ENGINEERING ELECTROMAGNETIC FIELDS AND WAVES

SECOND EDITION

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Engineering Electromagnetic Fields and Waves

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About the Author

Carl T. A. Johnk is Professor of Electrical and Computer Engineering at the University of Colorado at Boulder. Dr. Johnk received a BSEE degree from the University of Missouri at Rolla in 1942 and a Ph.D. in Electrical Engineering from the University of Illinois in 1954. His early teaching experience includes being an electric engineering faculty member at the University of Missouri at Rolla during the period 1946–49, after a stint in the Navy in World War II. He worked as engineer for Radio Corporation of America and the Emerson Electric Company in early warning radar and radar fire control systems before joining the Radio Direction Finding and Antenna Laboratories at the University of Illinois in Urbana in 1949. He has been a member of the faculty at the University of Colorado at Boulder since 1954 and has been a Visiting Professor at the University of Illinois in 1966 and at the Technische Hochschule in Aachen, West Germany, in 1975. He has served as chapter chairman of the Boulder–Denver Chapter of the Microwave Theory and Techniques Society of the IEEE. His research interests include aperture antenna synthesis, modeling of antenna structures above lossy regions, microwave measurements and systems, and microwave heating of organic materials. Dr. Johnk has received the Tau Beta Pi Outstanding Professor of the Year award in 1969, 1982, and 1984 and the Western Electric Award for Excellence in Instruction in 1976. He has been consultant to TRW and to the Denver Research Institute.

The first edition of this text has been used in a two-semester course sequence in the Department of Electrical and Computer Engineering at the University of Colorado with excellent success, covering Chapters 1 through 5 in the first semester and Chapters 6 through 10 (and a portion of 11) in the second semester. The original chapter arrangement and subject matter are largely retained in this second revised edition. Some changes and additions have been incorporated. A few topics have been moved into appendixes to smooth the main body of the text and to furnish greater flexibility and options in the design of course syllabi. In its present revised form, this text is suitable for either a two-semester electromagnetic fields and wave-transmission sequence or a more limited one-semester treatment.

The book has an ample number of worked-out examples, a few of them new, enabling the student to use this text as a self-study aid. An expanded set of problems, nearly all new, is included at the end of each chapter. To indicate the chapter section to which each group of problems belongs, section headings are given at appropriate intervals.

The present revision of this text retains the objectives of the first edition: to introduce Maxwell's equations early and to develop them into familiar, useful tools. This is accomplished in the first three chapters through graded exercises with applications to elementary static systems with simple symmetries in free space. The electricity and magnetism portion of a previous physics course taken by most students makes this early introduction of Maxwell's equations readily palatable. The temptation to present merely an expanded or extended version of that physics background is avoided here.

By the time the first three chapters are completed, the student will have developed and applied Maxwell's equations in their integral and differential forms to free space and material regions, with most of the exercises confined to static field examples. Exceptions to this are the basic, though reasonably comprehensive, discussions of uniform plane waves in empty space at the end of Chapter 2 and of plane waves in lossy regions in Chapter 3. This early inclusion of time-varying field solutions is found to be rewarding, not only from the point of view of gaining a more complete understanding of Maxwell's equations in the time domain, but also of acquiring a working knowledge of the concepts of loss tangent, complex permittivity, and skin effect—all related to a broader appreciation of the implications of the material constants. This also has the advantage of providing some background in plane-wave theory if Chapter 6 is omitted from the syllabus.

In the book, the important mathematical tools are developed in the first three chapters. The remaining chapters depend largely on the first three, making the sequence of presentation chosen for the remaining eight chapters relatively unimportant. (An exception is that Chapter 10 should follow all or part of Chapter 9.) This degree of flexibility allows the text to be used for either a two-semester course or for a one-semester course embodying a variety of chapter combinations. Parts of chapters can be used, as desired. Although other possibilities exist, four suggested chapter coverages, each suitable for a single-semester fields course with somewhat different objectives, are given here. The omission of certain noncritical sections, as indicated, provides the additional time needed for the remaining material.

