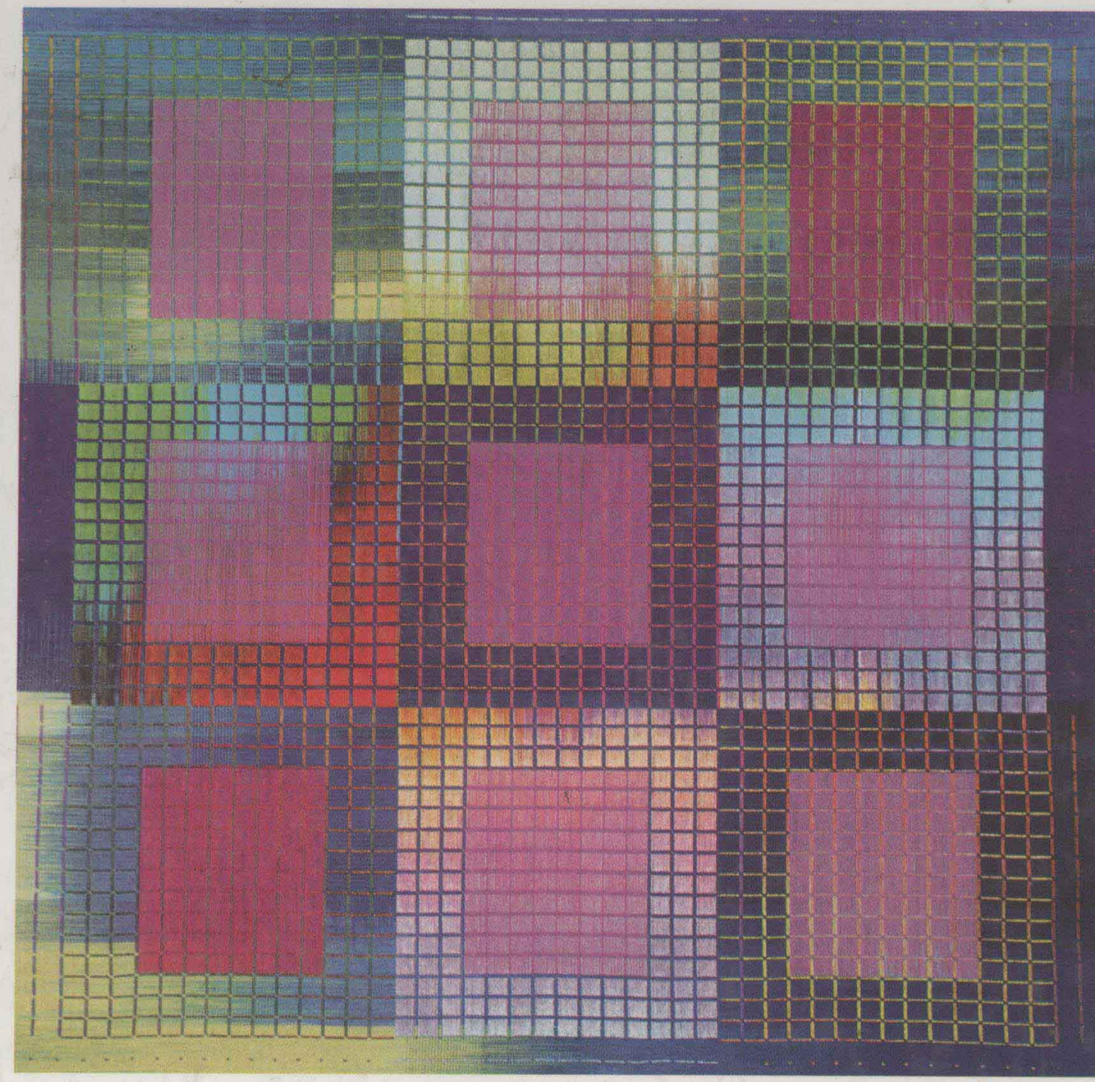
FIFTH EDITION

College Algebra



JEROME E. KAUFMANN

FIFTH EDITION

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Symbols

$=$	Is equal to
$\neq$	Is not equal to
$\approx$	Is approximately equal to
$>$	Is greater than
$\geq$	Is greater than or equal to
$<$	Is less than
$\leq$	Is less than or equal to
$a < x < b$	a is less than x and x is less than b
$0.\overline{34}$	The repeating decimal 0.343434. . .
LCD	Least common denominator
$\{a, b\}$	The set whose elements are a and b
$\{x \mid x \geq 2\}$	The set of all x such that x is greater than or equal to 2
$\varnothing$	Null set
$a \in B$	a is an element of set B
$a \notin B$	a is not an element of set B
$A \subseteq B$	Set A is a subset of set B
$A \nsubseteq B$	Set A is not a subset of set B
$A \cap B$	Set intersection
$A \cup B$	Set union
$\lvert x \rvert$	The absolute value of x
b^n	nth power of b
$\sqrt[n]{a}$	nth root of a
$\sqrt{a}$	Square root of a
i	Imaginary unit
$a + bi$	Complex number
$\pm$	Plus or minus
(a, b)	Ordered pair; first component is a and second component is b
f, g, h, etc.	Names of functions
$f(x)$	Functional value at x
$f \circ g$	The composition of functions f and g
f^{-1}	The inverse of the function f
$\log_b x$	Logarithm, to the base b, of x
$\ln x$	Natural logarithm (base e)
$\log x$	Common logarithm (base 10)
$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$	Two-by-three matrix
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$	Determinant
a_n	nth term of a sequence
S_n	Sum of n terms of a sequence
$\sum_{i=1}^{n}$	Summation from $i = 1$ to $i = n$
S_∞	Infinite sum
$n!$	n factorial
$P(n, n)$	Permutations of n things taken n at a time
