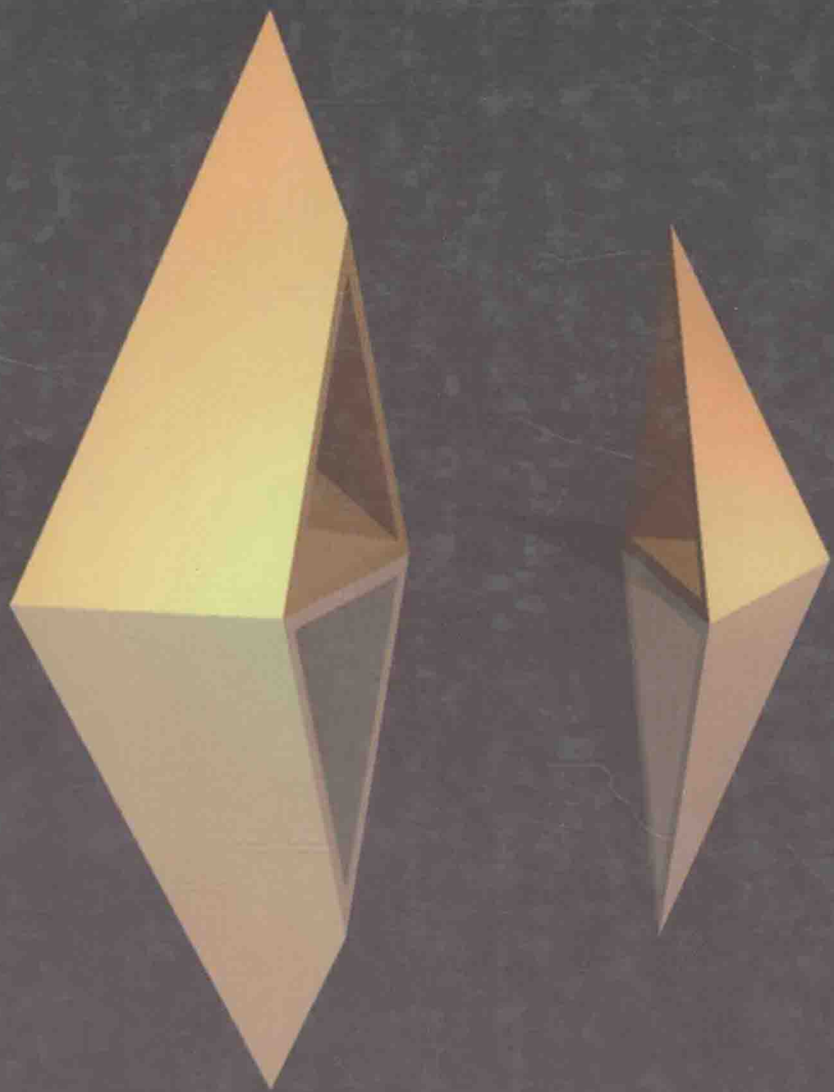
TRIGONOMETRY

FIFTH EDITION



LARSON ◆ HOSTETLER

Trigonometry

Fifth Edition

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The Pennsylvania State University
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A Word from the Authors

Welcome to *Trigonometry*, Fifth Edition. In this revision we focus on student success, accessibility, and flexibility.

Student Success: During the past 30 years of teaching and writing, we have learned many things about the teaching and learning of mathematics. We have found that students are most successful when they know what they are expected to learn and why it is important to learn. With that in mind, we have restructured the Fifth Edition to include a thematic study thread in every chapter.

Each chapter begins with a study guide called *How to Study This Chapter*, which includes a comprehensive overview of the chapter concepts (*The Big Picture*), a list of *Important Vocabulary* that is integral to learning *The Big Picture* concepts, a list of study resources, and a general study tip. The study guide allows students to get organized and prepare for the chapter.

An old pedagogical recipe goes something like this: "First I'm going to tell you what I'm going to teach you, then I will teach it to you, and finally I will go over what I taught you." Following this recipe, we have also included a set of learning objectives in every section that outlines what students are expected to learn, followed by an interesting real-life application that illustrates why it is important to learn the concepts in that section. Finally, the chapter summary (*What did you learn?*), which reinforces the section objectives, and the chapter *Review Exercises*, which are correlated to the chapter summary, provide additional study support at the conclusion of each chapter.

Our new *Student Success Organizer* supplement takes this study thread one step further, providing a content-based study aid.

Accessibility: Over the years we have taken care to write our texts for the student. We have paid careful attention to the presentation, using precise mathematical language and clear writing, to create an effective learning tool. We believe that every student can learn mathematics and we are committed to providing a text that makes the mathematics within it accessible to all students. In the Fifth Edition, we have revised and improved many text features designed for this purpose. The *Technology*, *Exploration*, and *Study Tip* features have been expanded. *Chapter Tests*, which give students an opportunity for self-assessment, now follow every chapter in the Fifth Edition. The exercise sets now include both *Synthesis* exercises, which check students' conceptual understanding, and *Review* exercises, which reinforce skills learned in previous sections and chapters. Also, students have access to several media resources that accompany this text—videotapes, *Interactive Trigonometry* CD-ROM, and a *Trigonometry* website—that provide additional text-specific support.

Flexibility: From the time we first began writing in the early 1970s, we have always viewed part of our authoring role as that of providing instructors with flexible teaching programs. The optional features within the text allow instructors with different pedagogical approaches to design their courses to meet both their instructional needs and the needs of their students. Instructors who stress applications and problem solving, or exploration and technology, or more traditional methods, will be able to use this text successfully. In addition, we provide several print and media resources to support instructors, including a new *Instructor Success Organizer*.

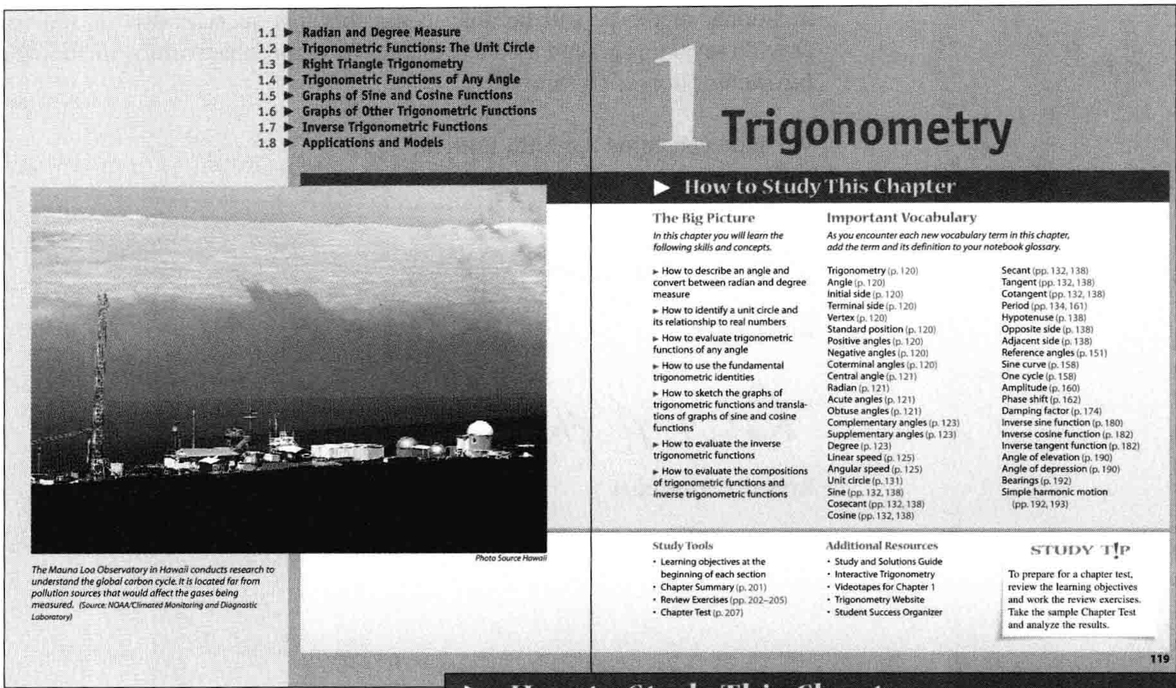
We hope you enjoy the Fifth Edition.

