PRINCIPLES AND APPLICATIONS OF SUPERCONDUCTING QUANTUM INTERFERENCE DEVICES

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PREFACE

When I was invited to act as editor of a book concerning SQUIDs, I considered the idea very stimulating as well as extremely challenging. Indeed, to what extent it was possible to make an interesting and comprehensive volume on the subject was a nontrivial question.

As a result, I believe that the contribution of excellent authors, together with the selection of the topics, guarantees for the high quality standard of the present book. However, it certainly cannot be considered exhaustive or conclusive, this matter continuously growing.

The volume lies halfway between a textbook and a multi-author collection of critical reviews. The latter aspect emerges in the introductory material of each chapter which can somewhat overlap. However, without preventing a large scale coherence of the whole text, this leads to rather self contained chapters. This choice can better provide "individual" messages for the various scientific communities interested in the physics of SQUIDs and their specific applications. Indeed the smooth introduction of each chapter allows the reader to skip, at least in a first reading, the other ones without a serious lack of continuity.

The first chapter provides an extensive background for the comprehension of the physics underlying the SQUID structures focusing mainly on the D.C. configuration and leaving classical and modern aspects of the R.F. devices to the second chapter. The description of very advanced technology and important aspects of device fabrication are given in chapter 3. In chapter 4, an extensive discussion on the most important topic of Biomagnetism, the Neuromagnetic studies, is given in detail. Chapter 5 deals with a further new challenging area, namely the SQUID technology for space applications. A description of the variety of applications of SQUIDs to the world of fundamental physics is contained in chapter 6. The last chapter gives an account of the status of high T_c Superconducting SQUIDs, which includes extremely interesting results and important notions for projections of this extremely stimulating topic.

As it will appear from the reading of the book, SQUID's technology has reached a degree of maturity which legitimates the great interest and research efforts devoted to the topic during the last two decades. This not only for the stimulating underlying physics but also for the concrete perspectives of industrial applications to which high-T_c materials add a further interest. Indeed, as stated by a great pioneer of the field, John Clarke, "*given the world-wide effort on the new superconductors there is every reason to be optimistic about the long-term future of SQUIDs based on these materials*".

I hope that the present book, providing both basic aspects and recent progresses of SQUIDs technology, might offer a realistic and stimulating picture of the state-of-the-art, contributing towards a further development of the field toward a variety of applications.

I gladly take the opportunity to thank, here, the several authors who have contributed to this volume and the directors of the various museums and organizations who kindly provided the photographs and slides used for the chapters separations. Thanks are also due to Mrs. Anna Maria Mazzarella for her valuable assistance throughout the editing.

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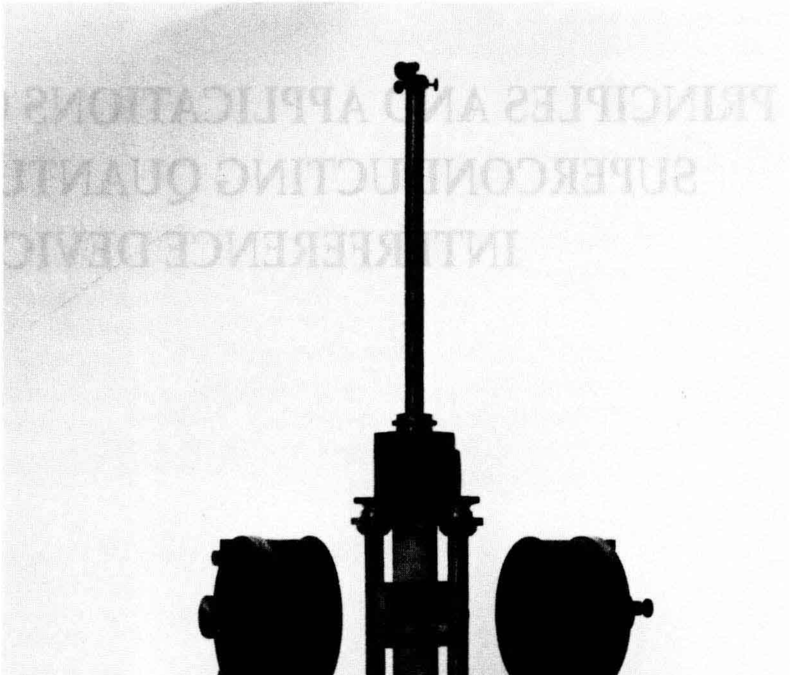

"WIEDEMANN'S GALVANOMETER"– Late 19th Century–overall height 62 cm. (*Courtesy of the Museum of the Department of Physical Sciences, University of Naples "Federico II".*)