1. A one-semester fields course, primarily emphasizing static fields, could cover the bulk of the first five chapters. Chapters 1, 2, and 3 require approximately six weeks in a three-credit-hour, one-semester course; plus Chapter 4 (possibly omitting Sections 4-9, 4-10, 4-12, and 4-16) and Chapter 5 (omitting Section 5-9 and selected portions of 5-10, 5-11, and 5-12).
2. A one-semester course with reduced emphasis on static fields but including transmission line applications would include Chapters 1, 2, 3, and portions of 4 and 5 (Sections 4-1 through 4-6 and 5-3, 5-7, and 5-11) followed by Chapter 9 (Sections 9-1 through 9-4 plus 9-7) and Chapter 10.
3. A single semester emphasizing electromagnetic power, waveguides, and antennas might include, after the first three chapters, selected portions of Chapter 6 (Sections 6-1 through 6-4 plus 6-9), Chapters 7 and 8 (Sections 8-1 through 8-4), and a portion of Chapter 11 (Sections 11-1 through 11-4).
4. A one-semester, in-depth course leading to further work in guided waves and optics with laser applications could include Chapters 1, 2, 3, 6 (Sections 6-1 through 6-4 and 6-9), 7, and 8.

Some features of the chapters including changes incorporated since the first edition are noted in the following.

Chapter 1 begins with a development of the rules of vector algebra, extended to the evaluation of pertinent line, surface, and volume integrals. These techniques are applied to both the interpretation and simple applications of the Maxwell integral laws, postulated for free space in Section 1-11. A short section has been added on coordinate transformations.

Chapter 2 concerns the development of the Maxwell equations for free space in differential form, after defining the div and curl operators. Section 2-11 on wave polarization effects has been added after the treatment of plane waves in free space.

Chapter 3 develops Maxwell's equations for materials by adding in polarization and conduction effects, developing boundary conditions in the process. Uniform plane wave solutions are developed to strengthen the concepts surrounding material parameters. Sections 4-3 and 4-14 of the first edition have been moved to more relevant locations in this chapter, as Sections 3-6 and 3-11.

Chapter 4 is a unified treatment of solutions of the Maxwell equations of electrostatics. New additions include examples of static line and surface charge distributions and two-dimensional finite-difference solutions of Laplace's equation. A long chapter, it is readily trimmed by choosing desired topics.

Chapter 5, concerning solutions of the Maxwell equations of static and quasistatic magnetic fields, is largely unaltered.

Chapter 6 extends the generic plane wave solutions of Chapters 2 and 3 to reflection and transmission at plane boundaries. A section on oblique incidence has been added. Optional sections involving the Smith chart and standing-wave concepts are included, with the theoretical development of the Smith chart now appearing in Appendix D to serve optional applications of the chart in either this chapter or Chapter 10, on transmission lines.

Chapter 7 is largely unaltered, giving an in-depth treatment of the Poynting theorem. Chapter 8 provides a detailed introduction to the mode theory of rectangular waveguides, with emphasis on the concept of the dominant mode. Chapter 9 has been revised by moving a good portion of the original section on transmission-line parameters into Appendix B, primarily for reference and completeness. The original Section 10-6 on nonsinusoidal waves on lossless lines now appears at the end of Chapter 9 in Section 9-7. By now it is clear that the instructor who prefers to cover transmission lines before waveguides may simply go to Chapters 9 and 10 before Chapter 8, since the topics of these last chapters can be ordered arbitrarily.

In Chapter 11, the details of the integration of the inhomogeneous wave equation has been moved to Appendix C. Sections on antenna directive gain and receiving antennas have been added.

Special mention should be made of the many helpful comments made by Ezekiel Bahar, David Chang, Edward Kuester, Leonard Lewin, Samuel Maley, Herbert Reno, and Henry J. Stalzer Jr. Feedback from many students has been very useful, and I am very appreciative of their enthusiastic response. The excellent typing efforts of Mrs. Marie Kindgren and Mrs. Mae Jean Ruehlman are acknowledged. Lastly, I would like to thank in advance any readers who forward corrections or suggestions for improvements.

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CHAPTER 9

TEM Waves on Two-Conductor Transmission Lines 457

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Vector Analysis and Electromagnetic Fields in Free Space

The introduction of vector analysis as an important branch of mathematics dates back to the midnineteenth century. Since then, it has developed into an essential tool for the physical scientist and engineer. The object of the treatment of vector analysis as given in the first two chapters is to serve the needs of the remainder of this book. In this chapter, attention is confined to the scalar and vector products as well as to certain integrals involving vectors. This provides a groundwork for the Lorentz force effects defining the electric and magnetic fields and for the Maxwell integral relationships among these fields and their charge and current sources. The coordinate systems employed are confined to the common rectangular, circular cylindrical, and spherical systems. To unify their treatment, the *generalized coordinate system* is used. This time-saving approach permits developing the general rules for vector manipulations, to enable writing the desired vector operation in a given coordinate system by inspection. This avoids the rederivation of the desired operation for each new coordinate system employed.

