Joint Ventures of Labor & Capital

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Preface

This book focuses on how competition among coalitions for members explains which profit-seeking joint ventures of labor and capital form and can survive. The nature, direction, and control of investments by joint ventures, the division of labor, and the determinants of wages are among the main topics. Core theory is the principal tool for studying them. It is used to explain under what conditions a corporation is a stable entity. It can also describe in terms of the nature and diversity of the projects the types of investors in these projects consistent with stable joint ventures. Core theory can answer several important questions about wages that are often ignored and much harder to answer without it. Among these are the relation between ability and wages as well as the relation between the size of a venture and the wage formula conducive to its stability.

The introduction provides a general description of the usefulness of core theory in economics in elementary terms. The first chapter uses core theory to study pure exchange, a situation in which there is trading but no production. The problem is to see when a competitive market has a core. This theory is applied to the market for shares of stock in chapters 2 and 3. Chapter 2 offers a theory of joint ventures using core theory. It describes the conditions for stability of these ventures in terms of the specialization of projects and the heterogeneity of the investors. These have implications about the holders of mutual funds as well as the owners of corporations. The internal stability of joint ventures is the subject of chapters 4 and 5. Chapter 4 describes when self-enforcing agreements can induce loyalty from the participants in a joint venture. Chapter 5 describes how superiors can remunerate their subordinates in order to bring about a harmony between their own interests and the self-interest of their subordinates. Chapter 6 applies core theory to determine wages. The last chapter analyzes the special problems of multiproduct firms. It finds optimal coalitions between the firms and their customers that can resolve empty core problems. Such optimal coalitions can explain some puzzling practices of the old motion picture industry. The key is how motion picture producers used these practices to cope with risk and uncertainty.

With the help of simple numerical examples the introduction shows how core theory is useful in economics and especially in industrial organization. It

presents the elements of core theory and describes many economic applications. Unrestricted contracting among individuals who are free to band together in groups lies at the heart of the competitive process. Depending on the particular application this theory can say what conditions are conducive to efficient outcomes. Sometimes, however, unrestricted freedom of contracting does not lead to efficient outcomes. When this happens the core is empty. Many of the most important challenges to core theory arise from empty cores. People facing empty cores try to devise suitable restrictions and rules in order to obtain efficient outcomes. Efficiency is when it is not possible for anyone to become better off without making somebody else worse off. Inefficiency entails deadweight losses. This means it would be possible to make somebody better off without making anybody else worse off. Core theory assumes that people arrange their affairs to avoid deadweight losses. These themes occur repeatedly in the various applications of core theory to joint ventures in this book.

Chapter 1 presents an application of core theory to pure exchange. Pure exchange is a situation in which traders exchange their initial holdings of things for what they prefer. There is no production in models of pure exchange. The first section describes a market with a finite number of traders, each with a concave valuation function of commodities. It gives two sufficient conditions for a nonempty core: first, the valuation functions of the traders are nondecreasing, and, second, the final allocations are strictly positive for all traders. The next section studies markets with infinitesimally small traders. This means individual traders have a negligibly small effect on the outcome. It shows that such markets have a core if the budget constraint for each trader is satisfied with equality. Consequently, under very general conditions a Walrasian equilibrium belongs to the core. A Walrasian equilibrium is what economic theory means by a competitive equilibrium. A Walrasian equilibrium has prices that can guide traders to the best final allocation of their initial resources. The third section offers various numerical examples that illustrate the uses and limits of the models. One of these is especially pertinent because it describes a market with a core but without a Walrasian equilibrium. This means there are cases in which a competitive equilibrium in the sense of a nonempty core can exist but cannot be attained using prices of a Walrasian equilibrium. The more general kind of competition in the sense of core theory describes terms of trade that can attain the best allocation of resources without prices to guide the traders. The last section explains why money is the yardstick. It derives the properties of a commodity that enable it to become the most widely used medium of exchange.