Ron Larson

Robert P. Hostetler

Features Highlights

Student Success Tools



1 Trigonometry

The Mauna Loa Observatory in Hawaii conducts research to understand the global carbon cycle. It is located far from pollution sources that would affect the gases being measured. (Source: NOAA/Climated Monitoring and Diagnostic Laboratory)

Photo Source Hawaii

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▶ "How to Study This Chapter"

The new chapter-opening study guide includes:

- *The Big Picture*—an objective-based overview of the main concepts of the chapter
- *Important Vocabulary*—mathematical terms integral to learning *The Big Picture* concepts
- *Study Tools*
- *Additional Resources*
- *Study Tip*

▶ How to Study This Chapter

The Big Picture

In this chapter you will learn the following skills and concepts.

- ▶ How to describe an angle and convert between radian and degree measure
- ▶ How to identify a unit circle and its relationship to real numbers
- ▶ How to evaluate trigonometric functions of any angle
- ▶ How to use the fundamental trigonometric identities
- ▶ How to sketch the graphs of trigonometric functions and translations of graphs of sine and cosine functions
- ▶ How to evaluate the inverse trigonometric functions
- ▶ How to evaluate the compositions of trigonometric functions and inverse trigonometric functions

Important Vocabulary

As you encounter each new vocabulary term in this chapter, add the term and its definition to your notebook glossary.

Trigonometry (p. 120)
Angle (p. 120)
Initial side (p. 120)
Terminal side (p. 120)
Vertex (p. 120)
Standard position (p. 120)
Positive angles (p. 120)
Negative angles (p. 120)
Coterminal angles (p. 120)
Central angle (p. 121)
Radian (p. 121)
Acute angles (p. 121)
Obtuse angles (p. 121)
Complementary angles (p. 123)
Supplementary angles (p. 123)
Degree (p. 123)
Linear speed (p. 125)
Angular speed (p. 125)
Unit circle (p. 131)
Sine (pp. 132, 138)
Cosecant (pp. 132, 138)
Cosine (pp. 132, 138)
Secant (pp. 132, 138)
Tangent (pp. 132, 138)
Cotangent (pp. 132, 138)
Period (pp. 134, 161)
Hypotenuse (p. 138)
Opposite side (p. 138)
Adjacent side (p. 138)
Reference angles (p. 151)
Sine curve (p. 158)
One cycle (p. 158)
Amplitude (p. 160)
Phase shift (p. 162)
Damping factor (p. 174)
Inverse sine function (p. 180)
Inverse cosine function (p. 182)
Inverse tangent function (p. 182)
Angle of elevation (p. 190)
Angle of depression (p. 190)
Bearings (p. 192)
Simple harmonic motion (pp. 192, 193)

Study Tools

- Learning objectives at the beginning of each section
- Chapter Summary (p. 201)
- Review Exercises (pp. 202–205)
- Chapter Test (p. 207)

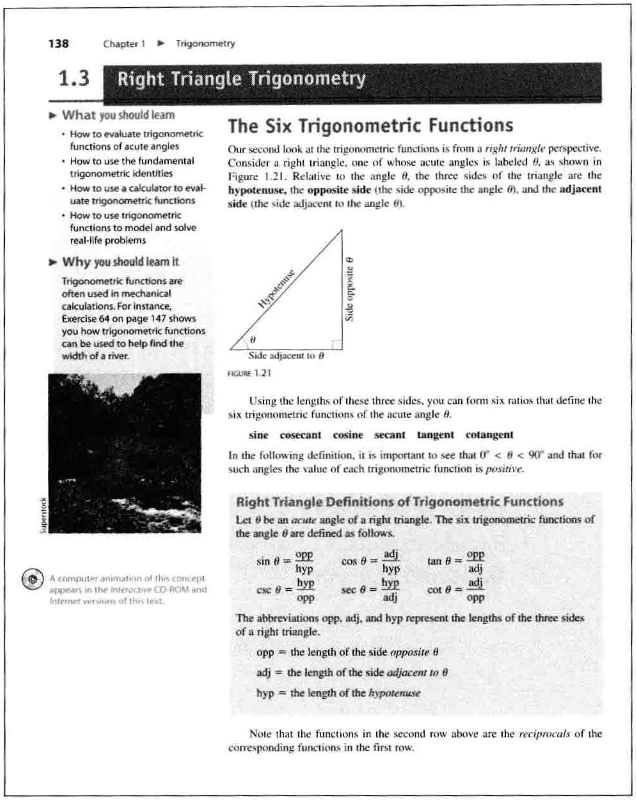
Additional Resources

- Study and Solutions Guide
- Interactive Trigonometry
- Videotapes for Chapter 1
- Trigonometry Website
- Student Success Organizer

STUDY TIP

To prepare for a chapter test, review the learning objectives and work the review exercises. Take the sample Chapter Test and analyze the results.

New Section Openers include:


138 Chapter 1 ▶ Trigonometry

1.3 Right Triangle Trigonometry

▶ What you should learn

- How to evaluate trigonometric functions of acute angles
- How to use the fundamental trigonometric identities
- How to use a calculator to evaluate trigonometric functions
- How to use trigonometric functions to model and solve real-life problems

▶ Why you should learn it

Trigonometric functions are often used in mechanical calculations. For instance, Exercise 64 on page 147 shows you how trigonometric functions can be used to help find the width of a river.

Superstock

A computer animation of this concept appears in the *Interactive* CD-ROM and *Internet* versions of this text.

The Six Trigonometric Functions

Our second look at the trigonometric functions is from a *right triangle* perspective. Consider a right triangle, one of whose acute angles is labeled θ, as shown in Figure 1.21. Relative to the angle θ, the three sides of the triangle are the **hypotenuse**, the **opposite side** (the side opposite the angle θ), and the **adjacent side** (the side adjacent to the angle θ).