Next are postulated the Maxwell integral relations for the electric and magnetic fields produced by charge and current sources in free space. Applying the vector rules developed earlier, their solutions corresponding to simple classes of symmetric static charge and current distributions are considered. The chapter concludes with a discussion of transformations among the three common coordinate systems.

1-1 SCALAR AND VECTOR FIELDS

A field is taken to mean a mathematical function of space and time. Fields can be classified as *scalar* or *vector* fields. A scalar field is a function having, at each instant in time, an assignable magnitude at every point of a region in space. Thus, the temperature field $T(x, y, z, t)$ inside the block of material of Figure 1-1 (*a*) is a scalar field. To each point $P(x, y, z)$ there exists a corresponding temperature $T(x, y, z, t)$ at any instant t in time. The velocity of a fluid moving inside the pipe shown in Figure 1-1 (*b*) illustrates a vector field. A variable direction, as well as magnitude, of the fluid velocity occurs in the pipe where the cross-sectional area is changing. Other examples of scalar fields are mass, density, pressure, and gravitational potential. A force field, a velocity field, and an acceleration field are examples of vector fields.

The mathematical symbol for a scalar quantity is taken to be any letter: for example, A, T, a, f. The symbol for a vector quantity is any letter set in boldface roman type, for example, **A**, **H**, **a**, **g**. Vector quantities are represented graphically by

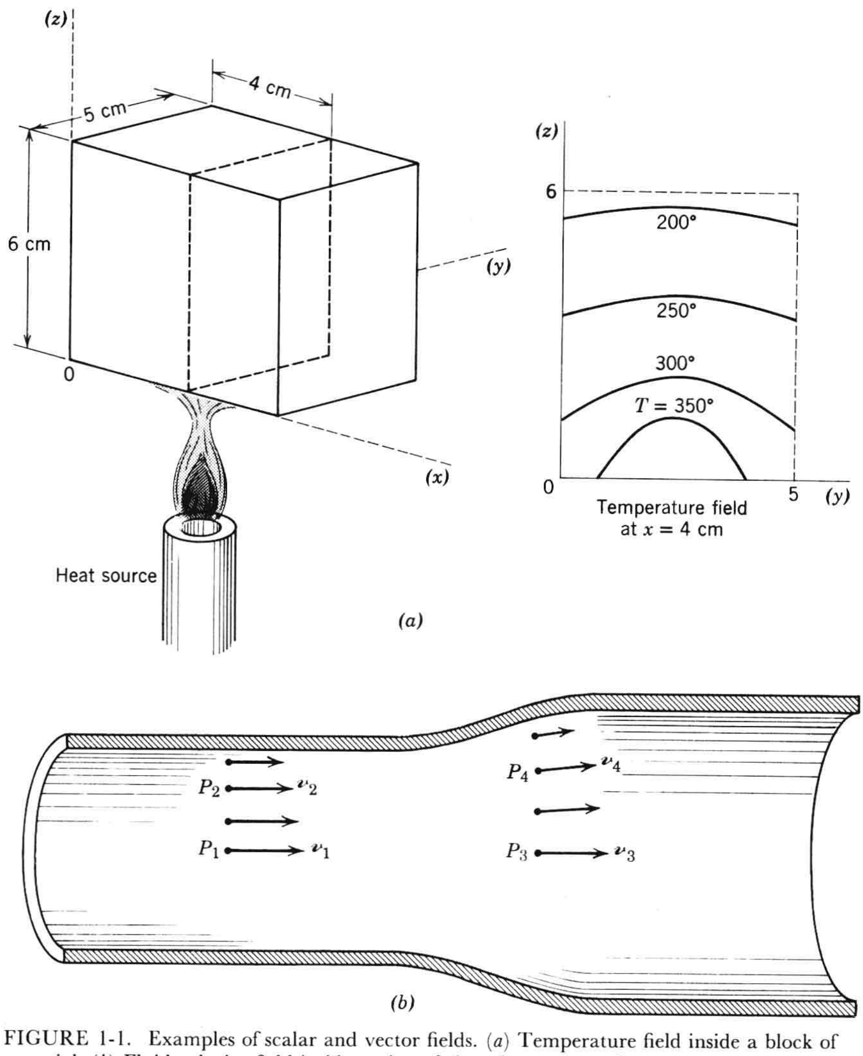

FIGURE 1-1. Examples of scalar and vector fields. (*a*) Temperature field inside a block of material. (*b*) Fluid velocity field inside a pipe of changing cross-section.

