on the Type of Vibration 110

5.1.6 Multilink Pendulums 115

5.2 Chelomei's Pendulum with a Vibrating Axis of Suspension 116

5.2.1 Brief Bibliographical Review 116

5.2.2 Pendulum with an Absolutely Rigid Rod 117

5.2.3 The Behavior of the Washer on a Vibrating Deformable Rod 124

5.3 The Follower-loaded Double Pendulum with a Vibrating Axis of Suspension 128

5.4 Some Generalizations and Applications. On Vibrational Stabilization and Destabilization, Vibrational Shift (Drift) 130

6 Rotor Mechanisms. Machine Aggregates 137

6.1 Unbalanced Rotor on a Vibrating Base — Vibrational Maintenance of Rotation 137

6.1.1 Brief Bibliography 137

6.1.2 Equation of Motion, the Main Equation of Vibrational Mechanics 139

6.1.3 Stationary Regimes of Rotation of the Rotor and Their Stability 143

6.1.4 Vibrational Maintenance of Rotation of the Rotor . . 144

6.1.5 Vibrational Capture of the Rotation of an Unbalanced Rotor of an Electric Engine. How the Efficiency of an Engine Can Be More than Unit 150

6.2 Devices "A Roller (Ball) in a Vibrating Cavity" and "A Ring on a Vibrating Rod (Hula-Hoop)". Vibrational Maintenance of the Planetary Motion 152

6.2.1 About the Devices under Consideration: a Brief Bibliographic Reference 152

6.2.2 Equation of Motion, the Main Equation of Vibrational Mechanics 154

6.2.3 The Discussion of Results, their Applications. The Critical Slot of Vibro-Inertial Crushing and Grinding Machines 156

6.3 Unbalanced Rotor (Mechanical Unbalanced Vibro-Exciter) in the Oscillatory System — Vibrational Retardation of Rotation, Zommerfield's Effect 158

6.3.1 On the Effects under Consideration: Brief Bibliographic Reference 158

6.3.2 Simplest Case: the Oscillatory Part of the System is Linear and has One Degree of Freedom 161

6.3.3 General Case: the Oscillatory Part of the System is Non-linear and has Several Degrees of Freedom . . . 165

6.3.4 On Certain Applications 167

6.4 Machine Aggregates 168

7 Self-synchronization of Mechanical Vibro-exciters 175

7.1 On the Phenomenon of Synchronization of Unbalanced Rotors (Mechanical Vibro-exciters). A Brief Review of Investigations 175

7.2 The Simplest Case: Self-synchronization of Vibro-exciters in the Linear Oscillatory System with One Degree of Freedom 180

7.2.1 Equations of Motion and Setting up the Problem . . 180

7.2.2 Main Equations of Vibrational Mechanics 184

7.2.3 Analysis of the Main Equations. Vibrational Torques, Partial Angular Velocities; Vibrational coupling between the rotors 187

7.2.4 Stationary Regimes of Synchronous Rotation and Their Stability. The Integral Sign of Stability (Extreme Property) of Synchronous Motions 190

7.2.5 A Case of Two Exciters. Once Again about the Phenomenon of Vibrational Maintenance of Rotation 194

7.2.6 About the Case of Almost Identical Exciters 195

7.3 The General Case. A Short Review of the Results of Investigations 197

7.3.1 Equations of Motion and the Main Equations of Vibrational Mechanics in the General Case 197

7.3.2 Potential Function and Integral Criterion of Stability 199

7.3.3 Short Review of the Results on the Theory of Synchronization of Mechanical Vibro-exciters 201

7.4 Steadiness of Phasing the Vibro-exciters and the Adaptive Property of Vibrational Machines in case of Self-synchronization 202

7.4.1 On the Concept of the Steadiness of Phasing 202

7.4.2 Estimation of Steadiness. On the Relative Strength of Vibrational Coupling between the Unbalanced Rotors 203

7.4.3 The Adaptive Property of Vibrational Machines with Self-synchronized Vibro-exciters 205

7.5 On Theoretic Investigation of the Devices with Self-synchronized Vibro-exciters. An Example of Using the Integral Criterion of Stability 206
7.6 On the Synthesis of the Devices with Self-synchronized Vibro-exciters 209
7.7 Regularities and Paradoxes of Self-synchronization of Unbalanced Vibro-exciters 210
7.8 Supplement: On the Phenomena of Synchronization of the Oscillatory and Rotational Motions in Nature and in Technology. The modern state of the problem. General Definition of synchronization 214

