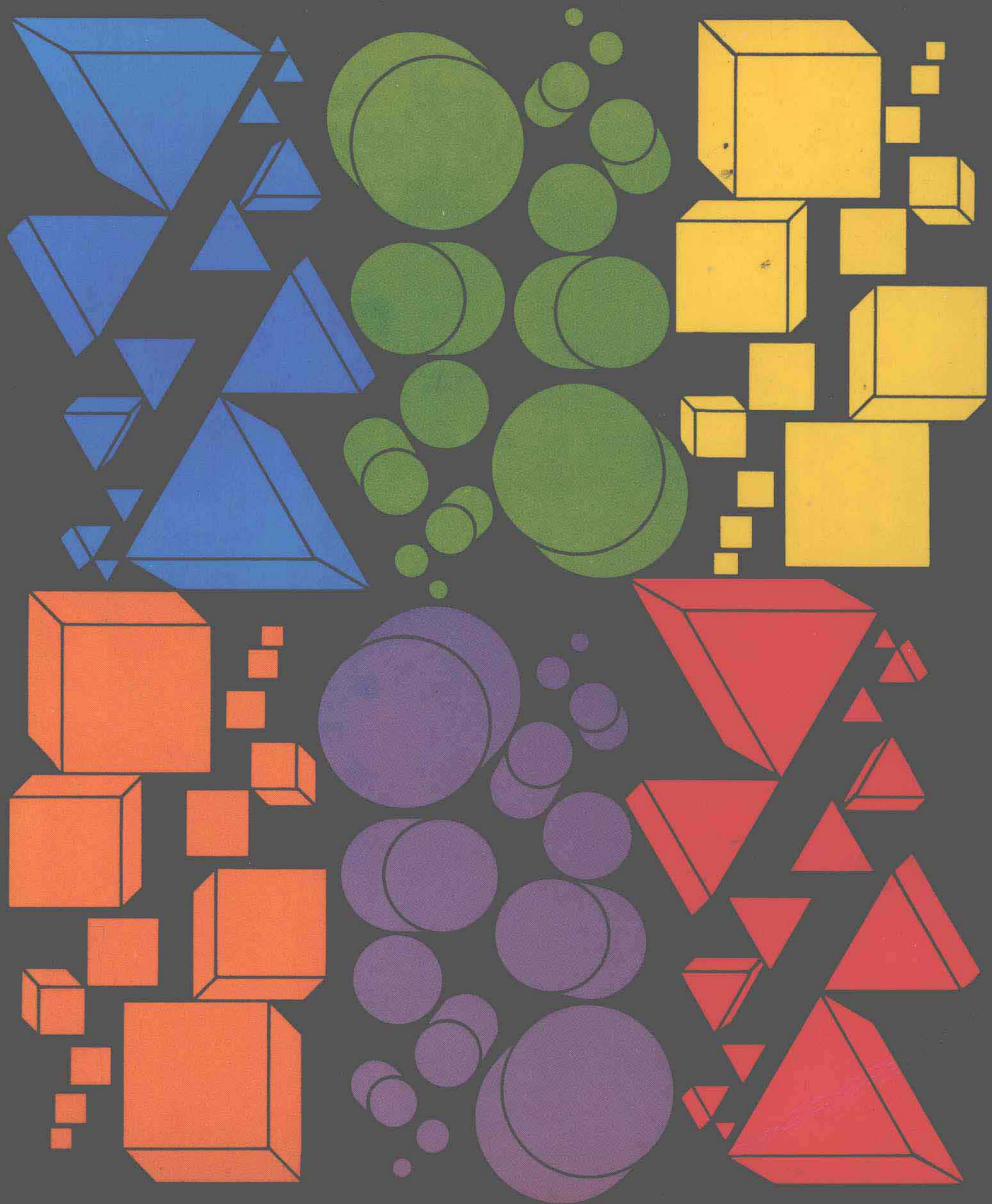
A Mathematics Activity Curriculum for Early Childhood and Special Education



A MATHEMATICS ACTIVITY CURRICULUM for Early Childhood and Special Education

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Macmillan Publishing Co., Inc.
New York
Collier Macmillan Publishers
London

Printed in the United States of America

Macmillan Publishing Co., Inc.
866 Third Avenue, New York, New York 10022

Collier Macmillan Canada, Ltd.