FIGURE 1.21

Using the lengths of these three sides, you can form six ratios that define the six trigonometric functions of the acute angle θ.

sine cosecant cosine secant tangent cotangent

In the following definition, it is important to see that $0° < \theta < 90°$ and that for such angles the value of each trigonometric function is *positive*.

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. The six trigonometric functions of the angle θ are defined as follows.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations opp, adj, and hyp represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ

adj = the length of the side *adjacent to* θ

hyp = the length of the *hypotenuse*

Note that the functions in the second row above are the *reciprocals* of the corresponding functions in the first row.

▶ "What you should learn"

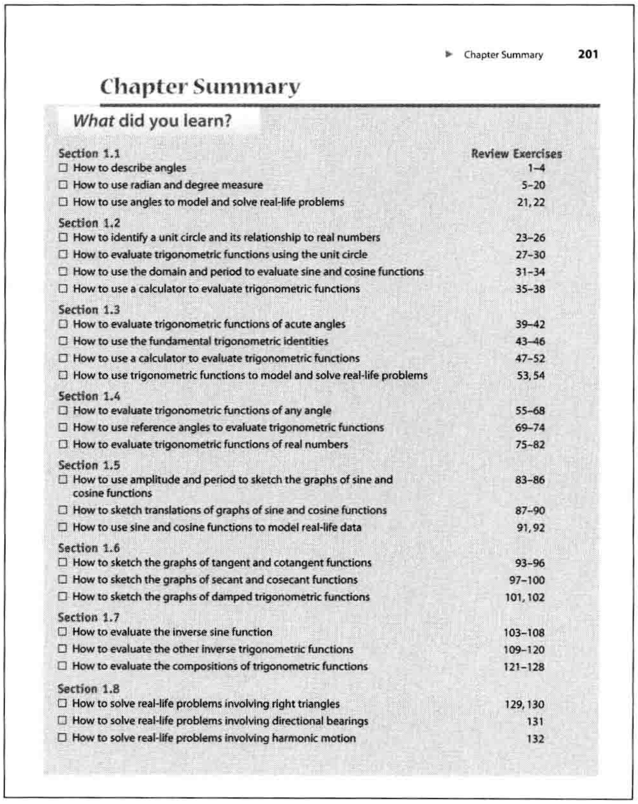
Objectives outline the main concepts and help keep students focused on *The Big Picture*.

▶ "Why you should learn it"

A real-life application or a reference to other branches of mathematics illustrates the relevance of the section's content.

▶ "What did you learn?" Summary

The chapter summary provides a concise, section-by-section review of the section objectives. These objectives are correlated to the chapter Review Exercises.

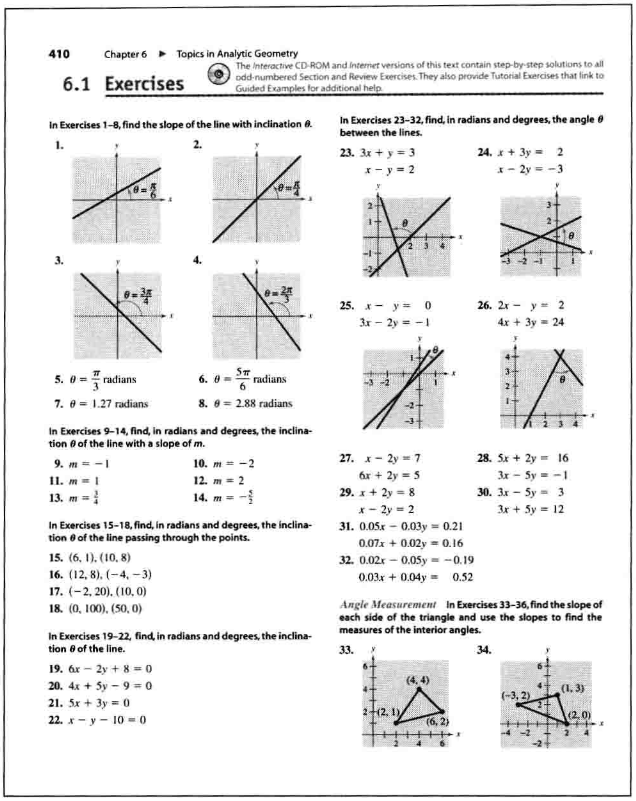

▶ Chapter Summary 201

Chapter Summary

What did you learn?

	Review Exercises
Section 1.1	
☐ How to describe angles	1–4
☐ How to use radian and degree measure	5–20
☐ How to use angles to model and solve real-life problems	21, 22
Section 1.2	
☐ How to identify a unit circle and its relationship to real numbers	23–26
☐ How to evaluate trigonometric functions using the unit circle	27–30
☐ How to use the domain and period to evaluate sine and cosine functions	31–34
☐ How to use a calculator to evaluate trigonometric functions	35–38
Section 1.3	
☐ How to evaluate trigonometric functions of acute angles	39–42
☐ How to use the fundamental trigonometric identities	43–46
☐ How to use a calculator to evaluate trigonometric functions	47–52
☐ How to use trigonometric functions to model and solve real-life problems	53, 54
Section 1.4	
☐ How to evaluate trigonometric functions of any angle	55–68
☐ How to use reference angles to evaluate trigonometric functions	69–74
☐ How to evaluate trigonometric functions of real numbers	75–82
Section 1.5	
☐ How to use amplitude and period to sketch the graphs of sine and cosine functions	83–86
☐ How to sketch translations of graphs of sine and cosine functions	87–90
☐ How to use sine and cosine functions to model real-life data	91, 92
Section 1.6	
☐ How to sketch the graphs of tangent and cotangent functions	93–96
☐ How to sketch the graphs of secant and cosecant functions	97–100
☐ How to sketch the graphs of damped trigonometric functions	101, 102
Section 1.7	
☐ How to evaluate the inverse sine function	103–108
☐ How to evaluate the other inverse trigonometric functions	109–120
☐ How to evaluate the compositions of trigonometric functions	121–128
Section 1.8	
☐ How to solve real-life problems involving right triangles	129, 130
☐ How to solve real-life problems involving directional bearings	131
☐ How to solve real-life problems involving harmonic motion	132

Revised Exercises and Applications



410 Chapter 6 ▶ Topics in Analytic Geometry

6.1 Exercises

The *Interactive* CD-ROM and *Internet* versions of this text contain step-by-step solutions to all odd-numbered Section and Review Exercises. They also provide Tutorial Exercises that link to Guided Examples for additional help.

In Exercises 1–8, find the slope of the line with inclination θ.

1. $\theta = \frac{\pi}{6}$
2. $\theta = \frac{\pi}{4}$
3. $\theta = \frac{3\pi}{4}$
4. $\theta = \frac{2\pi}{3}$
5. $\theta = \frac{\pi}{3}$ radians
6. $\theta = \frac{5\pi}{6}$ radians
7. $\theta = 1.27$ radians
8. $\theta = 2.88$ radians

In Exercises 9–14, find, in radians and degrees, the inclination θ of the line with a slope of m.

