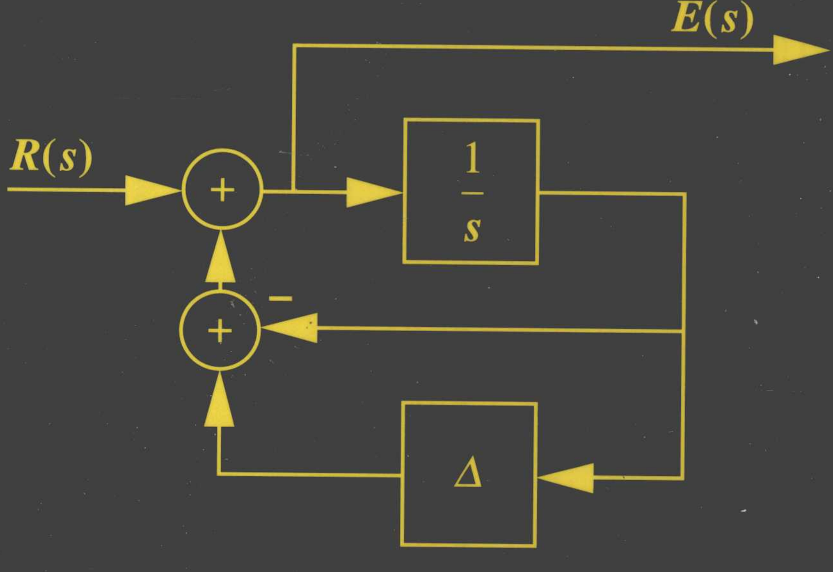
Linear Optimal Control

$\mathcal{H}_2$ and $\mathcal{H}_\infty$ Methods



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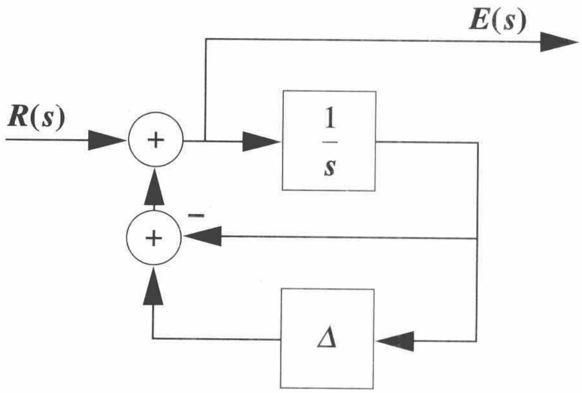

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PREFACE

The last ten to fifteen years have seen a resurgence of interest in control systems designed to meet specifications for robustness and disturbance rejection. Robust stability and robust performance tests, based on the structured singular value, have been developed. The robustness of the linear quadratic regulator and the linear quadratic Gaussian controllers have been analyzed. Loop transfer recovery was developed as a means to improve the robustness properties of the linear quadratic Gaussian controller. $\mathcal{H}_\infty$ control was developed to provide a means of incorporating frequency-domain specifications in control system designs. $\mathcal{H}_\infty$ control also provides an *ad hoc* means of incorporating robustness specifications into control system designs. Finally, μ-synthesis was developed as a powerful procedure for designing control systems that satisfy both robustness and performance specifications.

This book is being written to accomplish a number of tasks. First and foremost, I wish to make the new material on robustness, $\mathcal{H}_\infty$ control, and μ-synthesis accessible to both students and professionals. Second, I wish to combine these new results with previous work on optimal control to form a more complete picture of control system design and analysis. Third, I wish to incorporate recent results on robust stability and robust performance analysis into the presentation of linear quadratic Gaussian optimal control. Lastly, I wish to acquaint the reader with the CAD tools available for robust optimal controller design.

Special Features

This book has been written to provide students and professionals with access to relatively recent research results on robustness analysis, $\mathcal{H}_\infty$ optimal control, and μ-synthesis. In addition, this material is integrated with linear quadratic Gaussian ($\mathcal{H}_2$) optimal control results. The overall treatment is organized in a logical manner rather than along the lines of historical development. A number of more specific features enhance the value of this book as a teaching text.