8 Generalized Principle of Auto-balancing 219
8.1 Laval's Principle and its Generalization, Ensuing from the Theory of Self-synchronization of Mechanical Vibro-exciters and from Some Other Investigations 219
8.2 Application to the Theory of Group Foundations for the Unbalanced Machines 224
8.3 Supplement to the Theory of Auto-balance Devices 227

III Vibrational Mechanics of Processes (Vibrational Displacement and Shift) 231

9 The Main Models and General Regularities of Processes of Vibrational Displacement from the Position of Vibrational Mechanics 233
9.1 On the Effect of Vibrational Displacement, its Theory and Applications 233
9.2 Simplest Model of the Process of Vibrational Displacement 235
9.2.1 Motion of a Particle over a Rough Horizontal Plane under the Action of a Longitudinal Harmonic Force or the Longitudinal Vibration of the Plane 235
9.2.2 Analysis of the Solutions. Effect of the Seeming Vibrational Transformation of Dry Friction into Viscous. The Driving and the Vibro-transformed Vibrational Forces 239
9.3 Physical Mechanisms and Main Types of Asymmetry of the System, Causing Vibrational Displacement 240

9.4 More Complicated Main Models and Problems of the Theory of Vibrational Displacement 244
9.4.1 One-dimensional Motion of the Particle in the Resistant Medium under the Action of Vibration [95, 99, 198, 564, vol. 2 and 4] 244
9.4.2 The Problem in 9.4.1 in the Case of Harmonic Vibration 251
9.4.3 The Motion of a Particle over the Tilted Rough Plane, Performing Harmonic Oscillations in Two Mutually Perpendicular Directions 252
9.4.4 On Vibrational Transportation of a Body (Particle) Upwards along the Tilted Plane. The Limiting Angle of Rise 258
9.4.5 About Other Models and Problems 260

10 Effects of Vibrational Displacement in Technique, Technology and in Nature **261**
10.1 Vibrational Transportation 261
10.2 Vibrational Separation of the Components of Granular Mixtures 267
10.2.1 Factors, Determining the Efficiency of Using Vibration in the Processes of Separating the Components of Granular Mixtures 267
10.2.2 On Separation of Particles in a Layer of Granular Material under the Action of Vibration (Segregation, Stratification, Self-classification) 268
10.2.3 General Statement of the Problem on the Separation of Particles of the Granular Mixture in a Vibrating Vessel. Brief Characteristic of the State of the Problem 271
10.2.4 Motion of a Heavy Particle in the Medium with a Resistance of Dry Friction Type, Performing Circular Horizontal Oscillations. Pseudo-resonance Effect 273
10.2.5 Problem of 10.2.4 in the Case of Vertical Harmonic Oscillations of the Medium. The Effect of Floating up of a Heavy Particle in the Medium of Light Small Particles 277

10.2.6 Kinetics of Vibrational Separation of Multi-Component Mixture (the Continual Description) 280
10.2.7 Separation of Particles on the Vibrating Surfaces . . 287
10.2.8 On Separating the Particles in the Vibrational and Wave Fields, Created in the Rarefied Suspensions . . 291
10.3 Vibrational Sinking and Intrusion, Vibrational Cutting . . 292
10.4 Vibrational Transformation of Motion; Vibro-engines . . . 298
10.5 Vibrational Movement, Vibrational Coaches 303
10.5.1 Definitions. Preliminary Remarks 303
10.5.2 Vibrational Movement over a Rough Surface. Self-propelled Vibro-compactors of the Ground, Travel on Skateboards 304
10.5.3 Vibrational Movement in Fluid and in Gas. Vibro-flier, the movement of living organisms 306
10.5.4 Vibrational Movement in Heterogeneous Force Fields. Gravi-flier, Magneto-flier 311
10.6 The Vibro-jet Effect, Vibrational Pumps; Some More about Vibrational Coaches 313
10.6.1 On Vibro-jet Effect, Vibrational Pumps 313
10.6.2 Lishansky's Vibrational Pump; The Simplest vibroflighter: An Approximate Theory 317
10.6.3 Generating the Flow of Fluid by Means of a Vibrating Plate with Holes of Changing Cross-sections 319
10.6.4 Five Principle of Generating Vibrational Flows and Forces 322
10.6.5 On Vibrational Convection 324
10.6.6 On the Main Equations of Vibrational Hydromechanics and Hydraulic 324

11 Vibrational Shift (Drift) 325
11.1 On the Notion of Vibrational Shift (Drift) 325
11.2 Effect of Vibrational Shift in Applications; Special Features of the Effect in Systems with Dry Friction 325

IV Vibrorheology 329

12 On Rheology and Vibrorheology 331
12.1 Rheology as a Section of Mechanics 331

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