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MACROSCOPIC QUANTUM INTERFERENCE: DC-SQUID

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1 INTRODUCTION

Superconductivity is an example of quantum behavior on a macroscopic scale. According to the Bardeen, Cooper and Schrieffer (BCS) theory[1], a superconductor is a many body system made of electrons coupled in pairs. A bound pair (Cooper pair) has a charge $2e$, twice the charge of a single electron and an effective mass depending on the details of the materials. The two electron which form a pair are really spread over a distance larger than the mean distance between pairs. There are several pairs in the same space at the same time. A single pair is not a point particle, however all the electrons do, in some way, work in pairs, and therefore we can talk about the wave function for a pair. From a phenomenological point of view, all the pairs are described by the same wave function or order parameter which in general is a complex quantity of the kind:

$$\psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}e^{i\theta\mathbf{r}} \tag{1}$$

where $\rho(\mathbf{r})$ and $\theta(\mathbf{r})$ are real function of position $\mathbf{r}$. $\rho(\mathbf{r}) = \psi^*\psi$ represents the density of pairs at a given point $\mathbf{r}$ in the superconductor. $\theta(\mathbf{r})$ is the phase of the wave function and is the same for all the pairs. Since it is common to so many particles, its effects do not average out on a macroscopic scale, but are related to measurable quantities, providing the opportunity to observe quantum behavior on a macroscopic scale. Flux quantization and Josephson effect are good examples of the physical meaning and measurability of the phase of the wave function.

In normal metals usually this quantity is a meaningless parameter. In fact the coherence between the wave functions of the electron is destroied by the inelastic scattering with the lattice, also in a presence of a driving force like a vector potential or an electric field. Recently coherence effects have been observed also in normal metals by using very tiny loops at very low temperatures [2,3], showing that phase difference may be a measurable physical quantity in normal metals too.

One of the key prediction of the B.C.S. theory was the presence of an energy gap E_g in the excitation spectrum of quasiparticles in the superconducting state. $E_g = 2\Delta(T)$ is the minimum amount of energy required to break a pair to create two quasiparticles. $\Delta(T)$ increases from zero at the critical temperature T_c to the limiting value:

$$E_g = 2\Delta(0) = 3.528k_BT_c \tag{2}$$

for $T \to 0$.

As Gor'kov has shown there is a proportionality between the energy gap function $\Delta(T)$ and the order parameter ψ [4]. Since the superconducting state is characterized by a large number of pairs inside the volume of coherence, fluctuations of the coherence parameter are negligible. Therefore mean field theory is exactly applicable up to temperatures very close to T_c and the order parameter, i.e. $\Delta(T)$, has in that range the usual square root dependence, typical of second order phase transitions:

$$\Delta(T) = \Delta(0)1.74\sqrt{1 - \frac{T}{T_c}} \tag{3}$$

With the new superconducting ceramic materials the small values of the coherence length may cause severe limitations to the applicability of the mean field theory.

The present chapter deals with Josephson effect interferometers. Many good review papers on this subject can be found in the literature [5,6,7]. The references quoted in this chapter are only the ones needed to have a knowledge of the subject, a more complete list of papers can be found in references [6]. The working mechanism of such a kind of devices is a direct consequence of some well established properties of superconductivity: flux quantization and Josephson effect. These phenomena are briefly described in section 2 and 3. In section 4 the static behavior of two junction interferometer is discussed. The effect of LC resonances on the current voltage characteristics are also described. The dc-SQUID magnetometer is discussed in section 5.

2 FLUX QUANTIZATION

Lets consider a ring of superconducting material encircling a non superconducting region, as an example, an hollow superconducting cylinder. An external applied magnetic field will decrease exponential in the wall of the superconductor with a typical length λ, the penetration length of the material. This length far from the superconductive critical temperature is the order of $500 \div 1000\mathring{A}$ for most materials.

If the wall of the ring is large respect to λ, the following equation:

$$\mathbf{J} \equiv 0 \tag{4}$$

will be valid, with a good approximation, inside the ring.

J is the quantum mechanical current density that is different from zero only in the region where the magnetic field penetrates.