Chapter 2 begins with a discussion of a theory of joint ventures. A joint venture is a coalition of investors at an early stage in the life of a corporation before its shares can be traded on an organized public stock market. These early investors determine the size and direction of the projects of the joint venture. Coalitions of investors compete for new members, a process that determines

the cost of membership in a coalition. The theory implies that limited liability is necessary for a stable joint venture. Limited liability means that an investor cannot lose more than his initial stake in the joint venture. He is not liable for the debts of the joint venture. This differs from a partnership that requires all partners to be liable for its debts. The assets of a corporation are semiprivate. A semiprivate good is in contrast to a private good—an individual acting on his own can decide how much of a private good he wants. A semiprivate good is owned jointly by all the members of the coalition, who must agree on how much of it to have. For instance, a blast furnace is a semiprivate good to a corporation because all the owners (or their representatives), who may each own a different fraction of the total assets of the corporation, must agree on the total investment in blast furnaces. The return to an investor depends on his share of the total equity in the corporation. Usually, economists speak of a firm run by an entrepreneur. This ignores the nature of the organization that obtains the funds for carrying on its activities. In the conventional story corporations enter the scene only as a source of the different kinds of financial assets that are traded in the financial markets. These markets are divorced from the markets for actual goods and services. By using core theory and treating the investor as the basic entity, we can ask and answer questions about who owns corporations and mutual funds. For example, core theory solves puzzles about takeover bids out of reach of the standard theories.

In the second stage of its life, the shares of a joint venture can be traded publicly. The theory of pure exchange in chapter 1 applies to the market for shares of stock. When the corporation goes public, its original owners may sell their shares to anyone they please at mutually agreed-upon prices on an organized public stock market such as the New York Stock Exchange. Owing to limited liability, the owners of a corporation can sell their shares without the consent of the other owners. This is not true in a partnership. A partner cannot sell his share to an outsider without the consent of the other partners. A corporation is not like a private club. Private clubs usually do not allow members to sell their memberships to nonmembers without the approval of the present members.

A central feature of the theory is its emphasis on uncertainty. It treats the return to a joint venture as a random draw from a probability distribution. The size of the capital stock is a parameter of the probability distribution. Potential investors may differ in their views about the probability distribution, and their willingness to put funds at the disposal of the joint venture depends on these opinions.

The last section of this chapter describes empirical studies of investors' holdings. These include some material on the concentration of ownership in publicly traded corporations. This section also considers the major differences among investor types. Owing to current fashions in finance, there is little in-

terest in who owns corporations and, therefore, few studies of this subject. These few, lacking guidance from any theory, seldom contain material pertinent to the theory here. For instance, it is worth noting that the distribution of ownership of common stock is more concentrated than that of preferred stock. This is significant in view of the material given in chapter 2 but seems to be of no interest in the standard theory. The chapter concludes with an appendix containing some technical material on core theory relevant for the theory of joint ventures.

A key element in the theory of corporations as developed in chapter 2 is the individual investors' valuations of the prospects for a joint venture. Chapter 3 presents models of these valuations. The first section begins with the simplest case in which there is certainty. The next section treats the more difficult problem of how to value joint ventures when there is uncertainty about their prospects. Chapter 2 describes studies that imply that the valuation of a firm to its present owners is generally above its market value as determined in the stock market. This raises a question that a theory of valuation must answer—how to explain the difference between the market value of a firm as determined by the price of its shares of stock and the value of the firm as perceived by its present owners. Chapter 4 answers this question.

Chapter 4 opens with a description of the neoclassical theory of the optimal stock of capital for a profit-making firm in a perfectly competitive industry with certainty. It begins with a discussion of the effects of inflation on the value of a firm. It goes on to show when it is correct to say that the market value of a firm equals the price per share of its stock multiplied by the number of its shares outstanding. Sometimes this measure of the market value of a firm understates the value of the firm to its owners. The accuracy of the market value as a measure of the firm's prospects depends on the nature of the returns to scale of the firm. For constant returns to scale this measure of market value is correct. However, the value of the firm as measured by the price per share as determined in the stock market is downward biased for decreasing returns to scale.