The results and derivations are simplified by treating special cases whenever this can be done without compromising the clarity of results or methods. In addition, mathematical developments that provide little insight into key derivations, results, and/or applications are relegated to the appendix. This approach allows the reader to develop a solid grounding in the basics before tackling the mathematical subtleties required to derive the most general results. Practicing engineers can then augment their understanding using more advanced books and research papers, or use computer-aided design software to handle the more general cases. While this approach does simplify the derivations, the mathematical level of this text is still quite challenging.

The solutions of both the $\mathcal{H}_2$ (linear quadratic Guassian) control problem and the $\mathcal{H}_\infty$ control problem are based on a common variational approach. The variational approach adds more insight into the optimization process than completing the square. Using a common variational approach in both of these problems also tends to demystify the $\mathcal{H}_\infty$ theory.

The use of computer-aided design (CAD) tools is integrated into the presentation and problems. The CAD software employed is MATLAB® with the Control System Toolbox and the μ-Synthesis and Analysis Toolbox. Software is available via ftp for almost every example in the text. For examples that are done analytically, software is included for numerically checking the result. These software programs and their documentation provide a significant learning resource (and also a significant reference source), since virtually all optimal controller design and analysis is performed with the aid of CAD software.

A general treatment of performance, including transient performance, tracking performance, and disturbance rejection, is given up front along with a treatment of robustness. This organization provides a solid foundation in control system analysis. A thorough performance and robustness analysis can then be performed on the controllers developed subsequently. This approach develops an understanding of what each design does well and what each design does poorly, as opposed to simply showing that a design is optimal.

Tracking and disturbance rejection are presented in the linear quadratic Gaussian setting, as opposed to the linear quadratic regulator setting. Presenting this material in the linear quadratic regulator setting leaves many students unsure of how to incorporate estimation within a tracking system design or within a design tailored for disturbance rejection. The organization of this book solves this problem by treating the idiosyncrasies involving estimation in these systems.

The book concludes with a case study that compares a design obtained via linear quadratic Gaussian-loop transfer recovery with a design obtained via μ-synthesis. The insight gained through this comparison yields a better understanding of both design methodologies, and provides guidelines on when to apply which design method.

Computer exercises are included for each chapter. These computer exercises develop familiarity with current CAD software and allow the exploration of design options and "what if" questions concerning the results. My students have frequently told me that a significant part of their learning comes as a result of performing the computer exercises.

A symbol list is included to help the reader with the notation. When writing this text, I became painfully aware that the English language has only twenty-six letters, and the Greek language has even fewer. While much effort has been made to keep the notation simple and consistent, of necessity some symbols appear in multiple roles. In general, I have rendered time-domain functions in lowercase. Laplace-domain and Fourier-domain functions are rendered in uppercase. These practices are consistent with most introductory control texts and should be familiar to the reader. Matrices are given in bold type, while both scalars and vectors are not bolded. Most introductory books bold both vectors and matrices, but almost everything in this text is a vector, so I have instead used bolding to highlight matrices, a practice I feel will better serve the reader. There are some exceptions to these basic formatting rules in order to make the results in this book match those appearing widely in the open literature. In these cases, the notation used should be clearly delineated and not lead to confusion.

Supplemental material consists of a solutions manual available to instructors and software used for the examples, which is available via ftp at ftp://ftp.aw.com/cseng/authors/burl/loc/mfiles.

Prerequisites

Prerequisites include an introduction to control systems (classical control), probability, state-space linear systems, and a working knowledge of linear algebra. In addition, an introduction to random processes is desirable.