Library of Congress Cataloging in Publication Data

A Mathematics activity curriculum for early childhood and special education.

Includes index.
1. Mathematics – Study and teaching – Elementary.
I. Richardson, Lloyd I.
QA135.5.M3677 1980 372.7 79-15677

ISBN 0-02-399710-9

Printing: 1 2 3 4 5 6 7 8 Year: 0 1 2 3 4 5 6

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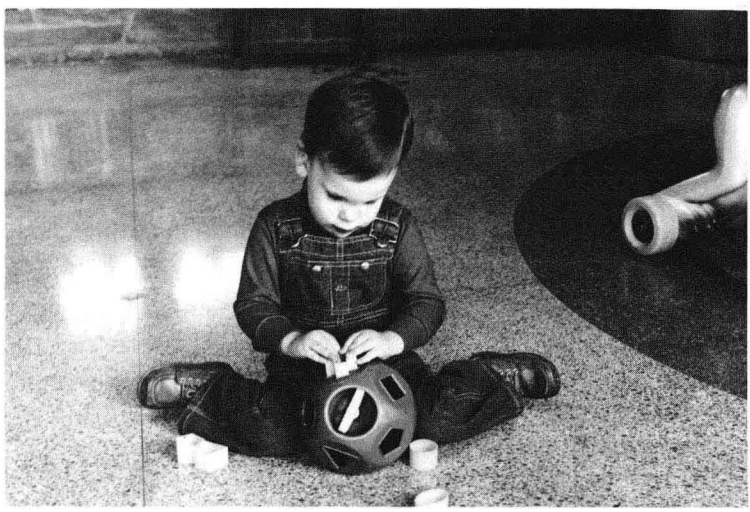
CHAPTER 1

Rationale and Organization

1.1 Introduction

The purpose of this book is to provide the teacher with a set of activity lessons with which to build a prenumber mathematics program and to supplement the early childhood math curriculum through grade 3. The titles for the chapters identified as prenumber concepts are Topology, Shape, Color, Size, Classification, and Seriation. Also a chapter entitled Numerals and one chapter on Numberness are included. The latter chapter is for those children who begin to conserve number early. This chapter offers an appropriate sequence for beginning one-to-one relationships and addition concepts for the teacher who does not regularly deal with a child who conserves. Chapters on Addition, Place Value, Subtraction, and Multiplication complete an early childhood mathematics program on the elementary school level. These chapters do not provide a comprehensive curriculum; rather, they are included to supplement an existing school text. Within each chapter, topics are sequenced to coincide with the child's development.



Early work with shapes and boundaries is a natural part of the young child's life.

The preschool teacher is provided a framework linking mathematical readiness to elementary school mathematics programs. When a child attends elementary school, mathematics and reading will be the primary content concerns. Although a great deal of information is available regarding preparation of a child for reading experiences, much less is available for mathematical readiness.

There seems to be a basic void in activity-oriented developmental lessons for children before starting the study of numerals and number. A comprehensive early childhood mathematics curriculum does not exist in a form usable by teachers. Certainly scattered activity suggestions are available as are books of a theoretical nature. Many early childhood books are aimed at language development, perceptual and motor skills, and cognitive development. The activities in this book draw from each of these areas, but their primary aim is longitudinal development of the child's mathematical abilities.

The term "mathematics" usually conjures up images of counting objects or adding and subtracting with numerals. Although crucial when the child studies arithmetical operations, these activities have a limited role in a prenumber math curriculum. Until a child can conserve number, these activities are learned only rotely and/or superficially. To clarify why conservation is a turning point in a child's development and why lack of it prevents successful math experiences with numeration, let's look at an example defining conservation of number. A child who does not conserve is influenced by appearances. A set in which objects are placed close together "appears" to have fewer objects than a set in which the objects are spread out.

A child would have difficulty working with numbers of objects when the groups of objects were forever changing their quantity just by being rearranged in space. One can only stress the importance of delaying arithmetic work with numbers until a child conserves.

Verbal challenges extend and expand the potential of each activity, if indeed the activity is appropriate for the child's level of maturation. Questions and comments that provide the teacher and child a mutual form of communication about experiences and that seek to encourage problem-solving abilities are especially important. Thinking up questions on the spur of the moment as the lesson progresses is difficult for most teachers; thus important questions are provided where the authors feel they are crucial to the activity. When appropriate, the authors provide direction to the teacher through questions posed.