The theory of corporations in chapter 2 says that the willingness of an investor to back a firm depends on how much value he attaches to it. Chapter 4 describes in detail the valuation of uncertain prospects. A novel aspect of this analysis is its emphasis on the uncertain duration of the firm's prospects. This leads to a theory of when a joint venture stops as a result of an autonomous voluntary decision of its owners even if the joint venture has no debt and no creditors who can force it into bankruptcy. The laws of bankruptcy need not be invoked to explain the demise of a firm.

The last section in this chapter presents evidence to test the validity of the preceding theory of the valuation of joint ventures. It begins with a derivation of an equation for stock prices using the theory in sections 1 and 2. Next it presents estimates of regressions to estimate this equation. The estimates use

monthly data for two periods: January 1920 to December 1940 and January 1947 to December 1991. The first includes the October 1929 stock market crash and the second the October 1987 stock market crash. The regressions for the first sample up to October 1929 and for the entire post–World War II period including the October 1987 crash all strongly support the predictions of the theory. Except for the period from October 1929 to March 1933, the Great Depression itself, the results for the period from April 1933 to December 1940 also support the theory.

Core theory given in chapter 2 as applied to joint ventures emphasizes the external forces acting on a coalition that threaten to disrupt it. The coalition can survive only if it can offer its members enough inducements to remain as members. It also faces internal problems that may threaten its survival. Chapter 4 tackles the problems facing a group of like-minded peers with a common interest who must rely on self-enforcement to maintain the group. Self-enforcement means no outsiders intervene to carry out the provisions of the agreement among the parties. In the economic situations to which this book applies core theory, conflict often arises between the group interest and the individual interest. While adherence to the agreement advances the long-term interest of the group, one who violates the agreement without giving notice of this may gain enough to break his promise even after subtracting the loss from the punishment that such treachery provokes. When punishment cannot deter treachery, an alliance cannot survive. We want to see when suitable punishment of violations of a self-enforcing agreement among the individuals can instill enough harmony between the group and an individual so that the alliance can survive internal disruptive forces.

Chapter 2 recognizes at the outset that investors in a joint venture hire people to act in the best interest of the owners. Chapter 4 deals with the problems that arise among peers in a joint venture. Chapter 5 takes up the problems that arise in a hierarchy. Those at the higher levels in a hierarchy must devise ways that will inspire those at the lower levels to advance the interests of their superiors. Delegation of power is pervasive and necessary in all large organizations. Since it is prohibitively costly for a principal to watch closely everything an agent does, the principal needs ways to pay the agent that will reduce these costs and will induce a self-seeking agent to act so as to move toward the principal's goals.

In the simplest case there is one principal and one agent, and even so most of the major issues are present. Analysis of this case is the topic of the second section of chapter 5. The next section assumes the principal has many agents. It suitably modifies the theory to reckon with the complications that ensue.

A major implication of this model applies to the type of remuneration for top corporate executives. In many corporations the managers own a very small fraction of the shares. Hence their return from these shares cannot induce them

to act in the interest of the shareowners. This is to say that a manager's action may have a big effect on the corporation but a small effect on his return as a shareowner. What does motivate managers is a method of remuneration in the form of rewards contingent on the results of their actions. The models in this chapter explain how these promises can align the interests of the managers and the owners.