Classical control, probability, and some linear algebra are part of the undergraduate education of most incoming engineering graduate students. An introduction to state-space linear systems is typically accomplished (along with converting a linear algebra background into working knowledge) by an introductory graduate course from a book such as Chen or Kailath. This book begins with a review of the relevant state-space linear systems material in a multivariable setting (Chapter 2). I usually only present the sections on singular value decomposition, principle gains, and internal stability, since this material is new to most of my students. But I recommend that my students skim the remainder of Chapter 2 as a review. I then ask them to inform me of any topics with which they are not familiar, and I provide references, when necessary, to bring students up to speed.

The book contains a terse but self-contained introduction to random processes (Chapter 3). This chapter also contains many state-space random process results that are not typically included in introductory random process courses. Therefore, I usually cover this chapter thoroughly.

Organization

This book consists of three parts. The first part covers the analysis of control systems. It contains a review of multivariable linear systems (Chapter 2), an introduction to vector random processes (Chapter 3), control system performance analysis (Chapter 4), and robustness analysis (Chapter 5). The performance analysis chapter includes transient performance analysis, tracking system analysis, and disturbance rejection. Cost functions are also presented as a means of quantifying performance analysis.

The robustness analysis chapter begins with a review of the Nyquist stability criterion. The Nyquist plot is used to develop the gain margin, the phase margin, and the downside gain margin. The stability robustness interpretation of these classical control stability margins is clearly illustrated. The small-gain theorem is presented as a means of determining stability robustness to unstructured perturbations. The structured singular value is then presented as a means of determining both stability and performance robustness to more general structured perturbations.

The second part of this book is devoted to $\mathcal{H}_2$, (i.e., linear quadratic Gaussian) optimal control. This part is divided into the linear quadratic regulator (Chapter 6), Kalman filtering (Chapter 7), and linear quadratic Gaussian control (Chapter 8).

The chapter on the linear quadratic regulator (LQR) begins with a brief introduction to optimization using variational theory. The results are then used to derive the LQR, both time-varying and steady-state. Application of the LQR is discussed along with cost function selection. Performance and robustness of the steady-state LQR are evaluated in some detail.

The chapter on Kalman filtering begins with an introduction to minimum mean square estimation theory and the orthogonality principle. The Kalman filter, both time-varying and steady-state, is then developed. Application of the Kalman filter is discussed in some detail. Kalman filter performance and robustness are also discussed.

The chapter on linear quadratic Gaussian (LQG) control begins with the development of the stochastic separation principle leading to the structure of the LQG controller. Performance and robustness of the LQG control system are discussed. Loop transfer recovery is presented as a means of increasing LQG robustness when needed. Tailoring the LQG control system for tracking and disturbance rejection is also discussed in this chapter.

The last part of this book is devoted to $\mathcal{H}_\infty$ control. Chapter 9 begins with an introduction to differential games. Differential game theory is used to derive the solution of the suboptimal $\mathcal{H}_\infty$ full information controller. The $\mathcal{H}_\infty$ output estimator is then derived using duality.

The $\mathcal{H}_\infty$ output feedback controller is presented in Chapter 10. The application of this controller to tracking systems, disturbance rejection, and robustness optimization is discussed in detail. The $\mathcal{D}$-$\mathcal{K}$ iteration algorithm for μ-synthesis is then presented. A significant case study is presented as a means of contrasting the μ-synthesis and the LQG loop transfer recovery design methodologies.

The generation of reduced-order controllers is presented in the final chapter. This chapter begins by showing that reduced-order controller approximation can often be evaluated using a frequency-weighted ∞-norm. The general properties of a desirable reduced-order approximation are then gleaned from a few examples. Pole-zero truncation and balance truncation are both presented as methods of generating reduced-order controllers.

Usage

This book is recommended for use as a text in a two-semester sequence covering linear optimal control. The entire book can be covered thoroughly in two semesters. A two-quarter course can also be formed from this material if the students are well prepared. The two-quarter course would also necessitate that the material be covered in less depth.

A one-semester course that covers robust optimal control can also be based on the material in this book. Such a course would be composed of a review of Chapter 2 followed by a thorough treatment of Chapters 4, 5, 9, 10, and 11.