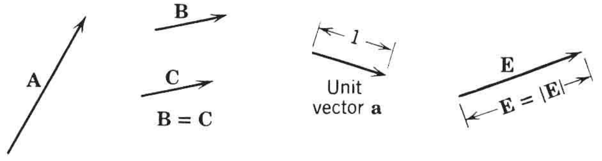

FIGURE 1-2. Graphic representations of a vector, equal vectors, a unit vector, and the representation of magnitude or length of a vector.

means of arrows, or directed line segments, as shown in Figure 1-2. The magnitude or length of a vector **A** is written $|\mathbf{A}|$ or simply A, a positive real scalar. The *negative* of a vector is that vector taken in an opposing direction, with its arrowhead on the opposite end. A *unit vector* is any vector having a magnitude of unity. The symbol **a** is used to denote a unit vector, with a subscript employed to specify a special direction. For example, $\mathbf{a}_x$ means a unit vector having the positive-x direction. Two vectors are said to be *equal* if they have the same direction and the same magnitude. (They need not be collinear, but only parallel to each other.)

1-2 VECTOR SUMS

The vector sum of **A** and **B** is defined in relation to the graphic sketch of the vectors, as in Figure 1-3. A physical illustration of the vector sum occurs in combining displacements in space. Thus, if a particle were displaced consecutively by the vector distance **A** and then by **B**, its final position would be denoted by the vector sum $\mathbf{A} + \mathbf{B} = \mathbf{C}$ shown in Figure 1-3(*a*). Reversing the order of these displacements provides the same vector sum **C**, so that

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \tag{1-1}$$

the commutative law of the addition of vectors. If several vectors are to be added, an associative law

$$(\mathbf{A} + \mathbf{B}) + \mathbf{D} = \mathbf{A} + (\mathbf{B} + \mathbf{D}) \tag{1-2}$$

follows from the definition of vector sum and from Figure 1-3(*b*).

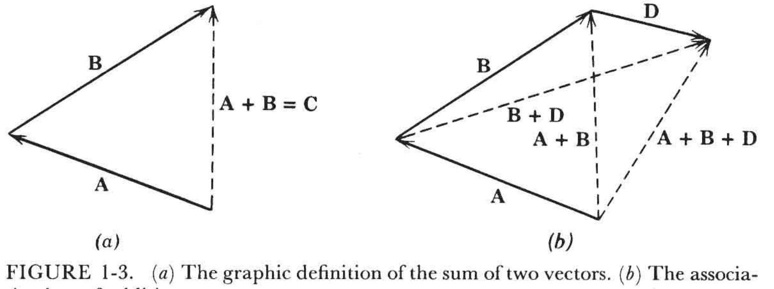

FIGURE 1-3. (*a*) The graphic definition of the sum of two vectors. (*b*) The associative law of addition.

1-3 PRODUCT OF A VECTOR AND A SCALAR

If a scalar quantity is denoted by u and if $\mathbf{B}$ denotes a vector quantity, their product $u\mathbf{B}$ means a vector having a magnitude u times the magnitude of $\mathbf{B}$, and having the same direction as $\mathbf{B}$ if u is a positive scalar, or the opposite direction if u is negative. The following laws hold for the products of vectors and scalars.

$$u\mathbf{B} = \mathbf{B}u \quad \text{Commutative law} \tag{1-3}$$

$$u(v\mathbf{A}) = (uv)\mathbf{A} \quad \text{Associative law} \tag{1-4}$$

$$(u + v)\mathbf{A} = u\mathbf{A} + u\mathbf{A} \quad \text{Distributive law} \tag{1-5}$$

$$u(\mathbf{A} + \mathbf{B}) = u\mathbf{A} + u\mathbf{B} \quad \text{Distributive law} \tag{1-6}$$

1-4 COORDINATE SYSTEMS

The solution of physical problems often requires that the framework of a coordinate system be introduced, particularly if explicit solutions are being sought. The system most familiar to engineers and scientists is the cartesian, or *rectangular* coordinate system, although two other frames of reference often used are the *circular cylindrical* and the *spherical* coordinate systems. The symbols employed for the independent coordinate variables of these orthogonal systems are listed as follows.