Chapter 6 applies core theory to the determination of wages among competing firms. It is noteworthy that Frank Knight in his *Risk, Uncertainty and Profit* (1921) presents a theory of wages, stemming from Edgeworth, much like one based on core theory. Competition for workers among the firms that are regarded as coalitions of workers raises the question of whether a worker's wage will equal his incremental contribution to the coalition that he joins. It is shown, contrary to the standard theory, that wages may be less than the worker's incremental contribution even with vigorous competition among firms for workers. Core theory also facilitates study of the relation between wages and ability. While the normal distribution seems to give a good fit to some measures of ability, it poorly fits wage distributions. The distribution of wages is often found to be highly skewed to the right, suggesting either that a normal distribution of ability is wrong or that the wage distribution reflects more than just the abilities of the workers. Core theory shows precisely how abilities and competition interact so as to determine the wage distribution as a function of workers' abilities. Core theory also illuminates an important relation between the size and stability of a firm depending on the nature of the wage formula. The last section of this chapter uses a more general production function than the preceding sections. It allows the productivity of a worker to depend on the identity and specific skills of the fellow workers. In this more general setting, core theory can derive many important new properties of wages determined in a competitive labor market.

Chapter 7 presents a theory of the best coalition for an industry for which the core would be empty. It extends the results for single-product Viner industries to multiproduct Viner industries. In a single-product Viner industry each firm has a U-shaped unit cost curve. When more than one firm is optimal, a Viner industry has no core. The situation is more complicated when firms make several products. Yet, it remains true that the core is empty for almost all rates of demand. This raises an important question. Given an empty core, what arrangements do firms and their customers contrive in order to secure an efficient outcome? The answer is a coalition that is best for all its members, both the firms and their customers. Such a coalition restricts entry and exit of both buyers and sellers. In practice this often means distinguishing between regular and transient buyers and sellers. People in an optimal coalition are the regular buyers and those sellers who make long-term arrangements with them like a *keiretsu* among Japanese firms. The people outside the optimal coalition are the

transients who buy and sell in spot markets usually on less favorable terms than the regulars who are inside the best coalition. Study of the motion picture industry in the period preceding World War II suggests that the theory of optimal coalitions can explain many of its practices, especially in view of the uncertainty about the success of individual motion pictures.

Core theory, the basic tool used in this book, focuses on many often ignored yet important problems in the theory of the firm. Now with the help of core theory we can see a relation between those who invest and the sorts of investments that they make. A gap between wages and marginal productivity is consistent with a competitive labor market. The nature of the wage agreement depends on the size of the enterprise. These are a sample of the problems that this book addresses with the help of core theory.

Finally, it will be helpful to describe the conventions in this book. The chapters in this book are divided into numbered sections and, sometimes, numbered subsections. Equations, theorems, and lemmas are numbered from (1) in each section. Reference to an equation or theorem in the same section has the equation, theorem, or lemma number. Reference to an equation, theorem, or lemma in another section of the same chapter has the section number, followed by a period and then the equation number. Thus, (2.11) means equation (11) in section 2. An equation in another chapter has three numbers: chapter number, period, section number, period, and equation number. Thus, (3.2.11) means chapter 3, section 2, equation (11). In this fashion the search necessary to find an equation or a theorem is minimized.

Some particular mathematical conventions in this book are as follows. The two notations for greater than or equal to are:

$x \geqq y$ for two vectors x and y means corresponding coordinates have the relation greater than or equal to, $\geq$, and equality is possible for each coordinate;

$x \geq y$ for two vectors means there is greater than for at least one pair of corresponding coordinates and equality for every pair of corresponding coordinates is not possible. The usage $\geq$ for two scalars is obvious.

The notation $x \circ y$ denotes the scalar product of the two vectors x and y. That is, $x \circ y = \Sigma x_i y_i$. The notation a^- means approach the scalar a from the left and a^+ means approach the scalar a from the right.

Several readers and listeners have given me helpful comments, criticism, and encouragement on the material in this book. George Bittlingmayer, Sheldon Kimmel, and John Matsusaka are at the forefront of this group together with many of the students in my classes. Alan Krueger, Carl Shapiro, Joseph Stiglitz, and Timothy Tyler were of great help to me in the article "Usefulness of Core Theory" that is the introduction to this book. The comments of James Coleman

helped improve the chapter "When an Alliance Can Survive." Louis Chan and Thomas Philipson's comments on multiproduct Viner industries made chapter 7 better. None of these should be held responsible for any errors or shortcomings of this book.