An additional one-semester course on linear quadratic Gaussian control can be taught using this material. This course would be composed of a review of Chapter 2, a thorough treatment of Chapter 3, the cost function presentation in Chapter 4, the material on unstructured perturbations and the small-gain theorem in Chapter 5, and a thorough treatment of Chapters 6, 7, and 8.

Acknowledgments

I would like to express my appreciation to several people who contributed to this project. I am greatly indebted to Roberto Cristi for providing me with many useful suggestions associated with his use of a preliminary version of this text in classes at the Naval Postgraduate School. I would also like to thank all my students at Michigan Technological University for finding errors (big and small) and for letting me know when an explanation was unsatisfactory. I am especially indebted to two students, Willem Van Marian and John Pakkala, who used special diligence in finding errors in earlier versions of this manuscript. My appreciation is also extended to the reviewers: Roberto Cristi, Naval Postgraduate School; Faryar Jabarri, University of California, Irvine; Charles P. Neuman, Carnegie Mellon University; and Ronald A. Perez, University of Wisconsin-Milwaukee for their valuable suggestions. Last, but not least, I wish to thank the editorial and production staff at Addison-Wesley, especially Royden Tonomura, Paul Becker, and Kamila Storr for their help in improving the manuscript and in bringing it to production.

LIST OF SYMBOLS

Symbol	Meaning
$1(t)$	Unit step function
α	Real part of pole, coefficient
$\alpha(s)$	Characteristic equation
$\mathbf{A}(t)$, $\mathbf{A}$	State matrix
a, b	First-order shaping filter coefficients
$\mathbf{A}_c$	Controller state matrix
$\mathbf{A}_{cl}$	Closed-loop system state matrix
adj(•)	Adjugate of a matrix
$\mathbf{A}_f$	State matrix for shaping filter
$\mathcal{B}$	Linear space on which a norm is defined (Banach space)
$\mathbf{B}(t)$, $\mathbf{B}$	Input matrix
$\mathbf{B}_{cl}$	Closed-loop system input matrix
$\mathbf{B}_{cr}$	Controller input matrix for the input r
$\mathbf{B}_f$	Input matrix for shaping filter
$\mathcal{BW}$	Bandwidth
$\mathbf{B}_w$	Input matrix for the input w
$\mathbf{B}_{w_0}$	Input matrix associated with a constant input
$c(t)$	Correction term in unbiased estimator
$\mathbf{C}(t)$, $\mathbf{C}$	Output matrix
$c(\bullet, \bullet, \ldots)$	Constraint equation
$\mathbf{C}_c$	Controller output matrix
$\mathbf{C}_{cl}$	Closed-loop system output matrix
$\mathbf{C}_e$	Output matrix associated with the output error
$\mathbf{C}_f$	Output matrix for shaping filter
$\mathbf{C}_x(\tau)$	Covariance function of the stationary random process $x(t)$
$\mathbf{C}_x(t_1, t_2)$	Covariance function of the random process $x(t)$
$\mathbf{C}_y$	Output matrix for the output y
$\mathbf{C}_z$	Output matrix for constructed measurement
$\mathcal{C}^{m \times n}$	Set of complex $m \times n$ matrices
$\mathcal{C}^n$	Set of complex n-dimensional vectors
$d(t)$	Output disturbance
$\mathbf{D}(t)$, $\mathbf{D}$	Input-to-output coupling matrix
$\mathbf{D}_{cl}$	Closed-loop system input-to-output coupling matrix
$\mathbf{D}_{cr}$	Controller input-to-output coupling matrix from r to u
den(s)	Transfer function denominator