1. Rectangular coordinates: (x, y, z)
2. Circular cylindrical coordinates: (ρ, ϕ, z)
3. Spherical coordinates: (r, θ, ϕ)

In Figure 1-4(a), the point P in space, relative to the origin 0, is depicted in terms of the coordinate variables of the three common orthogonal coordinate systems: as $P(x, y, z)$ in the rectangular system, as $P(\rho, \phi, z)$ in the circular cylindrical (or just "cylindrical") system, and as $P(r, \theta, \phi)$ in the spherical coordinate system. In the cylindrical and spherical systems, it is seen that the rectangular coordinate axes, labeled (x), (y), and (z), are retained to establish proper angular references. You should observe that the coordinate variable ϕ (the azimuth angle) is common to *both*

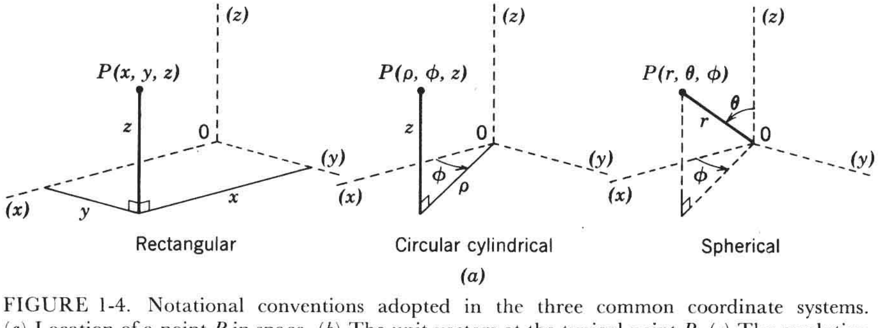

FIGURE 1-4. Notational conventions adopted in the three common coordinate systems. (a) Location of a point P in space. (b) The unit vectors at the typical point P. (c) The resolution of a vector $\mathbf{A}$ into its orthogonal components.

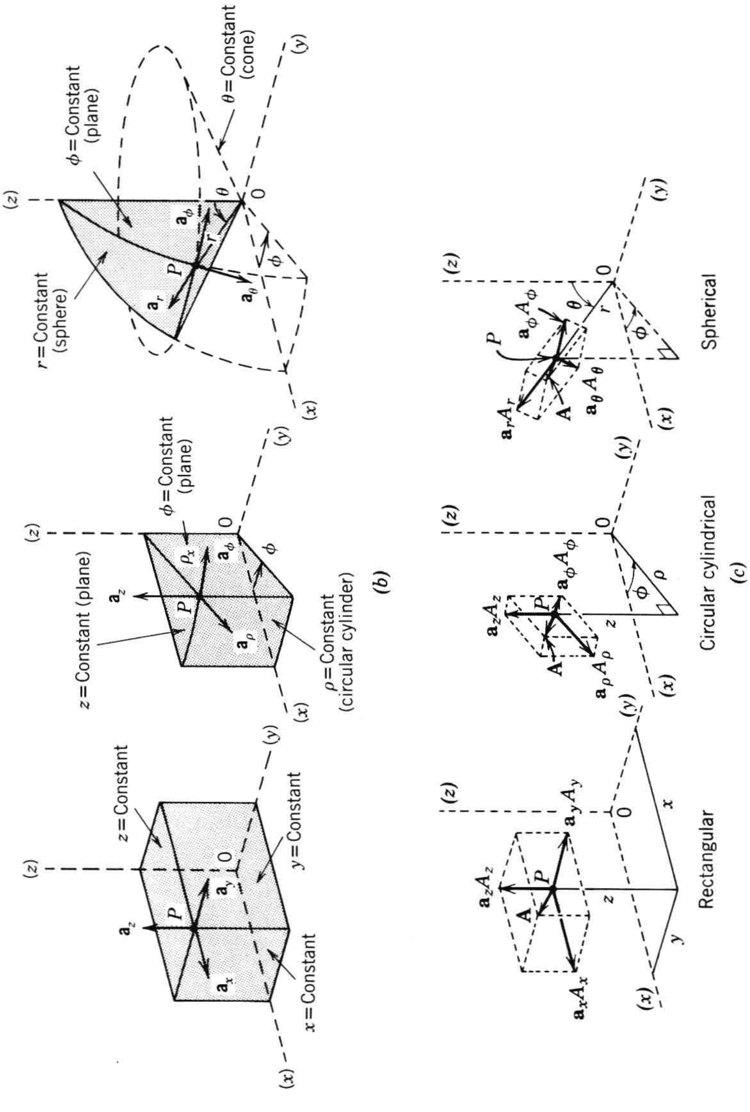

FIGURE 1-4 (continued)