Chapters dealing with prenumber math activities are appropriate for children ages $2\frac{1}{2}$ to around age 6 or 7. These activities will also be useful to the child enrolled in special education classes functioning on the prenumber level. Chapters dealing with arithmetical operations are most appropriate for children who have conserved number.

HOW TO USE THE BOOK

1.2 Chapter selection

The chapters have been sequenced to correspond to a child's development in acquiring math concept understanding. This sequence reflects current mathematics education research and the work of Piaget.

Basically, the book focuses on activities for the preoperational and concrete operational child (those readers not familiar with the terms are referred to Reys and Post, 1973). A child in the preoperational stage is characterized as very egocentric and is generally incapable of viewing a situation from more than one perspective. During this stage the child begins to move from viewing the world topologically (open-closed curves) to viewing the world Euclideanly (points, corners, lines) as in dealing with shapes. Once the child discriminates shapes, work with learning shapes should be provided.

According to research, the child discriminates color soon after shape—coming so close that either color or shape could be presented first. The child then becomes increasingly adept in working with progressively more difficult concepts of size, classification, seriation, and patterning. A child can work with numerals (chanting them, recognizing them, writing them) long before numeration activities can be comprehended appropriately. A separate chapter on numerals is therefore included before the chapter on numeration. One must keep in mind though that the abilities to use numerals in chanting or recognizing situations does not imply that a child can understand numberness. When the child can conserve, he/she moves from the preoperational to the concrete operational stage on the number concept.

The numeration chapter is intended for the conserver of number and concentrates on mathematical operations. It was decided that, even though sufficient numbers of elementary school mathematics programs concentrate on mathematical operations, exemplar activities should be included here that enhance acquisition and mastery of the mathematical operations. A final chapter deals with patterns and other math games.

Not all activities in a particular chapter are to be used before teaching activities from subsequent chapters. Primarily the chapter ordering follows

Reys, Robert E. and Thomas R. Post. *The Mathematics Laboratory*. Boston: Prindle, Weber & Schmidt, Inc., 1973.

the developmental sequences of children. As an example, shapes should be taught before size. Once the introductory activities from the shape chapter have been taught, the teacher may do introductory activities from the size chapter. Then activities from both the shape and size chapters may be taught at different times during the same day. Thus *all* the shape activities are *not* finished before doing *any* size activities.

An exception to mixing activities from two chapters is discussed in detail in the shape chapter. Basically naming shapes and colors can be confusing unless one or the other concept has been well learned prior to the introduction of the other.

1.3 Activity selection

The mathematical content is sequenced so that prerequisite topics are presented before topics utilizing the prerequisites. For example, in the topology chapter, the topic "open-closed" is presented before "inside-outside," as the latter topic assumes the concept of closure as a prerequisite skill. The mathematical content sequence appears as section headings within the chapter.

Vocabulary activities are included throughout and are necessary to student success. Learning the shape and color names is a mathematical discrimination, as abstract names are applied to properties of objects. This is analogous to applying the number name, numeral, to the property of the set of objects.

Activities were chosen and sequenced using certain guidelines selected from the work of Jerome Bruner and Robert Gagne. Bruner suggests that a pupil progresses through three modes in developing an understanding of a mathematical concept. The three modes are

enactive	a concrete model of the concept, usually involving manipulative or physical materials.
iconic	a semiabstract model, usually involving pictures of the situation.
symbolic	an abstract model of the concept.

These three modes have been incorporated throughout the development of each concept in this book. In every section, the authors have attempted to involve the pupil first in activities in the enactive mode with a progression to activity involvement in the symbolic mode when feasible. To facilitate selecting activities in one of these modes, the appendix catalogues every activity as to whether it is concrete, semiabstract, or abstract. Refer to the appendix for assistance in choosing an appropriate mode.

Incorporating the ideas of Gagne, the activities have been ordered so as to begin with the easiest task and progress sequentially to the problem-solving capability desired.

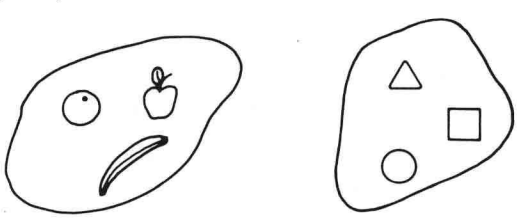
Bruner, Jerome S. *Toward a Theory of Instruction.* Cambridge: Harvard University Press, Belknap Press, 1963, 28.