Symbol	Meaning
$\det(\bullet)$	Determinant of a matrix
$\mathbf{D}_{yw}$	Input-to-output coupling matrix from w to y
$\mathfrak{D}_L(s)$	Diagonal scaling matrix (left $\mathfrak{D}$-scaling matrix)
$\mathfrak{D}_R(s)$	Diagonal scaling matrix (right $\mathfrak{D}$-scaling matrix)
$\mathbf{\Delta}(s)$	Normalized general perturbation
$\mathbf{\Delta}'(s)$	Unnormalized general perturbation
$\mathbf{\Delta}_a(s)$	Additive perturbation
$\mathbf{\Delta}_{fi}(s)$	Input feedback perturbation
$\mathbf{\Delta}_{fo}(s)$	Output feedback perturbation
$\mathbf{\Delta}_{G_{yr}}$	Change in the closed-loop transfer function due to a perturbation
$\mathbf{\Delta}_i(s)$	Input-multiplicative perturbation
$\mathbf{\Delta}_{\max}(j\omega)$	Frequency-dependent perturbation bound
$\mathbf{\Delta}_o(s)$	Output multiplicative perturbation
$\mathbf{\Delta}_p$	Normalized perturbation augmented with performance block
$\tilde{\mathbf{\Delta}}$	Specific perturbation
$\bar{\mathbf{\Delta}}$	Set of perturbations with a given block diagonal structure
$\bar{\mathbf{\Delta}}_s$	Set of block diagonal perturbations (square blocks)
$\Delta J(\bullet, \delta\bullet)$	Increment of J
d_i	$\mathfrak{D}$-scale
$\delta J(\bullet, \delta\bullet)$	Variation of J
δx	Variation of x, differential of x
δ_{ij}	Kronecker delta function
$\delta(t)$	Dirac delta (impulse) function
ε	Small, positive constant
$e(t)$	Tracking error, estimation error
e_I	Integral of the tracking error
$E[\bullet]$	Expectation operator
ϕ	Phase perturbation
$\mathbf{\Phi}(t, t_0)$, $\mathbf{\Phi}(t)$	State-transition matrix of a time-varying, time-invariant system
$\mathbf{\Phi}$, $\mathbf{\Phi}(T)$	Discrete-time state matrix
$\mathbf{F}$, $\mathbf{F}(t, \tau)$	Linear estimator weights
$f_{x,y}(x, y)$	Joint density function
$f_{x\|y}(x \mid y)$	Conditional density function
$\phi_{\max}$	Maximum-phase perturbation
$\phi_{\min}$	Minimum-phase perturbation
$f_x(x; t)$	Density function of the random process $x(t)$
g	Uncertain gain
γ	∞-norm performance bound
$\mathbf{G}(s)$	Laplace transfer function of a generic system or a plant
$\mathbf{G}(t)$	Kalman gain
$\mathbf{g}(t)$	Impulse response matrix (generic system)
$\tilde{\mathbf{G}}_c(s)$	Reduced-order controller transfer function
$\mathcal{G}(\bullet)$	Linear system (possibly time-varying)
$\mathbf{\Gamma}$, $\mathbf{\Gamma}(T)$	Discrete-time input matrix
$\mathbf{G}_0(s)$	Nominal plant transfer function
$\mathbf{G}_1(t)$	State feedback gain for normalized measurement
$\mathbf{G}_{ab}(s)$	Transfer function from input b to output a
$\mathbf{G}_{cl}(s)$	Closed-loop system transfer function
$\mathbf{g}_{cl}(s)$	Closed-loop system impulse response
G_{Δ}	System with input $w - \gamma^{-2}\mathbf{B}_w^T\mathbf{P}x$ and output $u + \mathbf{B}_w^T\mathbf{P}x$
$\mathbf{G}_l(s)$	Loop transfer function
GM^+	Gain margin
Gm^-	Downside gain margin