$P(n, r)$	Permutations of n things taken r at a time
$C(n, r)$	Combinations of n things taken r at a time or r-element subsets taken from a set of n elements
$P(E)$	Probability of an event E
$n(E)$	Number of elements in the event space E
$n(S)$	Number of elements in the sample space S
E'	The complement of set E
E_v	Expected value
$P(E \mid F)$	Conditional probability of E given F

Properties of Absolute Value

$|a| \geq 0$

$|a| = |-a|$

$|a - b| = |b - a|$

$|a^2| = |a|^2 = a^2$

Interval Notation

(a, ∞)

$(-\infty, b)$

(a, b)

$[a, \infty)$

$(-\infty, b]$

$(a, b]$

$[a, b)$

$[a, b]$

Set Notation

$\{x | x > a\}$

$\{x | x < b\}$

$\{x | a < x < b\}$

$\{x | x \geq a\}$

$\{x | x \leq b\}$

$\{x | a < x \leq b\}$

$\{x | a \leq x < b\}$

$\{x | a \leq x \leq b\}$

Multiplication Patterns

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b\binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$

Properties of Logarithms

$\log_b b = 1$

$\log_b 1 = 0$

$\log_b rs = \log_b r + \log_b s$

$\log_b\left(\frac{r}{s}\right) = \log_b r - \log_b s$

$\log_b r^p = p(\log_b r)$

Properties of Exponents and Radicals

$b^n \cdot b^m = b^{n+m}$ $\qquad \frac{b^n}{b^m} = b^{n-m}$

$(b^n)^m = b^{mn}$

$(ab)^n = a^n b^n$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Factoring Patterns

$a^2 - b^2 = (a + b)(a - b)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Equations Determining Functions

Linear function: $f(x) = ax + b$

Quadratic function: $f(x) = ax^2 + bx + c$

Polynomial function: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

Rational function: $f(x) = \frac{g(x)}{h(x)}$, where g and h are polynomial functions