9. $m = -1$
10. $m = -2$
11. $m = 1$
12. $m = 2$
13. $m = \frac{3}{4}$
14. $m = -\frac{5}{2}$

In Exercises 15–18, find, in radians and degrees, the inclination θ of the line passing through the points.

15. $(6, 1), (10, 8)$
16. $(12, 8), (-4, -3)$
17. $(-2, 20), (10, 0)$
18. $(0, 100), (50, 0)$

In Exercises 19–22, find, in radians and degrees, the inclination θ of the line.

19. $6x - 2y + 8 = 0$
20. $4x + 5y - 9 = 0$
21. $5x + 3y = 0$
22. $x - y - 10 = 0$

In Exercises 23–32, find, in radians and degrees, the angle θ between the lines.

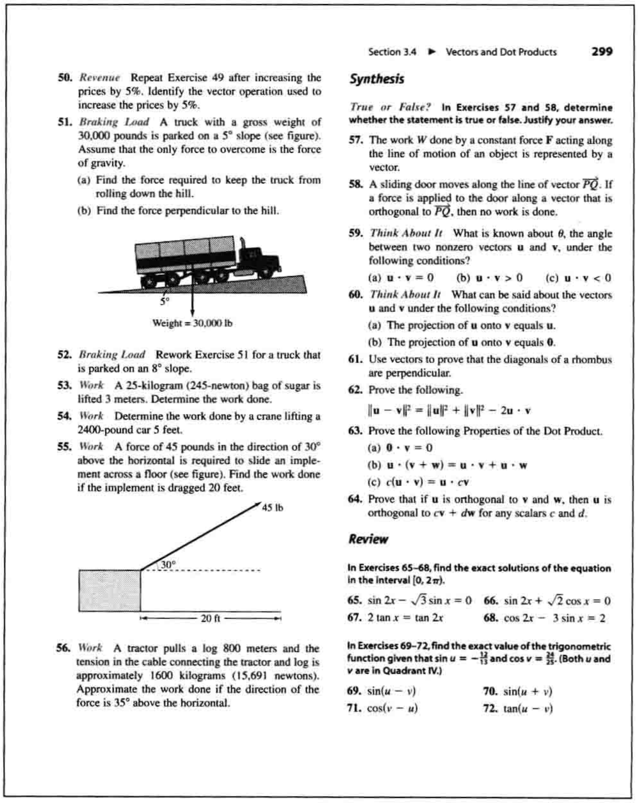
23. $3x + y = 3$, $x - y = 2$
24. $x + 3y = 2$, $x - 2y = -3$
25. $x - y = 0$, $3x - 2y = -1$
26. $2x - y = 2$, $4x + 3y = 24$
27. $x - 2y = 7$, $6x + 2y = 5$
28. $5x + 2y = 16$, $3x - 5y = -1$
29. $x + 2y = 8$, $x - 2y = 2$
30. $3x - 5y = 3$, $3x + 5y = 12$
31. $0.05x - 0.03y = 0.21$, $0.07x + 0.02y = 0.16$
32. $0.02x - 0.05y = -0.19$, $0.03x + 0.04y = 0.52$

Angle Measurement **In Exercises 33–36, find the slope of each side of the triangle and use the slopes to find the measures of the interior angles.**

33. $(2, 1), (4, 4), (6, 2)$
34. $(-3, 2), (1, 3), (2, 0)$

▶ Exercises

- Each exercise set contains a variety of computational, conceptual, and applied problems.
- Each exercise set is carefully graded in difficulty to allow students to gain confidence as they progress.
- Each exercise set now concludes with two new types of exercises:
  - *Synthesis* exercises promote further exploration of mathematical concepts, critical thinking skills, and writing about mathematics. These exercises require students to synthesize the main concepts presented in the section and chapter.
  - *Review* exercises reinforce previously learned skills and concepts.



Section 3.4 ▶ Vectors and Dot Products 299

50. *Revenue* Repeat Exercise 49 after increasing the prices by 5%. Identify the vector operation used to increase the prices by 5%.
51. *Braking Load* A truck with a gross weight of 30,000 pounds is parked on a 5° slope (see figure). Assume that the only force to overcome is the force of gravity.
    (a) Find the force required to keep the truck from rolling down the hill.
    (b) Find the force perpendicular to the hill.

    5° Weight = 30,000 lb
52. *Braking Load* Rework Exercise 51 for a truck that is parked on an 8° slope.
53. *Work* A 25-kilogram (245-newton) bag of sugar is lifted 3 meters. Determine the work done.
54. *Work* Determine the work done by a crane lifting a 2400-pound car 5 feet.
55. *Work* A force of 45 pounds in the direction of 30° above the horizontal is required to slide an implement across a floor (see figure). Find the work done if the implement is dragged 20 feet.

    45 lb 30° 20 ft
56. *Work* A tractor pulls a log 800 meters and the tension in the cable connecting the tractor and log is approximately 1600 kilograms (15,691 newtons). Approximate the work done if the direction of the force is 35° above the horizontal.

Synthesis

True or False? **In Exercises 57 and 58, determine whether the statement is true or false. Justify your answer.**

57. The work W done by a constant force $\mathbf{F}$ acting along the line of motion of an object is represented by a vector.
58. A sliding door moves along the line of vector $\overrightarrow{PQ}$. If a force is applied to the door along a vector that is orthogonal to $\overrightarrow{PQ}$, then no work is done.
59. *Think About It* What is known about θ, the angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, under the following conditions?
    (a) $\mathbf{u} \cdot \mathbf{v} = 0$ (b) $\mathbf{u} \cdot \mathbf{v} > 0$ (c) $\mathbf{u} \cdot \mathbf{v} < 0$
60. *Think About It* What can be said about the vectors $\mathbf{u}$ and $\mathbf{v}$ under the following conditions?
    (a) The projection of $\mathbf{u}$ onto $\mathbf{v}$ equals $\mathbf{u}$.
    (b) The projection of $\mathbf{u}$ onto $\mathbf{v}$ equals $\mathbf{0}$.
61. Use vectors to prove that the diagonals of a rhombus are perpendicular.
62. Prove the following.
    $\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v}$
63. Prove the following Properties of the Dot Product.
    (a) $\mathbf{0} \cdot \mathbf{v} = 0$
    (b) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
    (c) $c(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot c\mathbf{v}$
64. Prove that if $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $c\mathbf{v} + d\mathbf{w}$ for any scalars c and d.

Review

In Exercises 65–68, find the exact solutions of the equation in the interval $[0, 2\pi)$.

65. $\sin 2x - \sqrt{3} \sin x = 0$
66. $\sin 2x + \sqrt{2} \cos x = 0$
67. $2 \tan x = \tan 2x$
68. $\cos 2x - 3 \sin x = 2$

In Exercises 69–72, find the exact value of the trigonometric function given that $\sin u = -\frac{12}{13}$ and $\cos v = \frac{24}{25}$. (Both u and v are in Quadrant IV.)