Symbol	Meaning
g_{max}	Maximum uncertain gain
g_{min}	Minimum uncertain gain
$\mathbf{G}_s(s)$	Stable part of a transfer function
$\mathbf{G}_u(s)$	Unstable part of a transfer function
$\mathcal{H}$	Hardy space
$\mathbf{H}(s)$	Perturbed closed-loop system
$\mathbf{I}$	Identity matrix
$\inf(\bullet)$	Infimum
j	Square root of -1, discrete index
$J(\bullet)$	Cost function, objective function
J_2	2-norm cost function
$J_a(\bullet, p)$	Augmented cost function, augmented objective function
J_γ	Objective function for suboptimal control
$J_{\gamma a}$	Augmented objective function for suboptimal control
J_{LQR}	LQR cost function
J_{SR}	Stochastic regulator cost function
J_{SS}	Cost for suboptimal steady-state control
J_{TV}	Cost for time-varying optimal control
k	Discrete time index
$\mathbf{K}(s)$, $\mathbf{K}$	Controller transfer function, controller gain matrix
$\mathbf{K}_r$	Feedforward control gain (tracking input)
$\mathbf{K}_w$	Feedforward control gain (disturbance feedforward)
$\mathbf{\Lambda}$	Diagonal matrix containing eigenvalues
$\mathcal{L}$	Laplace transform
$\mathcal{L}_2$	Space of signals with finite 2-norms
λ	Eigenvalue
$l_i \times n_i$	Matrix dimensions
$\mathbf{L}_c$	Controllability grammian
$\lambda_i(\bullet)$	ith eigenvalue of a matrix
lim	Limit
$\mathbf{L}_o$	Observability grammian
$m(t)$	Measured output
$\max\{\bullet\}$	Maximum operator
$\mathbf{M}_{ij}$	Element of the matrix $\mathbf{M}$ (row i, column j)
$\min\{\bullet\}$	Minimum operator
$m_x(t)$	Mean of the random process $x(t)$
$m_{x\|y}$	Conditional mean
$\tilde{m}(t)$	Kalman innovations process
$\mu_{\bar{\Delta}}(\bullet)$	Structured singular value
$\mathcal{N}$	Observability matrix
n	Discrete-time index, matrix dimension
N_c	Number of encirclements of the point minus one by the Nyquist locus
N_p	Number of right half-plane poles of $\{1 + G(s)K(s)\}$
$\mathbf{N}_s$	Square version of a matrix
$\mathbf{N}(s)$	Nominal closed-loop transfer function (standard form)
num(s)	Transfer function numerator
n_v	Dimension of the vector v
N_z	Number of right half-plane zeros of $\{1 + G(s)K(s)\}$
$\mathbf{P}(s)$	Plant transfer function in standard form
$\mathbf{P}(t)$, $\mathbf{P}$	Riccati solution (LQR, $\mathcal{H}_\infty$ full information control)
p, $p(t)$	Lagrange multiplier
$p_1(t)$	Pulse function
p_i	Poles of a system

Symbol	Meaning
PM	Phase margin
$\mathfrak{Q}$	Controllability matrix
θ	Angle, phase
$\mathbf{Q}(t)$	State weighting function
$\mathbf{Q}_R$	Combined state and control weighting matrix
$\mathbf{R}$	Factor in Cholsky decomposition of controllability grammian
R	Rank
ρ	Measurement weighting coefficient (output LTR)
$\mathbf{R}(t)$	Control weighting function
$r(t)$	Reference input
$\rho(\bullet)$	Spectral radius
rank(•)	Rank of a matrix
Ric(•)	Ricci operator
$\mathbf{R}_x(\tau)$	Correlation function of the stationary random process $x(t)$
$\mathbf{R}_x(t_1, t_2)$	Correlation function of the random process $x(t)$
$\mathbf{R}_w^{(d)}(n)$	Discrete-time correlation function of the random sequence $w(k)$
$\Re^{m \times n}$	Set of real $m \times n$ matrices
$\Re^n$	Set of real n-dimensional vectors
$\mathbf{S}$	Matrix of singular values
s	Laplace variable
σ_i	Singular value
$\bar{\sigma}$	Maximum singular value
$\underline{\sigma}$	Minimum singular value
sup(•)	Supremum
$\mathbf{S}_w(\omega)$	Spectral density of the random process $w(t)$
$\mathbf{\Sigma}_x(t)$	Correlation matrix of the random process $x(t)$
$\mathbf{\Sigma}_{xy}$	Cross-correlation matrix
$\mathbf{\Sigma}_w^{(d)}(n)$	Discrete-time correlation matrix of the random sequence $w(k)$
T	Sampling time (for discrete-time systems)
t	Time
τ	Time variable; time difference in correlation function
$\mathbf{T}$, $\mathbf{T}(t)$	Transformation matrix for change of basis
t_0	Initial time
T_1, T_2	Components of matrix fraction decomposition
τ_c	Correlation time
t_f	Final time
tr(•)	Trace operator
T_s	Settling time
$\mathbf{U}$	Matrix of left singular vectors
$u(t)$	Control input; generic system input
u_1	Normalized control input
$\bar{u}_i$	Upper bound on the ith element of the control
$\mathbf{V}$	Matrix of right singular vectors
v	Generic vector, measurement error, measurement noise
$v(t)$	Measurement noise
v_1	White shaping filter input, normalized measurement input
v_2	Noise on constructed measurement
v_k	kth element of the vector v
v^T	Transpose of the vector v
$v^\dagger$	Conjugate transpose of v
$\mathbf{\Omega}$	$(s\mathbf{I} - \mathbf{A}_{11})^{-1}$
$\mathbf{W}$	Weighted matrix
ω	Frequency