Contents

Introduction: The Usefulness of Core Theory in Economics

Core theory furnishes a useful framework for studying a wide variety of economic problems. It has an undeserved reputation of being too abstract, owing mainly to the manner in which it is employed in the theory of general equilibrium. In fact, core theory is a highly flexible way of looking at practical economic problems, especially problems in industrial organization. This introduction seeks to show how simple numerical examples can illustrate the idea of the core and, in turn, how the core can illustrate basic principles of economics.

Principles of the Core

The theory of the core begins with the assumption that there are n individuals who can do something either all together, individually, or in small groups. For economic applications, a typical example is trade in a market, where all individuals may trade with each other in a single market or in submarkets or some may decide not to trade at all. The theory assumes that the individuals can measure the results of their actions. For the example of trade in a market, it is traditional to assume that an individual measures the outcome by the utility from the bundle of commodities. Alternatively, an individual can measure the gains from trade in terms of money. For a buyer, this is the maximum amount the buyer would have been willing to pay for the quantities purchased minus the amount actually paid. For a seller, it is the actual receipts minus the amount the seller would have been willing to accept for what was sold. Thus the theory of the core has three elements: n individuals; the various groups they can form, called coalitions; and functions that measure the results of the actions taken by the individuals and coalitions.

There are some outcomes that the whole group of individuals cannot improve. These are the outcomes such that it is not possible to make one person better off without making at least one other person worse off. Such outcomes are called Pareto optimal. They involve no deadweight loss. The originator of this theory, F. Y. Edgeworth (1881), went on to obtain remarkable results showing how competition among many traders and coalitions leads toward a Pareto-

efficient outcome.[1] Edgeworth called the mechanism that produces this result "recontracting." The approach is to consider all possible coalitions of traders, recognizing that any coalition of traders will only participate in the market as a whole if and only if they can do at least as well as they could by themselves in their own coalition. To put it another way, the best outcomes available to a coalition set lower bounds on what its members would be willing to accept as participants in the whole market. For example, the complete set of 1-person coalitions implies the constraint that each individual will not trade in the market unless the trade makes that person better off. Next consider all 2-person coalitions. In this case, any trades must make the individuals at least as well off as they would be in choosing any possible 2-person partnership. When this logic is extended from 3-person on up to the n-person coalition, there is a total of $2^n - 1$ possible coalitions, with each coalition placing a constraint on the outcome of trade. The larger the number of traders, the smaller is the range of outcomes without deadweight losses. Under certain conditions the terms of trade that can satisfy all these constraints constitute a competitive equilibrium.

Core theory examines this process systematically. Outcomes that are unacceptable to some coalition because it can do better for its members are said to be *dominated.* The set of undominated outcomes constitutes the core. Depending on the number of individuals and the process of recontracting, the core will sometimes consist of a range of outcomes, sometimes a single outcome, and sometimes the core will not exist at all.

An Example of Pure Exchange with a Nonempty Core

A simple example can illustrate these concepts. Say there are three individuals and the first two are potential buyers of a house from the third. The potential seller will not sell the house for less than $100,000. Buyer 1 will not pay more than $120,000 and buyer 2 not more than $150,000. (From now on units are understood to be in $1,000.) Let x denote the return to the seller, y_1 the gain to buyer 1, and y_2 the gain to buyer 2. In case it turns out the owner of the house sells it, x is the price of the house. The owner can ensure $x \geq 100$ because retaining the house is an option worth at least 100. For the potential buyers, $y_1 \geq 0$ and $y_2 \geq 0$, because each buyer can refuse to make a purchase and thereby can ensure a net gain of zero, no matter what anyone else does. These three inequalities are the constraints for the three 1-person coalitions.

A coalition of both buyers can do the same as either one of them separately, and the coalition of this pair will have the same lower bound on the sum of their gains. As a result, the relevant constraint for the coalition of both buyers is that $y_1 + y_2 \geq 0$.