69. $\sin(u - v)$
70. $\sin(u + v)$
71. $\cos(v - u)$
72. $\tan(u - v)$

▶ Real-Life Applications

- A wide variety of real-life applications, many using current, real data, are integrated throughout examples and exercises.
- The icon indicates an example that involves a real-life application.

Mathematical Modeling

Sine and cosine functions can be used to model many real-life situations, including electric currents, musical tones, radio waves, tides, sunrises, and weather patterns.

Example 7 ▶ Finding a Trigonometric Model

Throughout the day, the depth of water at the end of a dock varies with the tides. The table shows the depths (in meters) at various times during the morning.

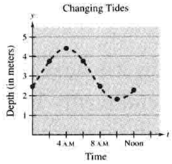


FIGURE 1.47

t (time)	Midnight	2 A.M.	4 A.M.	6 A.M.	8 A.M.	10 A.M.	Noon
y (depth)	2.55	3.80	4.40	3.80	2.55	1.80	2.27

a. Use a trigonometric function to model this data.
b. Find the depths at 9 A.M. and 3 P.M.
c. A boat needs at least 3 meters of water to moor at the dock. During what times in the afternoon can it safely dock?

Solution

a. Begin by graphing the data, as shown in Figure 1.47. You can use either a sine or cosine model. Suppose you use a cosine model of the form

$$y = a\cos(bt - c) + d.$$

The amplitude is given by

$$a = \frac{1}{2}[(\text{high}) - (\text{low})] = \frac{1}{2}(4.4 - 1.8) = 1.3.$$

The period is

$$p = 2[(\text{low time}) - (\text{high time})] = 2(10 - 4) = 12$$

which implies that $b = 2\pi/p \approx 0.524$. Because high tide occurs 4 hours after midnight, you can conclude that $c/b = 4$, so $c \approx 2.094$. Moreover, because the average depth is $\frac{1}{2}(4.4 + 1.8) = 3.1$, it follows that $d = 3.1$. So, you can model the depth with the function

$$y = 1.3\cos(0.524t - 2.094) + 3.1.$$

b. The depths at 9 A.M. and 3 P.M. are as follows.

$$y = 1.3\cos(0.524 \cdot 9 - 2.094) + 3.1 \approx 1.97 \text{ meters} \quad \text{9 A.M.}$$

$$y = 1.3\cos(0.524 \cdot 15 - 2.094) + 3.1 \approx 4.23 \text{ meters} \quad \text{3 P.M.}$$

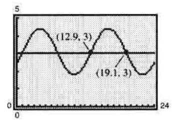


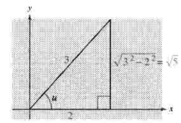
FIGURE 1.48

c. To find out when the depth y is at least 3 meters, you can graph the model with the line $y = 3$, as shown in Figure 1.48. From the graph, it follows that the depth is at least 3 meters between 12:54 P.M. ($t \approx 12.9$) and 7:06 P.M. ($t \approx 19.1$).

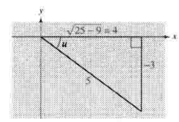
▶ Algebra of Calculus

- Special emphasis is given to the algebraic techniques used in calculus.
- Algebra of Calculus examples and exercises are integrated throughout the text.
- The symbol indicates an example or exercise in which the Algebra of Calculus is featured.

Example 6 shows how to use right triangles to find exact values of compositions of inverse functions. Then, Example 7 shows how to use triangles to convert a trigonometric expression into an algebraic expression. This conversion technique is used frequently in calculus.



(a)



(b)

FIGURE 1.65

Example 6 ▶ Evaluating Compositions of Functions

Find the exact value.

a. $\tan\left(\arccos\frac{2}{3}\right)$ **b.** $\cos\left[\arcsin\left(-\frac{3}{5}\right)\right]$

Solution

a. If you let $u = \arccos\frac{2}{3}$, then $\cos u = \frac{2}{3}$. Because $\cos u$ is positive, u is a *first*-quadrant angle. You can sketch and label angle u as shown in Figure 1.65(a). Consequently,

$$\tan\left(\arccos\frac{2}{3}\right) = \tan u = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{5}}{2}.$$

b. If you let $u = \arcsin\left(-\frac{3}{5}\right)$, then $\sin u = -\frac{3}{5}$. Because $\sin u$ is negative, u is a *fourth*-quadrant angle. You can sketch and label angle u as shown in Figure 1.65(b). Consequently,

$$\cos\left[\arcsin\left(-\frac{3}{5}\right)\right] = \cos u = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}.$$

Example 7 ▶ Some Problems from Calculus

Write each of the following as an algebraic expression in x.

a. $\sin(\arccos 3x),\ 0 \le x \le \frac{1}{3}$ **b.** $\cot(\arccos 3x),\ 0 \le x \le \frac{1}{3}$

Solution

If you let $u = \arccos 3x$, then $\cos u = 3x$. Because

$$\cos u = \frac{3x}{1} = \frac{\text{adj}}{\text{hyp}}$$

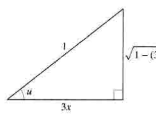


FIGURE 1.66

you can sketch a right triangle with acute angle u, as shown in Figure 1.66. From this triangle, you can easily convert each expression to algebraic form.

a. $\sin(\arccos 3x) = \sin u = \frac{\text{opp}}{\text{hyp}} = \sqrt{1 - 9x^2},\quad 0 \le x \le \frac{1}{3}$

b. $\cot(\arccos 3x) = \cot u = \frac{\text{adj}}{\text{opp}} = \frac{3x}{\sqrt{1 - 9x^2}},\quad 0 \le x \le \frac{1}{3}$

In Example 7, similar arguments can be made for x-values lying in the interval $\left[-\frac{1}{3}, 0\right]$.

The symbol indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

FEATURES

Flexibility and Accessibility

Exponential Growth and Decay

Example 1 ▶ Population Increase

Estimates of the world population (in millions) from 1992 through 2000 are shown in the table. The scatter plot of the data is shown in Figure 5.23. (Source: U.S. Bureau of the Census, International Data Base)

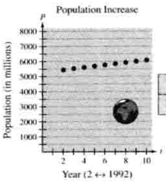


FIGURE 5.23

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Population	5445	5527	5607	5688	5767	5847	5926	6005	6083

An exponential growth model that approximates this data is

$$P = 5304e^{0.013819t}, \quad 2 \le t \le 10$$

where P is the population (in millions) and $t = 2$ represents 1992. Compare the values given by the model with the estimates given by the U.S. Bureau of the Census. According to this model, when will the world population reach 6.5 billion?

Solution

The following table compares the two sets of population figures. The graph of the model is shown in Figure 5.24.