Exponential function: $f(x) = b^x$, where $b > 0$ and $b \neq 1$

Logarithmic function: $f(x) = \log_b x$, where $b > 0$ and $b \neq 1$

FIFTH EDITION

College Algebra

area A
perimeter P
length l

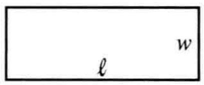
width w
surface area S
altitude (height) h

base b
circumference C
radius r

volume V
area of base B
slant height s

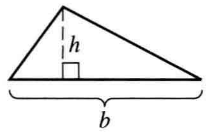
Rectangle

$A = lw \qquad P = 2l + 2w$



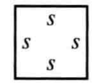
Triangle

$A = \frac{1}{2}bh$



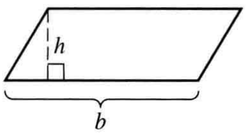
Square

$A = s^2 \qquad P = 4s$



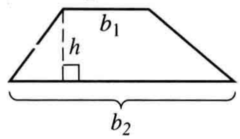
Parallelogram

$A = bh$



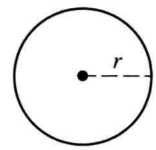
Trapezoid

$A = \frac{1}{2}h(b_1 + b_2)$

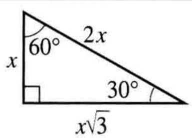


Circle

$A = \pi r^2 \qquad C = 2\pi r$

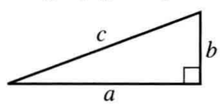


30°– 60° Right Triangle

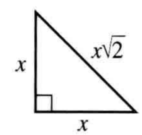


Right Triangle

$a^2 + b^2 = c^2$

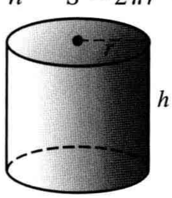


Isosceles Right Triangle



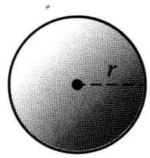
Right Circular Cylinder

$V = \pi r^2 h \qquad S = 2\pi r^2 + 2\pi rh$



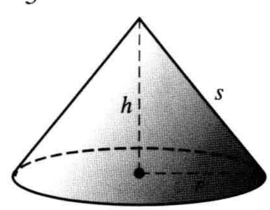
Sphere

$S = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$



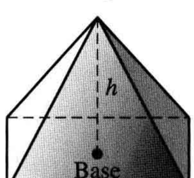
Right Circular Cone

$V = \frac{1}{3}\pi r^2 h \qquad S = \pi r^2 + \pi rs$



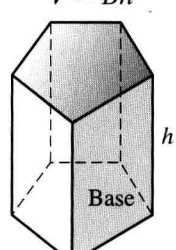
Pyramid

$V = \frac{1}{3}Bh$



Prism

$V = Bh$



Formulas

Quadratic formula:	The roots of $ax^2 + bx + c = 0$, where $a \neq 0$, are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
Distance formula for 2-space:	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a line:	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
Midpoint of a line segment:	$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
Simple interest:	$i = Prt$ and $A = P + Prt$
Compound interest:	$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$
nth term of an arithmetic sequence:	$a_n = a_1 + (n - 1)d$
Sum of n terms of an arithmetic sequence:	$S_n = \dfrac{n(a_1 + a_n)}{2}$
nth term of a geometric sequence:	$a_n = a_1 r^{n-1}$
Sum of n terms of geometric sequence:	$S_n = \dfrac{a_1 r^n - a_1}{r - 1}$
Sum of infinite geometric sequence:	$S = \dfrac{a_1}{1 - r}$
Number of permutations of n things:	$P(n, n) = n!$
Number of r-element permutations taken from a set of n elements:	$P(n, r) = \underbrace{n(n - 1)(n - 2)\cdots}_{r \text{ factors}}$
Number of r-element combinations taken from a set of n elements:	$C(n, r) = \dfrac{P(n, r)}{r!}$

Preface

College Algebra, Fifth Edition is written for students who need a college algebra course to serve as a prerequisite for the calculus sequence, for the finite math/calculus sequence, or to satisfy a liberal arts requirement. Sample outlines for these three types of courses are included at the end of this preface.

Four major ideas unify this text: solving equations and inequalities, solving problems, developing graphing techniques, and developing and using the concept of a function.