Symbol	Meaning
$\mathbf{W}(t)$	Weighted function
$w(t)$	Disturbance input
w_0	Constant disturbance input
$\mathbf{W}_0(t)$	Output weighted function
w_1	White shaping filter input, normalized plant input
w_f	Fictitious noise for LTR
$\mathbf{W}_i(t)$	Input-weighted function
$\hat{x}$	Estimate of x
$\hat{x}_\varepsilon$	Estimate of x given data through $x - \varepsilon$
$X(s)$	Laplace transform of $x(t)$
$x(t)$	State
x^*	Extremal of x
x_0	Initial state
x_d	Desired state
x_d	Desired state (tracking systems)
x_f	Final state
$\tilde{x}(t)$	State after change of basis or coordinate translation, adjoint state
$\mathbf{\Xi}_x(t)$	Covariance matrix of the random process $x(t)$
$\mathbf{\Xi}_{xy}$	Cross-covariance matrix
$\mathbf{\Psi}$	Eigenvector matrix
$\mathbf{Y}(t)$	Reference output weighting function
$y(t)$	Reference output; generic system output
y_1	Normalized reference output
y_w	Reference output due to w
$\mathfrak{Y}$	Hamiltonian matrix (Kalman filter)
$\mathfrak{Y}_\infty$	Hamiltonian matrix ($\mathscr{H}_\infty$ estimation)
ζ	Damping ratio
$\mathbf{Z}(t)$	Output error weighting function
$z(t)$	Constructed measurement (nonwhite measurement noise)
$z(t)$	Coordinate translation of the state
$z(t)$	Transformed state
z_i	Zeros of a system
$\mathfrak{X}$	Hamiltonian matrix (LQR)
$\mathfrak{X}_\infty$	Hamiltonian matrix (full information control)
$[a, b)$	Real interval, closed at a and open at b
∞	Infinity
$\lVert\bullet\rVert_2$	2-norm (vector, signal, or system)
$\lVert\bullet\rVert$	Euclidean vector norm, generic norm
$\lVert\bullet\rVert_{2,[t_0,t_f]}$	Finite-time signal 2-norm
$\lVert\bullet\rVert_{\infty,[t_0,t_f]}$	Finite-time signal ∞-norm
$\lVert\bullet\rVert_E$	Vector Euclidian norm
$\lVert\bullet\rVert_{W(t)}$	Weighted signal 2-norm
$\lVert\bullet\rVert_W$	Weighted vector 2-norm
$\lVert\bullet\rVert_\infty$	∞-norm (vector, signal, or system)
$\angle\{\bullet\}$	Argument of a complex number
$\otimes$	Convolution operator
$\in$	Element of a set
$\lvert\bullet\rvert$	Magnitude of a complex number

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