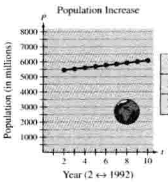


FIGURE 5.24

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Population	5445	5527	5607	5688	5767	5847	5926	6005	6083
Model	5453	5529	5605	5683	5763	5843	5924	6006	6090

To find when the world population will reach 6.5 billion, let $P = 6500$ in the model and solve for t.

$$5304e^{0.013819t} = P \qquad \text{Write original model.}$$
$$5304e^{0.013819t} = 6500 \qquad \text{Let } P = 6500.$$
$$e^{0.013819t} \approx 1.22549 \qquad \text{Divide each side by 5304.}$$
$$\ln e^{0.013819t} \approx \ln 1.22549 \qquad \text{Take natural log of each side.}$$
$$0.013819t \approx 0.203341 \qquad \text{Inverse Property}$$
$$t \approx 14.71 \qquad \text{Divide each side by 0.013819}$$

According to the model, the world population will reach 6.5 billion in 2004.

Technology Some graphing utilities have curve-fitting capabilities that can be used to find models that represent data. If you have such a graphing utility, try using it to find a model for the data given in Example 1. How does your model compare with the model given in Example 1?

An exponential model increases (or decreases) by the same percent each year. What is the annual percent increase for the model in Example 1?

Exploration

- Before introduction of selected topics, *Exploration* engages students in active discovery of mathematical concepts and relationships, often through the power of technology.
- *Exploration* strengthens students' critical thinking skills and helps them develop an intuitive understanding of theoretical concepts.
- *Exploration* is an optional feature and can be omitted without loss of continuity in coverage.

▶ Additional Features

Carefully crafted learning tools designed to create a rich learning environment can be found throughout the text. These learning tools include Study Tips, Historical Notes, Writing About Mathematics, Chapter Projects, Chapter Review Exercises, Chapter Tests, Cumulative Tests, and an extensive art program.

Technology

- Point-of-use instructions for using graphing utilities appear in the margins, encouraging the use of graphing technology as a tool for visualization of mathematical concepts, for verification of other solution methods, and for facilitation of computations.
- The use of technology is optional in this text. This feature and related exercises can easily be omitted without loss of continuity in coverage. Exercises that require the use of a graphing utility are identified by the symbol ▦.

▶ Examples

- Each example was carefully chosen to illustrate a particular mathematical concept or problem-solving skill.
- All examples contain step-by-step solutions, most with side-by-side explanations that lead students through the solution process.

Exploration

Complete the table:

$i^1 = i$	$i^7 =$
$i^2 = -1$	$i^8 =$
$i^3 = -i$	$i^9 =$
$i^4 = 1$	$i^{10} =$
$i^5 =$	$i^{11} =$
$i^6 =$	$i^{12} =$

What pattern do you see? Write a brief description of how you would find i raised to any positive integer power.

Many of the properties of real numbers are valid for complex numbers as well. Here are some examples.

Associative Properties of Addition and Multiplication
Commutative Properties of Addition and Multiplication
Distributive Property of Multiplication Over Addition

Notice below how these properties are used when two complex numbers are multiplied.

$$(a + bi)(c + di) = a(c + di) + bi(c + di) \qquad \text{Distributive Property}$$
$$= ac + (ad)i + (bc)i + (bd)i^2 \qquad \text{Distributive Property}$$
$$= ac + (ad)i + (bc)i + (bd)(-1) \qquad i^2 = -1$$
$$= ac - bd + (ad)i + (bc)i \qquad \text{Commutative Property}$$
$$= (ac - bd) + (ad + bc)i \qquad \text{Associative Property}$$

Rather than trying to memorize this multiplication rule, you should simply remember how the Distributive Property is used to multiply two complex numbers. The procedure is similar to multiplying two polynomials and combining like terms (as in the FOIL Method).

The *Interactive* CD-ROM and *Internet* versions of this text offer a built-in graphing calculator, which can be used in the Examples, Explorations, Technology notes, and Exercises.

Example 2 ▶ Multiplying Complex Numbers

a. $4(-2 + 3i) = 4(-2) + 4(3i)$ Distributive Property
$= -8 + 12i$ Simplify.

b. $(i)(-3i) = -3i^2$ Multiply.
$= -3(-1)$ $i^2 = -1$
$= 3$ Simplify.

c. $(2 - i)(4 + 3i) = 8 + 6i - 4i - 3i^2$ Product of binomials
$= 8 + 6i - 4i - 3(-1)$ $i^2 = -1$
$= (8 + 3) + (6i - 4i)$ Group like terms.
$= 11 + 2i$ Write in standard form.

d. $(3 + 2i)(3 - 2i) = 9 - 6i + 6i - 4i^2$ Product of binomials
$= 9 - 4(-1)$ $i^2 = -1$
$= 9 + 4$ Simplify.
$= 13$ Write in standard form.

e. $(3 + 2i)^2 = 9 + 6i + 6i + 4i^2$ Product of binomials
$= 9 + 4(-1) + 12i$ $i^2 = -1$
$= 9 - 4 + 12i$ Simplify.
$= 5 + 12i$ Write in standard form.

Supplements

Resources

Website (*college.hmco.com*)
Many additional text-specific study and interactive features for students and instructors can be found at the Houghton Mifflin website. These features include, but are not limited to, the following.

- Glossary
- Video clips
- Graphing calculator emulator
- Sample chapters
- Presentation slides

For the Student

Student Success Organizer

Study and Solutions Guide by Dianna L. Zook (Indiana University/Purdue University–Fort Wayne)

Graphing Technology Guide by Benjamin N. Levy and Laurel Technical Services

Instructional Videotapes by Dana Mosely

Instructional Videotapes for Graphing Calculators by Dana Mosely

For the Instructor

Instructor's Annotated Edition

Instructor Success Organizer

Complete Solutions Guide by Dianna L. Zook (Indiana University/Purdue University–Fort Wayne), Laurel Technical Services, and Mike Jones

Test Item File

Problem Solving, Modeling, and Data Analysis Labs by Wendy Metzger (Palomar College)

Computerized Testing (Windows, Macintosh)

Instructor's CD-ROM

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If you have suggestions for improving this text, please feel free to write to us. Over the past two decades we have received many useful comments from both instructors and students, and we value these comments very much.

Ron Larson
Robert P. Hostetler

An Introduction to Graphing Utilities

Graphing utilities such as graphing calculators and computers with graphing software are very valuable tools for visualizing mathematical principles, verifying solutions to equations, exploring mathematical ideas, and developing mathematical models. Although graphing utilities are extremely helpful in learning mathematics, their use does not mean that learning algebra is any less important. In fact, the combination of knowledge of mathematics and the use of graphing utilities allows you to explore mathematics more easily and to a greater depth. If you are using a graphing utility in this course, it is up to you to learn its capabilities and to practice using this tool to enhance your mathematical learning.

In this text there are many opportunities to use a graphing utility, some of which are described below.

Some Uses of a Graphing Utility

A graphing utility can be used to

- check or validate answers to problems obtained using algebraic methods.
- discover and explore algebraic properties, rules, and concepts.
- graph functions, and approximate solutions to equations involving functions.
- efficiently perform complicated mathematical procedures such as those found in many real-life applications.
- find mathematical models for sets of data.

In this introduction, the features of graphing utilities are discussed from a generic perspective. To learn how to use the features of a specific graphing utility, consult your user's manual or the website for this text found at *college.hmco.com*. Additionally, keystroke guides are available for most graphing utilities, and your college library may have a videotape on how to use your graphing utility.