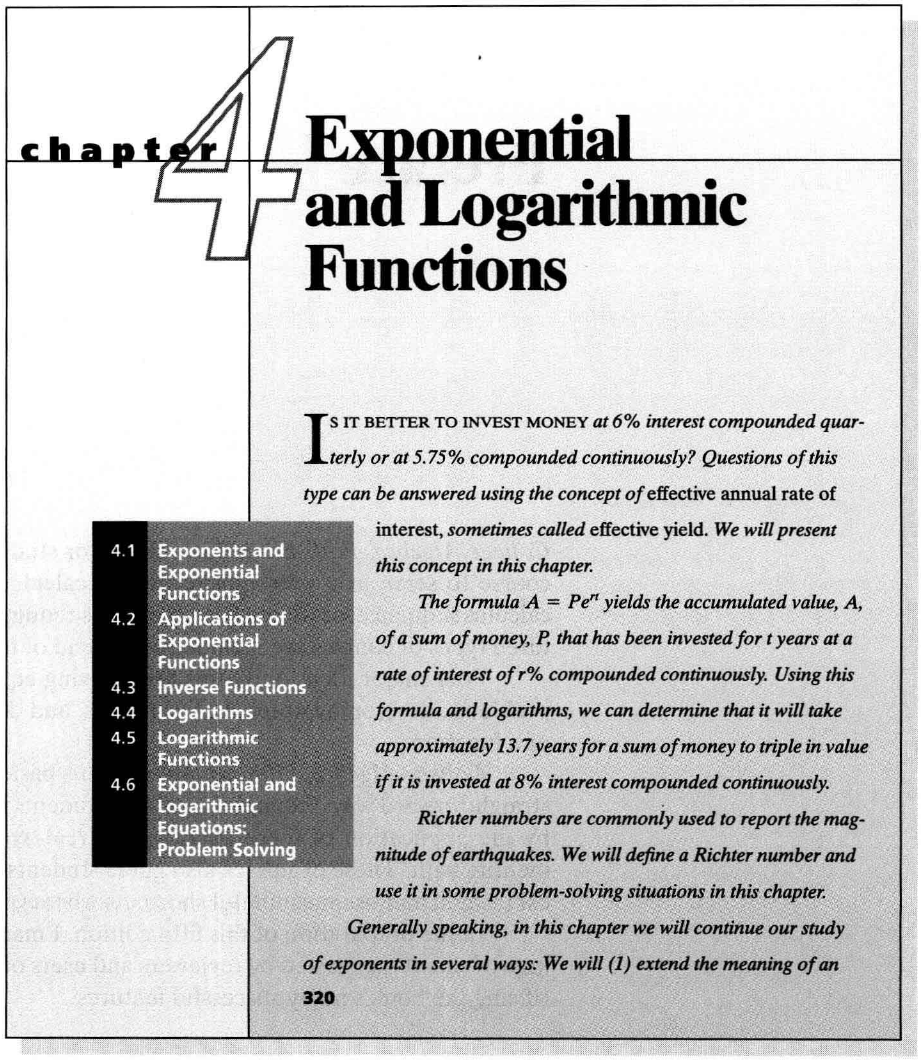
College Algebra, Fifth Edition presents basic concepts of algebra in a simple, straightforward way. Examples motivate students and reinforce algebraic concepts by the application of these examples to real-world situations that students can identify with. These examples also guide students to organize their work in a logical fashion and use meaningful shortcuts whenever appropriate.

In the preparation of this fifth edition, I made a special effort to incorporate improvements suggested by reviewers and users of the earlier editions without sacrificing the book's many successful features.

New in This Edition

- Four more Cumulative Review Problem Sets have been added. There are now such problem sets at the ends of Chapters 2, 3, 4, 5, 6, and 8. *All* answers for Chapter Review Problem Sets, Chapter Tests, and Cumulative Review Problem Sets appear at the back of the text.