By introducing the vector potential **A** the expression for the current density is derived from the generalization of Schröedinger equation and the use of the local continuity condition [8]:

$$\mathbf{J} = \frac{1}{2}\left\{\left[\frac{\hbar\nabla\psi}{im} - \frac{2e\mathbf{A}\psi}{m}\right]^{*}\psi + \psi^{*}\left[\frac{\hbar\nabla\psi}{im} - \frac{2e\mathbf{A}\psi}{m}\right]\right\} \tag{5}$$

Inserting expression (1) inside (5), the general equation of the superconducting current density is obtained:

$$\mathbf{J} = \frac{\hbar}{m}(\nabla\theta - \frac{2e}{\hbar}\mathbf{A})\rho \tag{6}$$

From expression (6) it follows:

$$\hbar\nabla\theta = 2e\mathbf{A} \tag{7}$$

The round line integral of both member of equation (7) in a curve Γ that encircle the hole (see Fig.1) is:

$$\hbar\oint_{\Gamma}\nabla\theta\cdot\mathbf{dl} = 2e\oint_{\Gamma}\mathbf{A}\cdot\mathbf{dl} \tag{8}$$

From the condition of the single valued of the wave function ψ:

$$\hbar n 2\pi = 2e\Phi \tag{9}$$

where $\Phi = \oint_{\Gamma}\mathbf{A}\cdot\mathbf{dl}$ is the magnetic flux inside the ring and n is an integer.

From the last expression:

$$\Phi = n\Phi_{o} \tag{10}$$

That means that trapped flux inside any hole contoured by a superconductor is an integer multiple of a fundamental quantity the flux quantum defined by:

$$\Phi_{o} = \frac{h}{2e}. \tag{11}$$

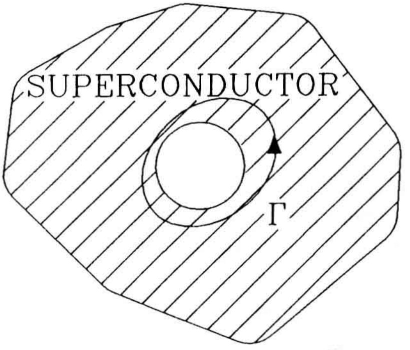

Figure 1: Hollow superconductor. The curve Γ encircles a normal region.

More generally, if the path of integration is taken in a region where the current density is different from zero, from equation (6) it follows that:

$$\nabla\theta = \frac{2\pi}{\Phi_o}(\mathbf{A} + \frac{m}{2e^2\rho}\mathbf{J}) \tag{12}$$

where expression (11) has been taken into account.

By integrating (12) on a closed loop Γ it follows that:

$$n\Phi_o = (\oint_\Gamma \mathbf{A}\cdot\mathbf{dl} + \frac{m}{2e^2\rho}\oint_\Gamma \mathbf{J}\cdot\mathbf{dl}) \tag{13}$$

The quantity on the left side of (13) is called *fluxoid*. The fluxoid is a integer multiple of the flux quantum. This equation takes into account the finite penetration of the magnetic field inside a superconductor. Equation (13) reduces to equation (10) in the particular case of a path of integration (Γ) far from the surface region.

The fluxoid quantization is equivalent to the second Kirchhoff rule for a superconducting network.

From equation (12) it follows that the phase gradient is related to the current density that is a measurable physical quantity. From that it follows that the absolute value of phase is not, observable but if the phase gradient is known everywhere the phase, is known except for a constant.

It is interesting to note that to demonstrate the fluxoid quantization we used only general principles of single valued wave function of the superconductor and the quantum mechanical expression of current density and no particular hypothesis on the material has been used.

Flux quantization was first predicted by F. London [9] and experimentally verified later [10,11]. It is interesting to note, that since that time the concept of a pair had not yet been established, the value predicted by London was twice that later observed experimentally. More careful experimental measurements has performed by Goodman and Deaver [12], the experimental value was: $\Phi_\circ = 2.07 \cdot 10^{-15}\ Wb$ in good agreement with expression (11) confirming that the current in a superconductor is transported by Cooper pairs.

In a superconducting ring of inductance L, two state having a difference in trapped flux of a flux quantum have a difference in circulating screening current of:

$$\Delta i = \frac{\Phi_\circ}{L} \tag{14}$$

their energy difference is:

$$\Delta E = \frac{1}{2} L (\Delta i)^2 = \frac{1}{2} \frac{\Phi_\circ^2}{L} \tag{15}$$

To be the flux quantization an observable phenomenon, this energy difference ΔE must be larger than $\frac{K_B T}{2}$. Otherwise thermal fluctuation will activate jumping between contiguous states. This correspond to a condition on the maximum inductance of devices based on flux quantization. Typical value of the limit inductance

$$L_m = \frac{\Phi_\circ^2}{2 k_B T}$$

for different temperature are shown in table I:

Table I: Limit inductance vs temperature

L_m (nH)	1550	155	15.5	1.55
$T(K)$	.1	1	10	100