The Equation Editor

Many graphing utilities are designed to act as "function graphers." In this course, you will study functions and their graphs in detail. You may recall from previous courses that a function can be thought of as a rule that describes the relationship between two variables. These rules are frequently written in terms of x and y. For example, the equation $y = 3x + 5$ represents y as a function of x.

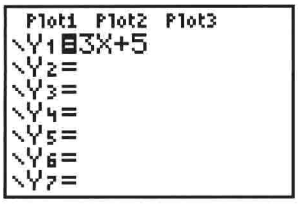

FIGURE 1

Many graphing utilities have an equation editor that requires an equation to be written in "$y =$" form in order to be entered, as shown in Figure 1. (You should note that your equation editor screen may not look like the screen shown in Figure 1.) To determine exactly how to enter an equation into your graphing utility, consult your user's manual.

The Table Feature

Most graphing utilities are capable of displaying a table of values with x-values and one or more corresponding y-values. These tables can be used to check solutions of an equation and to generate ordered pairs to assist in graphing an equation.

To use the *table* feature, enter an equation into the equation editor in "$y =$" form. The table may have a setup screen, which allows you to select the starting x-value and the table step or x-increment. You may then have the option of automatically generating values for x and y or building your own table using the *ask* mode. In the *ask* mode, you enter a value for x and the graphing utility displays the y-value.

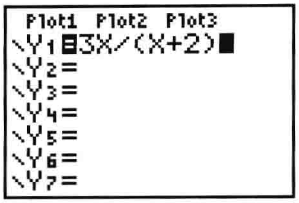

FIGURE 2

For example, enter the equation

$$y = \frac{3x}{x + 2}$$

into the equation editor, as shown in Figure 2. In the table setup screen, set the table to start at $x = -4$ and set the table step to 1. When you view the table, notice that the first x-value is -4 and each value after it increases by 1. Also notice that the Y_1 column gives the resulting y-value for each x-value, as shown in Figure 3. The table shows that the y-value when $x = -2$ is ERROR. This means that the variable x may not take on the value -2 in this equation.

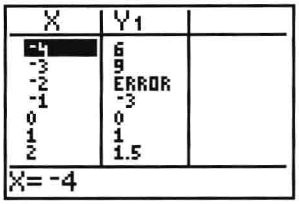

FIGURE 3

With the same equation in the equation editor, set the table to *ask* mode. In this mode you do not need to set the starting x-value or the table step, because you are entering any value you choose for x. You may enter any real value for x—an integer, fraction, decimal, irrational number, and so forth. If you enter $x = 1 + \sqrt{3}$, the graphing utility may rewrite the number as a decimal approximation, as shown in Figure 4. You can continue to build your own table by entering additional x-values in order to generate y-values.

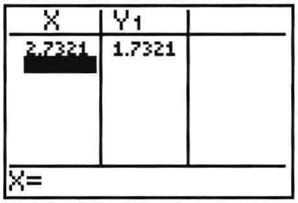

FIGURE 4

If you have several equations in the equation editor, the table may generate y-values for each equation.

Creating a Viewing Window

A **viewing window** for a graph is a rectangular portion of the coordinate plane. A viewing window is determined by the following six values.

Xmin = the smallest value of x
Xmax = the largest value of x
Xscl = the number of units per tick mark on the x-axis
Ymin = the smallest value of y
Ymax = the largest value of y
Yscl = the number of units per tick mark on the y-axis

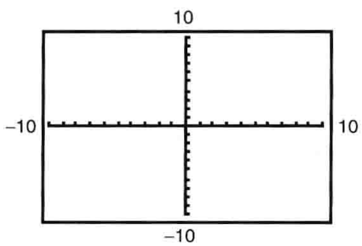

FIGURE 5

When you enter these six values into a graphing utility, you are setting the viewing window. Some graphing utilities have a standard viewing window, as shown in Figure 5.

By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure 6 shows four different viewing windows for the graph of

$$y = 0.1x^4 - x^3 + 2x^2.$$

Of these, the view shown in part (a) is the most complete.

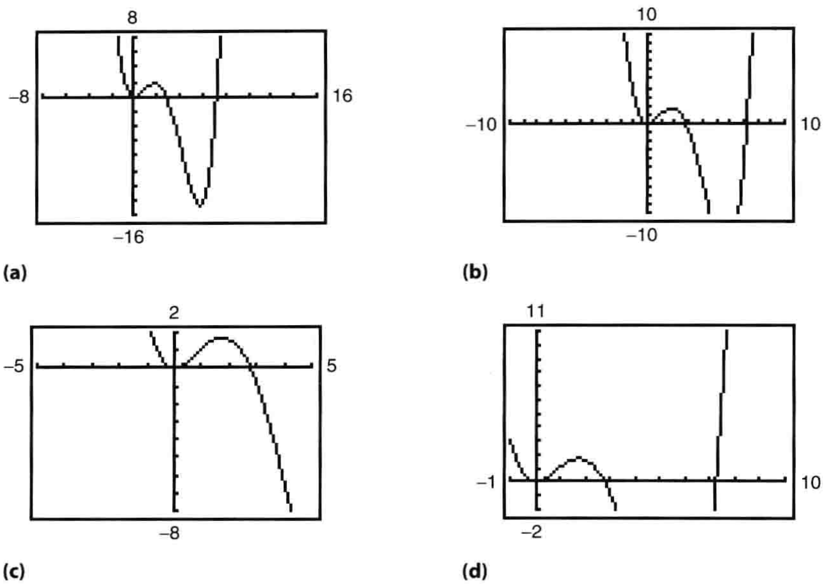

FIGURE 6

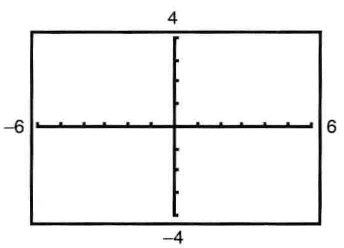

FIGURE 7

On most graphing utilities, the display screen is two-thirds as high as it is wide. On such screens, you can obtain a graph with a true geometric perspective by using a **square setting**—one in which

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{2}{3}.$$

One such setting is shown in Figure 7. Notice that the x and y tick marks are equally spaced on a square setting, but not on a standard setting.

To see how the viewing window affects the geometric perspective, graph the semicircles $y_1 = \sqrt{9 - x^2}$ and $y_2 = -\sqrt{9 - x^2}$ in a standard viewing window. Then graph y_1 and y_2 in a square window. Note the difference in the shapes of the circles.

Zoom and Trace Features

When you graph an equation, you can move from point to point along its graph using the *trace* feature. As you trace the graph, the coordinates of each point are displayed, as shown in Figure 8. The *trace* feature combined with the *zoom* feature allows you to obtain better and better approximations of desired points on a graph. For instance, you can use the *zoom* feature of a graphing utility to approximate the x-intercept(s) of a graph [the point(s) where the graph crosses the x-axis]. Suppose you want to approximate the x-intercept(s) of the graph of $y = 2x^3 - 3x + 2$.