- All chapter introductions have been rewritten using applications to lead into the concepts presented in the chapter.



chapter 4

Exponential and Logarithmic Functions

IS IT BETTER TO INVEST MONEY *at 6% interest compounded quarterly or at 5.75% compounded continuously? Questions of this type can be answered using the concept of* effective annual rate of interest, *sometimes called* effective yield. *We will present this concept in this chapter.*

The formula $A = Pe^{rt}$ yields the accumulated value, A, of a sum of money, P, that has been invested for t years at a rate of interest of r% compounded continuously. Using this formula and logarithms, we can determine that it will take approximately 13.7 years for a sum of money to triple in value if it is invested at 8% interest compounded continuously.

Richter numbers are commonly used to report the magnitude of earthquakes. We will define a Richter number and use it in some problem-solving situations in this chapter.

Generally speaking, in this chapter we will continue our study of exponents in several ways: We will (1) extend the meaning of an

320

- The Cartesian Coordinate system and the use of graphing utilities are briefly introduced in Section 0.1. *This allows us to use the graphing calculator as a teaching tool early in the text.* Graphing Calculator Activities have been added to some of the problem sets in Chapters 0 and 1.

Again let's pause for a moment and take another look at the relationship between the solutions of an algebraic equation and the x intercepts of a geometric graph. Figure 1.5 shows a graph of $y = 2x^2 + 6x - 3$. Note that one x intercept is between -4 and -3, and the other x intercept is between 0 and 1. The solution $\frac{-3 - \sqrt{15}}{2} \approx -3.4$, and the solution $\frac{-3 + \sqrt{15}}{2} \approx 0.4$. So our geometric analysis appears to agree with our algebraic solutions.

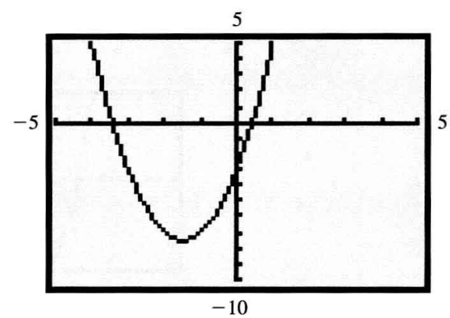


Figure 1.5

- Graphs are used at times in Chapter 0 to *give visual support for the manipulation of algebraic expressions.* Students do not need a graphing calculator to benefit from the graphs.

Probably the best way to check a factoring problem is to make sure the conditions for a polynomial to be completely factored are satisfied and the product of the factors equals the given polynomial. We can also give some visual support to a factoring problem by graphing the given polynomial and its completely factored form on the same set of axes, as shown for Example 10 in Figure 0.19. Note that the graphs for $Y_1 = 24x^2 + 2x - 15$ and $Y_2 = (6x + 5)(4x - 3)$ appear to be identical.

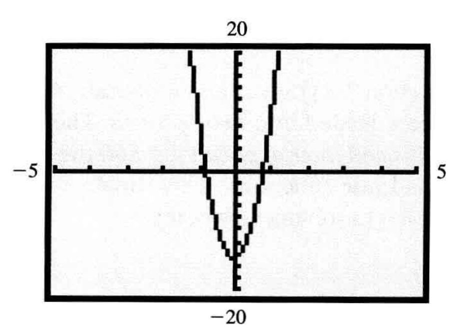


Figure 0.19

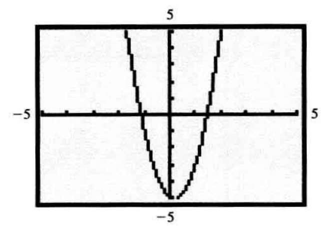
- A graphical analysis of approximating solution sets is introduced in Chapter 1. Then a graphical approach is used to both lend visual support to an algebraic approach and sometimes *to predict approximate solutions before an algebraic approach is shown.*

EXAMPLE 5 Solve each of the following by using the quadratic formula.

a. $3x^2 - x - 5 = 0$ **b.** $25n^2 - 30n = -9$ **c.** $t^2 - 2t + 4 = 0$

Solutions

a. $y = 3x^2 - x - 5$ (See Figure 1.6.)



