INCOMPRESSIBLE FLOW

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Preface

This book is written as a textbook for students beginning a serious study of fluid dynamics, or for students in other fields who want to know the main ideas and results in this discipline. A reader who judges the scope of the book by its title will be somewhat surprised at the contents. The contents not only treat incompressible flows themselves, but also give the student an understanding of how incompressible flows are related to the general compressible case. For example, one cannot appreciate how energy interactions occur in incompressible flows without first understanding the most general interaction mechanisms. I subscribe to the philosophy that advanced students should study the structure of a subject as well as its techniques and results. The beginning chapters are devoted to building the concepts and physics for a general, compressible, viscous fluid flow. These chapters taken by themselves constitute the fundamentals that one might study in any course concerning fluid dynamics. Beginning with Chapter 6 our study is restricted to fluids that obey Newton's viscosity law. Only when we arrive at Chapter 10 do we find a detailed discussion of the assumptions that underlie the subject of incompressible flow. Thus, roughly half the book is fundamentals, and the rest is incompressible flow.

Applied mathematicians have contributed greatly to the study of fluid mechanics, and there is a tendency to make a text into a sampler of known mathematical solutions. A conscious effort was made in writing the book to strike an even balance among physics, mathematics, and practical engineering information. The student is assumed to have had calculus and differential equations; the text then takes on the task of introducing tensor analysis in index notation, as well as various special methods of solving differential equations that have been developed in fluid mechanics. This includes an introduction to several computer methods and the method of asymptotic expansions.

The book places heavy emphasis on dimensional analysis, both as a subject in itself and as an instrument in any analysis of flow problems. The advanced worker knows many shortcuts in this area, but the student needs to study the foundations and details in order to be convinced that these shortcuts are valid. Vorticity, vortex lines, and the dynamics of vorticity also receive an expanded treatment, which is designed to bring the serious student more information than is customary in a textbook. It is apparent that advanced workers in fluid mechanics must be able to interpret flow patterns in terms of vorticity as well as in the traditional terms of forces and energy.

The study of how changes in the Reynolds number influence flow patterns occupies a large part of the book. Separate chapters describe flows at low, moderate, and high Reynolds numbers. Because of their practical importance, the complementary subjects of inviscid flows and boundary-layer flows are treated extensively. Introductory chapters on stability and turbulence are also given. These last two subjects are so large as to constitute separate fields. Nevertheless, a beginning student should have an overview of the rudiments and principles.

The book is not meant to be read from front to back. The coverage is rather broad so that the instructor may select those chapters and sections that suit his or her goals. For example, I can imagine that many people, considering the level and background of their students, will skip Chapter 2 on thermodynamics or Chapter 3 on tensor index notation. I placed these chapters at the beginning, rather than in an appendix, with the thought that the student would be likely to review these subjects even if they were not formally assigned as a part of the course. Students who want more information about any chapter will find a supplemental reading list at the back of the book.

A chapter usually begins with an elementary approach suitable for the beginning student. Subsections that are marked by an asterisk contain more advanced material, which either gives a deeper insight or a broader viewpoint. These sections should be read only by the more advanced student who already has the fundamentals of the subject well in hand. Likewise, the problems at the end of each chapter are classified into three types: (A) problems that give computational practice and directly reinforce the text material, (B) problems that require a thoughtful and more creative application of the material, and finally (C) more difficult problems that extend the text or give new results not previously covered.

Several photographs illustrating fluid flow patterns have been included. Some illustrate a simplified flow pattern or single physical phenomenon. Others were chosen precisely because they show a very complicated flow that contrasts with the simplified analysis of the text. The intent is to emphasize the nonuniqueness and complexity possible in fluid motions. In most cases only the major point about a photograph is explained. The reader will find a complete discussion in the original references.

Writing this book has been a long project. I would like to express my appreciation for the encouragement that I have received during this time from my family, students, colleagues, former teachers, and several anonymous reviewers. The people associated with John Wiley & Sons should also be mentioned: At every stage their professional attitude has contributed to the quality of this book.

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1 Continuum Mechanics

The science of fluid dynamics describes the motions of liquids and gases and their interaction with solid bodies. There are many ways to further subdivide fluid dynamics into special subjects. The plan of this book is to make the division into compressible and incompressible flows. Compressible flows are those where changes in the fluid density are important. The major specialty concerned with compressible flows is called gas dynamics. It deals with high-speed flows where density changes are large and wave phenomena frequently occur. Incompressible flows, of either gases or liquids, are flows where density changes in the fluid are not an important part of the physics. The study of incompressible flow includes such subjects as hydraulics, hydrodynamics, aerodynamics, and boundary-layer theory. It also contains the background information for such special subjects as hydrology, lubrication theory, stratified flows, turbulence, rotating flows, and biological fluid mechanics. Incompressible flow not only occupies the central position in fluid dynamics, but is also fundamental to the practical subjects of heat and mass transfer.

Figure 1-1 shows a photograph of a ship's propeller being tested in a water tunnel. The propeller is rotating, and the water flow is from left to right. The most prominent feature of this photograph is the line of vapor which leaves the tip of each blade and spirals downstream. The vapor is not itself important, but it marks a region of very low pressure in the core of a vortex which leaves the tip of each blade. This vortex would exist even if the pressure were not low enough to form water vapor. The convergence of the vapor lines into a smaller spiral indicates that the flow is faster behind the propeller than in front of the propeller.

A photograph of an airplane in level flight is shown in Fig. 1-2. A smoke device has been attached to the wing tip so that the core of the vortex formed there is made visible. The vortex trails nearly straight back behind the aircraft. From the sense of the vortex we may surmise that the wing is pushing air down on the inside while air rises outside the tip.

There are obviously some differences in these two situations. The wing moves in a straight path, whereas the ship's propeller blades are rotating. The propeller operates in water, a nearly incompressible liquid, whereas the wing operates in air, a very compressible gas. The densities of these two fluids differ by a factor of 800 : 1. In spite of these obvious differences, these two flows are governed by the same laws, and their fluid dynamics are very similar. The purpose of the wing is to lift the airplane, while the purpose of the propeller is to produce thrust on the boat. The density of the air as well as that of the