1. Carl Menger (1871, chaps. 4 and 5) gives perhaps the first rigorous account of a competitive market. There is a direct line from Menger to Böhm-Bawerk ([1891] 1930) to the game theory of von Neumann and Morgenstern (1947).

There are two more coalitions involving pairs of traders. Each is a coalition between the seller and either one of the buyers. A trade between the seller and buyer 1 must ensure them a return equal to the larger of the two values {100, 120}. The first value comes from the fact that the seller will only participate if the gain is at least 100; thus the gain to the coalition of the two must also be at least 100. But what determines the minimum gain is the willingness of buyer 1 to pay 120. Given the existence of an outside, higher offer for the house, with the possibility of reselling it to buyer 1 afterward, the coalition between the seller and buyer 1 would reject any offer below the valuation either places on the house because each member of this coalition can bid for the house in competition with an outside offer. Similarly, the coalition between the seller and buyer 2 will demand a return of at least 150 because this is the larger of the two values {100, 150} applicable to this coalition. These conditions set two additional constraints on the outcome: $x + y_1 \geq 120$ and $x + y_2 \geq 150$.

Lastly, the valuation for the coalition of all three traders equals the maximum of {100, 120, 150}. To put it another way, the cumulative return for the coalition of the whole must be at least as much as the most generous buyer is willing to pay. This condition puts an upper bound on the sum of the returns, given by $x + y_1 + y_2 \leq 150$.

Any triplet $\{x, y_1, y_2\}$ that can satisfy all these inequalities is undominated and is said to be in the core of the market. As an example, consider the triplet {115, 0, 35}, where buyer 2 purchases the house at a price of 115. This triplet is not in the core; it does not satisfy the inequality that the gains to the coalition of the seller and buyer 1 must sum to at least 120. Thus, the seller and buyer 1 can form a coalition leading to the triplet {118, 2, 30} in which buyer 1 buys the house from the seller for 118 and resells it to buyer 2 for 120 so that buyer 2 gains 30.

But this set of trades is not in the core either, because it does not fulfill the inequality that the gains to the coalition of the seller and buyer 2 must be at least 150. The imputation {120, 0, 30}, in which the seller deals directly with buyer 2, *is* in the core. It is undominated and satisfies all the constraints. More generally, all imputations in which $y_1 = 0$, $x + y_2 = 150$, $x \geq 120$, and $y_2 \geq 0$ are undominated and form the core of the market. Thus, any outcome where buyer 2 buys the house for at least 120, but no more than 150, is in the core, and nothing else is in the core.

When a nonempty core exists, it means that any trader or group of traders prefers the outcome determined by the whole market to those they could get in any possible submarket involving a subset of traders. These submarkets present feasible alternatives that place limits on the prices that can emerge from the market as a whole. When the market has a nonempty core, it can survive all possible competing alternatives. In core theory, coalitions compete for members by making offers to individuals to induce them to join the coalition. The grand coalition, which includes all the members, can survive only by offer-

ing terms that are at least as good as any feasible offer coming from a subcoalition.

Although the example presented here illustrates only the simplest case of pure exchange, it can be extended into a comprehensive analysis of nearly everything there is to say on this topic. Because there is only one seller in the example, competition is present only between the two buyers. More complicated examples would have many sellers and buyers. The core is not empty for the more general case in which there are m sellers and n buyers each of whom seeks or offers at most one unit of the commodity. It can be shown that all those who sell the commodity must get the same price. This common price is determined by the constraints that emerge as a result of the terms that various coalitions could arrange by dealing among themselves. A still more general model allows the buyers and the sellers to seek or offer more than one unit of the commodity. If the demand schedules of the buyer are downward sloping and the supply schedules of the sellers are upward sloping then there is a nonempty core. Therefore, it remains true that each individual prefers the terms determined in the whole market to those that subsets of traders could agree upon by confining trade among themselves.