One intercept is between −2 and −1, and the other is between 1 and 2.

Figure 1.6

We need to think of $3x^2 - x - 5 = 0$ as $3x^2 + (-x) + (-5) = 0$; thus $a = 3$, $b = -1$, and $c = -5$. We then substitute these values into the quadratic formula and simplify:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-5)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{61}}{6}$$

The solution set is $\left\{\frac{1 \pm \sqrt{61}}{6}\right\}$. (You should evaluate these solutions to be sure they agree with the intercepts.)

- Various aspects of problem solving have been emphasized in different sections throughout the text.

- Section 3.2 (Linear and Quadratic Functions) of the previous edition has been divided into two sections. The new Section 3.2 presents *linear functions and their applications,* and the new Section 3.3 deals exclusively with quadratic functions. This should make for a stronger approach from a problem solving viewpoint.

EXAMPLE 5 The cost for burning a 60-watt light bulb is given by the function $c(h) = 0.0036h$, where h represents the number of hours that the bulb is burning.

a. How much does it cost to burn a 60-watt bulb for 3 hours per night for a 30-day month?
b. Graph the function $c(h) = 0.0036h$.
c. Suppose that a 60-watt light bulb is left burning in a closet for a week before it is discovered and turned off. Use the graph from part b to approximate the cost of allowing the bulb to burn for a week. Then use the function to find the exact cost.

Solutions

a. $c(90) = 0.0036(90) = 0.324$ The cost, to the nearest cent, is \$0.32.
b. Because $c(0) = 0$ and $c(100) = 0.36$, we can use the points (0, 0) and (100, 0.36) to graph the linear function $c(h) = 0.0036h$ (see Figure 3.21).

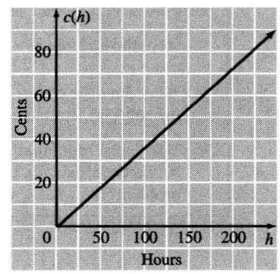


Figure 3.21

c. If the bulb burns for 24 hours per day for a week, it burns for 24(7) = 168 hours. Reading from the graph, we can approximate 168 on the horizontal axis, read up to the line, and then read across to the vertical axis. It looks as though it will cost approximately 60 cents. Using $c(h) = 0.0036h$, we obtain exactly $c(168) = 0.0036(168) = 0.6048$. ■

- Reviewers suggested several places where a page, a paragraph, a sentence, an example, or a solution to a problem could be rewritten to further clarify the intended meaning. *Sometimes we included a* **Remark** *to add a little flavor to the discussion.*

REMARK: If you are interested in finding out more about George Polya and his insights into problem solving, check the Internet. For example, the website http://www.google.com has some interesting information about his problem-solving techniques.

Other Special Features

- Icons found throughout the text point students to material contained on the Brooks/Cole Website and the Interactive Video Skillbuilder CD-ROM.

- A **Chapter Test** appears at the end of each chapter. Along with the Chapter Review Problem Sets, these practice tests should provide the students with ample opportunity to prepare for the "real" examinations.

- Problems called **Thoughts into Words** are included in every problem set except the review exercises. These problems are designed to encourage students to express in written form their thoughts about various mathematical ideas. For examples, see Problem Sets 0.5, 1.2, 1.3, and 5.6.

Thoughts into words

73. Give a step-by-step description of how you would solve the formula $F = \frac{9}{5}C + 32$ for C.

74. What does the phrase "declare a variable" mean in the steps involved in solving a word problem?

75. Why must potential answers to word problems be checked back in the original statement of the problem?

76. From a consumer's viewpoint, would you prefer that retailers figure their profit on the basis of the cost or the selling price? Explain your answer.

77. Some people multiply by 2 and add 30 to estimate the change from a Celsius reading to a Fahrenheit reading. Why does this give an estimate? How good is the estimate?