the cylindrical and the spherical systems, with the x-axis taken as the $\phi = 0$ reference, ϕ being generated in the positive sense from (x) toward (y). (By the "right-hand rule," if the thumb of the right hand points in the positive z-direction, the fingers will indicate the positive-ϕ sense.) The *radial* distance in the cylindrical system is ρ, measured perpendicularly from the z-axis to the desired point P; in the spherical system, the radial distance is r, measured from the origin 0 to the point P, with θ denoting the desired declination angle measured positively from the reference z-axis to r, as shown in Figure 1-4(a). The three coordinate systems shown are so-called "right-handed" systems, properly definable after first discussing the unit vectors at P.

A. Unit Vectors and Coordinate Surfaces

To enable expressing any vector $\mathbf{A}$ at the point P in a desired coordinate system, three orthogonal unit vectors, denoted by $\mathbf{a}$ and suitably subscripted, are defined at P in the positive-increasing sense of each of the coordinate variables of that system. Thus, as noted in Figure 1-4(b), $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ are the mutually perpendicular unit vectors of the rectangular coordinate system, shown at $P(x, y, z)$ as dimensionless arrows of unit length originating at P and directed in the positive x, y, and z senses respectively. Note that the disposition of these unit vectors at the point P corresponds to a *right-handed* coordinate system, so-called because a rotation from the unit vector $\mathbf{a}_x$ through the smaller angle toward $\mathbf{a}_y$ and denoted by the fingers of the right hand, corresponds to the thumb pointing in the direction of $\mathbf{a}_z$. Similarly, in the cylindrical coordinate system of that figure, the unit vectors at $P(\rho, \phi, z)$ are $\mathbf{a}_\rho$, $\mathbf{a}_\phi$, $\mathbf{a}_z$ as shown, pointing in the positive ρ, ϕ, and z senses; at $P(r, \theta, \phi)$ in the spherical system, the unit vectors $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_\phi$ are shown in the positive directions of the corresponding coordinates there. These are also right-handed coordinate systems, since on rotating the fingers of the right hand from the first-mentioned unit vector to the second, the thumb points in the direction of the last unit vector of each triplet.

Notice from Figure 1-4(b) that the only *constant* unit vectors in these coordinate systems are $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$. The unit vectors $\mathbf{a}_\rho$ and $\mathbf{a}_\phi$ in the circular cylindrical system, for example, will *change* (in direction, not magnitude) as the angle ϕ rotates P to a new location. Thus, in certain differentiation or integration processes involving unit vectors, most unit vectors should not be treated as constants (see Example 1-1 in Section 1-6).

In Figure 1-4(b), it is instructive to notice how the point P, in any of the coordinate systems, can be looked on as the intersection of three *coordinate surfaces*. A coordinate surface (not necessarily planar) is defined as that surface formed by simply setting the desired coordinate variable equal to a constant. Thus, the point $P(x, y, z)$ in the figure is the intersection of the three coordinate surfaces $x =$ constant, $y =$ constant, $z =$ constant (in this case planes), those constants depending on the desired location for P. (Any *two* such coordinate surfaces intersect orthogonally to define a line; while the perpendicular intersection of the line with the third surface pinpoints P.) The unit vectors at $P(x, y, z)$ are thus perpendicular to their corresponding coordinate surfaces (e.g., $\mathbf{a}_x$ is perpendicular to the surface $x =$ constant). Because the coordinate surfaces are mutually perpendicular, so are the unit vectors.

Similar observations at $P(\rho, \phi, z)$ in the cylindrical coordinate system are applicable. P is the intersection of the three orthogonal coordinate surfaces $\rho =$ constant (a right circular cylindrical surface), $\phi =$ constant (a semi-infinite plane), and $z =$ constant (a plane), to each of which the corresponding unit vectors are perpendicular, thus making $\mathbf{a}_\rho$, $\mathbf{a}_\phi$, $\mathbf{a}_z$ orthogonal as well. Equivalent comments apply to the unit vectors $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_\phi$ at $P(r, \theta, \phi)$ in the spherical coordinate system of Figure 1-4(b),