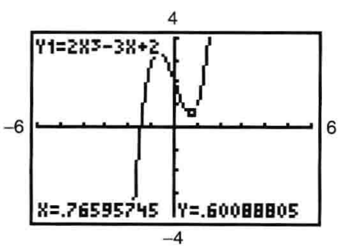

FIGURE 8

Begin by graphing the equation, as shown in Figure 9(a). From the viewing window shown, the graph appears to have only one x-intercept. This intercept lies between -2 and -1. By zooming in on the intercept, you can improve the approximation, as shown in Figure 9(b). To three decimal places, the solution is $x \approx -1.476$.

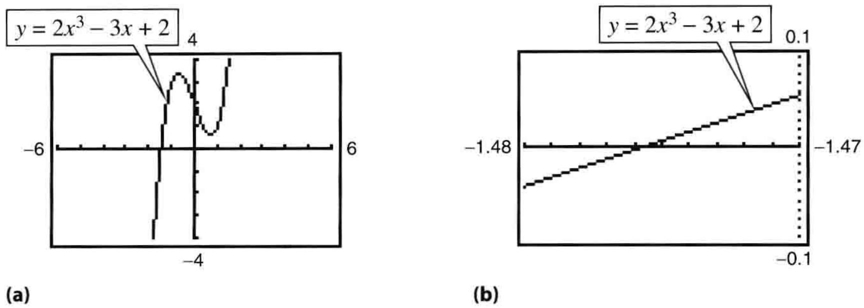

FIGURE 9

Here are some suggestions for using the *zoom* feature.

1. With each successive zoom-in, adjust the x-scale so that the viewing window shows at least one tick mark on each side of the x-intercept.
2. The error in your approximation will be less than the distance between two scale marks.
3. The *trace* feature can usually be used to add one more decimal place of accuracy without changing the viewing window.

Figure 10(a) shows the graph of $y = x^2 - 5x + 3$. Figures 10(b) and 10(c) show "zoom-in views" of the two x-intercepts. From these views, you can approximate the x-intercepts to be $x \approx 0.697$ and $x \approx 4.303$.

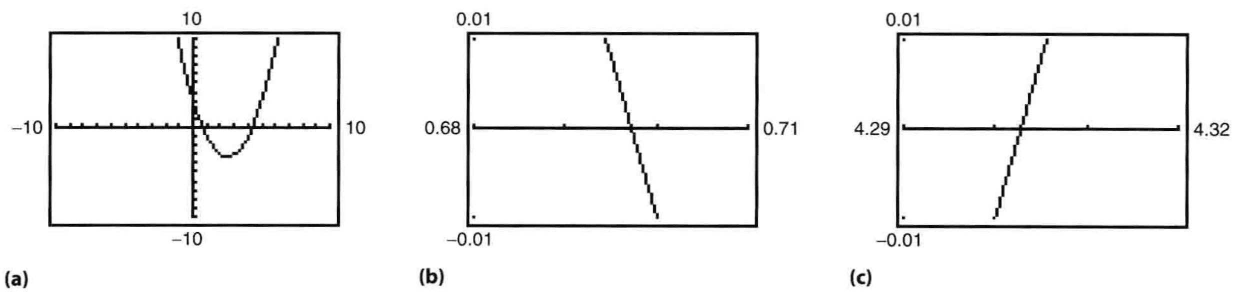

FIGURE 10

Zero or Root Feature

Using the *zero* or *root* feature, you can find the real zeros of functions of the various types studied in this text—polynomial, exponential, logarithmic, and trigonometric functions. To find the zeros of a function such as $f(x) = \frac{3}{4}x - 2$, first enter the function as $y_1 = \frac{3}{4}x - 2$. Then use the *zero* or *root* feature, which may require entering lower and upper bound estimates of the root, as shown in Figure 11.

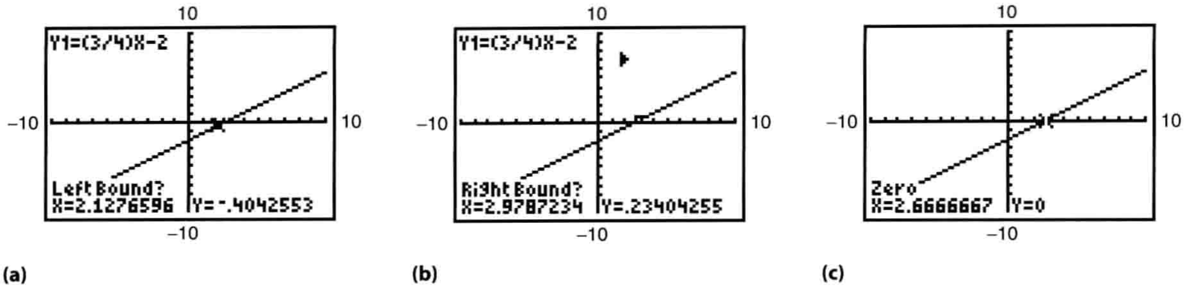

FIGURE 11

In Figure 11(c), you can see that the zero is $x = 2.6666667 \approx 2\frac{2}{3}$.

Intersect Feature

To find the points of intersection of two graphs, you can use the *intersect* feature. For instance, to find the points of intersection of the graphs of $y_1 = -x + 2$ and $y_2 = x + 4$, enter these two functions and use the *intersect* feature, as shown in Figure 12.

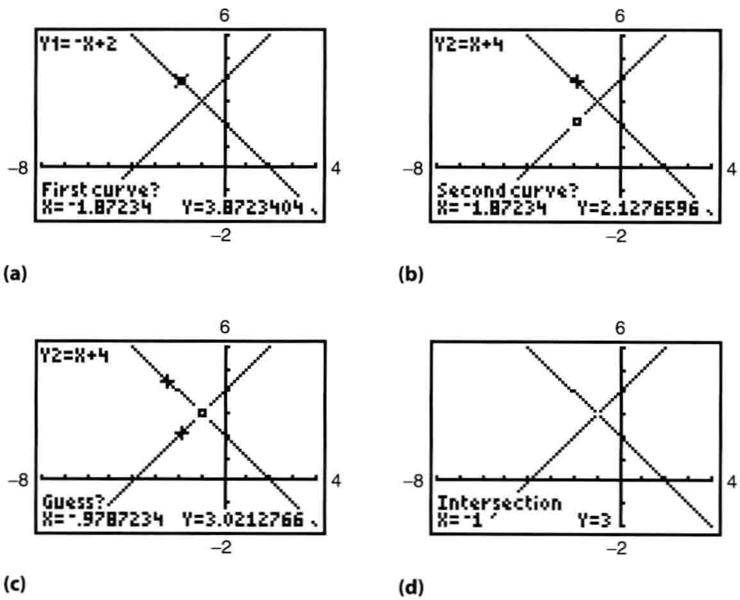

FIGURE 12

From Figure 12(d), you can see that the point of intersection is $(-1, 3)$.