- Many problem sets contain a special group of problems called **Further Investigations**, which lend themselves to small-group work. These problems encompass a variety of ideas: some are proofs, some exhibit different approaches to topics covered in the text, some bring in supplementary topics and relationships, and some are more challenging problems. Note that, although these problems add variety and flexibility to the problem sets, they can be omitted entirely without disrupting the continuity of the text. For examples, see Problem Sets 1.1, 1.2, 2.1, and 2.3.

Further investigations

67. Verify that for any three consecutive integers, the sum of the smallest and the largest is equal to twice the middle integer.

68. Verify that no four consecutive integers can be found such that the product of the smallest and the largest is equal to the product of the other two integers.

69. Some algebraic identities provide a basis for shortcuts to do mental arithmetic. For example, the identity $(x + y)(x - y) = x^2 - y^2$ indicates that a multiplication problem such as (31)(29) can be treated as $(30 + 1)(30 - 1) = 30^2 - 1^2 = 900 - 1 = 899$.

For each of the following, use the given identity to provide a way of mentally performing the indicated computations. Check your answers with a calculator.

a. $(x + y)(x - y) = x^2 - y^2$: (21)(19); (39)(41); (22)(18); (42)(38); (47)(53)

b. $(x + y)^2 = x^2 + 2xy + y^2$: $(21)^2$; $(32)^2$; $(51)^2$; $(62)^2$; $(43)^2$

c. $(x - y)^2 = x^2 - 2xy + y^2$: $(29)^2$; $(49)^2$; $(18)^2$; $(38)^2$; $(67)^2$

d. $(10t + 5)^2 = 100t^2 + 100t + 25 = 100t(t + 1) + 25$: $(15)^2$; $(35)^2$; $(45)^2$; $(65)^2$; $(85)^2$

- As recommended in the standards produced by NCTM and AMATYC, **problem solving** is an integral part of this text. With problem solving as its focus, Chapter 1 pulls together and expands on a variety of approaches to the process of solving equations and inequalities. Polya's four-phase plan is used as a basis for developing a variety of problem solving strategies. Applications of radical equations are a part of Section 1.5, and applications of slope are in Section 2.3. Functions are introduced in Chapter 3 and are immediately used to solve problems. *Exponential and logarithmic functions become problem solving* tools in Chapter 4. Systems of equations provide more problem solving power in Chapter 6. Problem solving is the unifying theme of Chapters 9 and 10.

Is it better to invest at 6% interest compounded quarterly or at 5.75% compounded continuously? To answer such a question, we can use the concept of **effective yield** (sometimes called *effective annual rate of interest*). The effective yield of an investment is the simple interest rate that would yield the same amount in 1 year. Thus, for the *6% compounded quarterly* investment, we can calculate the effective yield as follows:

$$P(1 + r) = P\left(1 + \frac{0.06}{4}\right)^4$$

$$1 + r = \left(1 + \frac{0.06}{4}\right)^4 \qquad \text{Multiply both sides by } \frac{1}{P}.$$

$$1 + r = (1.015)^4$$

$$r = (1.015)^4 - 1$$

$$r \approx 0.0613635506$$

$$r = 6.14\% \qquad \text{to the nearest hundredth of a percent}$$

Likewise, for the 5.75% *compounded continuously* investment we can calculate the effective yield as follows:

$$P(1 + r) = Pe^{0.0575}$$

$$1 + r = e^{0.0575}$$

$$r = e^{0.0575} - 1$$

$$r \approx 0.0591852707$$

$$r = 5.92\% \qquad \text{to the nearest hundredth of a percent}$$

Therefore, comparing the two effective yields, we see that it is better to invest at 6% compounded quarterly than to invest at 5.75% compounded continuously.

Problems have been chosen so that a variety of problem solving strategies can be introduced. Sometimes alternate solutions are shown for the same problem (see Problem 3 of Section 1.4), while at other times different problems of the same type are used to illustrate different approaches (see Problems 6, 7, and 8 of Section 1.4). *No attempt is made to dictate a specific problem solving technique. Instead my goal is to introduce the students to a large variety of techniques.*

As you tackle word problems throughout this text, keep in mind that our primary objective is to expand your repertoire of problem-solving techniques. We have chosen problems that provide you with the opportunity to use a variety of approaches to solving problems. Don't fall into the trap of thinking "I will never be faced with this kind of problem." That is not the issue; the development of problem-solving techniques is the goal. In the examples we are sharing some of our ideas for solving problems, but don't hesitate to use your own ingenuity. Furthermore, don't become discouraged—all of us have difficulty with some problems. Give each your best shot!