However, with multiunit traders, a wider range of alternatives is consistent with the core constraints than if each trader were replaced by an equivalent set of single-unit traders. This is true because multiunit traders do not make or tender offers for their commodities one unit at a time unless the forces of competition compel them to. In particular, it need no longer be true that a single price must prevail for the commodity throughout the market in this case. Different sellers could get different receipts per unit, and different buyers could pay different prices per unit. In these cases core theory shows how the sizes of the traders could affect the outcome.

A still more general model allows the traders to deal in bundles of continuously divisible commodities. There is a nonempty core in this case if the valuation functions of the traders are continuously increasing concave functions of quantities. Chapter 1 on pure exchange derives this result. The most general analysis assumes a continuum of traders. Think of each trader as indexed by a real number and suppose there are as many traders as there are real numbers in the unit interval. An individual trader is infinitesimally small and has a correspondingly small effect on the outcome of trade. A nonempty core exists for this market under very general conditions on the preferences of the traders. Even with many "crazy" traders, who violate assumptions of rational choice such as transitivity or revealed preference, there is a nonempty core if the "sane" traders are sufficiently more numerous than the crazy ones.[2]

2. Models of pure exchange using core theory are in Scarf and Debreu 1963. For a continuum of traders, see Aumann 1964, 1966 and Hildenbrand 1974. Simpler versions are in Telser 1972 and 1988.

Examples of an Empty Core

Sometimes it is not possible to satisfy the conditions for a nonempty core. This happens when the lower bounds on the terms that the coalitions would be willing to accept cannot all be met by the grand coalition. In this event, the core is empty. It means a fully competitive market fails to bring about a Pareto-optimal result.

Consider an example that is most easily visualized as applying to the cost conditions of the airline industry. Two sellers each operate an airline; they have one airplane apiece. The first airline, A_1, has a small airplane that can carry only two passengers at a total cost of 85. Or, to put it another way, not making the trip will save a cost of 85. (In the preceding example, this condition corresponds to the seller of the house retaining it, as if selling it to himself at the minimal price he would be willing to take for it, which is 100 in that example.) The second airline, A_2, has a bigger airplane that can carry up to 3 passengers at a total cost of 150. It can avoid this cost entirely by not making the trip. Note that the costs of the airlines are not dependent on whether they fly partly or entirely full but only on whether they make the trip at all. In this example, let us agree to ignore both fixed and variable costs. This does not affect the validity of the results, and it simplifies the arithmetic.

Let there be three potential travelers: B_1, who is willing to pay at most 55 for the trip; B_2, who is willing to pay at most 60 for the trip; and B_3, who is willing to pay up to 70. The total number of coalitions in this example is $2^{2+3} - 1 = 31$. Let us again adopt the terminology that x represents the returns to the sellers while y represents the gains to the buyers. For starters, consider the 1-person coalitions; these have a buyer or a seller acting alone. To make a deal the sellers must receive enough to cover the costs of a trip: that is, $x_1 \geq 85$ and $x_2 \geq 150$. The buyers must all perceive themselves as better off by making a purchase; therefore, y_1, y_2, and $y_3 \geq 0$.

Plainly, no coalition of a seller with only one buyer can cover the cost of a trip. Also, a coalition of either two or three buyers cannot gain more than zero. The remaining 2-person coalition, the two airlines without any passengers, cannot get more than the sum of what they could each get by themselves. Therefore, the interesting possibilities involve a seller with at least two buyers.

Consider the alternatives for the small airline A_1 with the three possible pairs of buyers. Again the return for each possible coalition is determined by the maximum of what the various purchasers might pay. For example, the potential gains for the coalition A_1, B_1, B_2 would be determined by the maximum of what the two buyers would pay and the seller would demand, that is, the maximum of $\{55 + 60,\ 85\}$, which equals 115. Similarly, the gains for the coalition A_1, B_1, B_3 would be the maximum of $\{55 + 70,\ 85\}$, or 125, and the gains for the coalition A_1, B_2, B_3 would be the maximum of $\{60 + 70,\ 85\}$, or 130.