- Specific graphing ideas (intercepts, symmetry, restrictions, asymptotes, and transformations) are introduced and used throughout Chapters 2, 3, 4, 5, and 8. In Section 3.5 the extensive work with graphing parabolas from Section 3.3 is used to motivate definitions for translations, reflections, stretchings, and shrinkings. These transformations are then applied to the graphs of $f(x) = x^3$, $f(x) = x^4$, $f(x) = \sqrt{x}$, and $f(x) = |x|$. Furthermore, in later chapters the *transformations are applied to graphs of exponential, logarithmic, polynomial, and rational functions.*

As you graph exponential functions, don't forget to use your previous graphing experience. For example, consider the following functions.

1. The graph of $f(x) = 2^x + 3$ is the graph of $f(x) = 2^x$ *moved up three units.*
2. The graph of $f(x) = 2^{x-4}$ is the graph of $f(x) = 2^x$ *moved to the right four units.*
3. The graph of $f(x) = -2^x$ is the graph of $f(x) = 2^x$ *reflected across the x axis.*
4. The graph of $f(x) = 2^x + 2^{-x}$ is symmetric with respect to the y axis because $f(-x) = 2^{-x} + 2^x = f(x)$.

The graphs of these functions are shown in Figure 4.4.

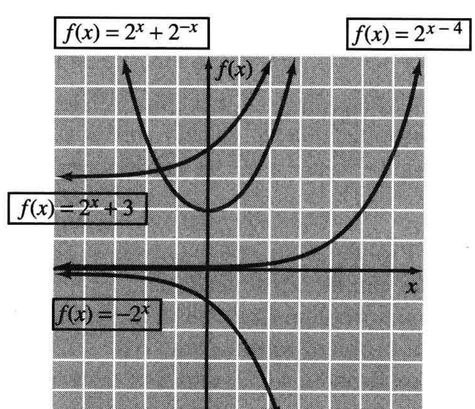


Figure 4.4

- The use of a graphing utility is introduced in Section 0.1. From then on, graphing calculator examples are incorporated, as appropriate, throughout the text. *These examples usually reinforce ideas presented in the section* and are written so that students without graphing calculators can read and benefit from them. The graphing ideas mentioned previously provide a sound basis for efficient use of a graphing utility.

Figure 3.39 shows the result we got when we used a graphing calculator to graph the three functions of Example 3 on the same set of axes. This gives us a visual interpretation of the conclusions drawn regarding the x intercepts and vertices.

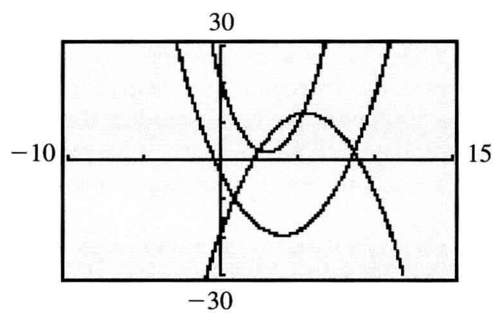


Figure 3.39

- Beginning with Problem Set 0.1, a group of problems called **Graphing Calculator Activities** is included in many of the problem sets. These activities, which are good for either individual or small group work, have been designed to reinforce concepts (see, for example, Problem Set 4.5) as well as lay groundwork for concepts about to be discussed (see, for example, Problem Set 2.2). Some of these activities ask students to predict shapes and locations of graphs based on previous graphing experiences, and then to use a graphing utility to check their predictions (see, for example, Problem Set 3.5). The graphing calculator is also used as a problem solving tool (see, for example, Problem Set 5.5). When students do these activities, they should become familiar with the capabilities and limitations of a graphing utility.