Gagne, Robert. *The Condition of Learning,* 2nd ed. New York: Holt, Rinehart & Winston, 1970.

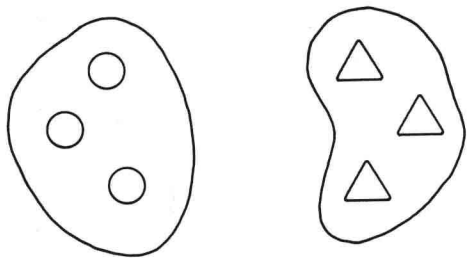
The authors have attempted to sequence the lessons so that work with a mathematical concept progresses from familiar objects to mathematically symbolic objects. Incorporated, when appropriate, is the assumption that when a pupil encounters a mathematical concept, the concept should first be embodied in an activity utilizing materials and objects familiar to the child (taken from his environment). Second, presentation of the concept should utilize mathematically symbolic objects (shapes and numerals) but allow manipulation and touching. Third, the presentation should utilize these symbolic objects in a mathematically symbolic setting. Perhaps an example is in order. In the chapter on numeration a presentation on one-to-one correspondence (all in the enactive mode of Bruner) would progress as follows:

1. set of hats (cowboy hat, fireman hat, nurse cap, baseball hat) with set of people (cowboy, fireman, nurse, baseball player) (both sets familiar, from environment)

2. set of fruit (orange, apples, bananas) matched with shapes including circle, square, triangle (set familiar to the child with mathematically symbolic objects)



3. set of circular shapes matched with sets of triangles



1.4 Activity format

The format chosen for activity description includes concept, materials, group size, procedure, questions, notes, and diagram (where appropriate).

Concept: Briefly describes the objective of the activity.

Materials: Describes the supplies necessary to conduct the activity.

Group size: Indicates the authors' thinking when conceptualizing the activity. This does not preclude using the activity with larger or smaller groups.

Procedure: Explains the presentation of the activity and coordination of materials presentation with teacher questions.

Questions: Lists significant questions to ask during the activity.

Diagram: Drawing of the activity to facilitate construction from materials or special feature for organizing the materials if verbal description could be confusing.

Note space: An area for the user to make notes regarding modifications or special items to be remembered. Once the activity is completed, variations or pertinent modifications observed by the user should be marked in the space below the activity for future reference. When the activity is again used (with another class, another pupil, or another sibling), the user will have a record of past experience and desired modifications.

Activities included in this book have been field tested in a preschool or special education setting. The questions posed and procedures described are, in many cases, those that resulted from the field testing. Each activity has been given a name for facilitating communication both within the book and with the children.

1.5 Using the appendix

All the activities included in the book are listed in the appendix. The purposes of the appendix are to aid the teacher in choosing appropriate activities to fit the needs of a special child, to facilitate making lesson plans, and to provide an index of activities by name and number. The appendix may assist teachers in special education who need to identify skills when writing Individual Educational Plans (IEPs).

The appendix is divided into three types of information—the mode of the activity presentation (concrete, semiabstract, abstract), the learning mode (visual, auditory, and tactile/kinesthetic), and the mode of the child's response (verbal, gross motor, fine motor, or with writing and drawing). Any one or all of these types of information may be included to develop lesson plans.

A complete description of this appendix is included in the appendix itself, and the reader should refer to this section.

CHAPTER 2

Topology

2.1 Introduction

There are many possible geometries in mathematics, one of which is topology. Traditionally people are familiar with Euclidean plane geometry because it is the usual geometry taught in schools. However, the young child first views the world from a topological point of view, then matures to viewing the world Euclideanly.

Topology is the study of intrinsic qualitative geometric properties without regard for number or measurement. These properties are independent of location, shape, or size. The properties do not change when the object under consideration is bent or stretched, as long as no tearing occurs. Consider the rubber band in Figure 2–1 as it is stretched.

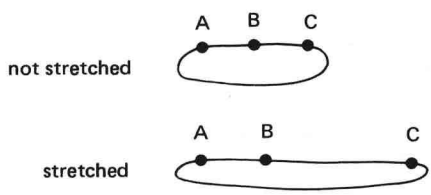


Figure 2.1

Note that the property of B between A and C is preserved when the rubber band is stretched.

As another example consider the closed circuit formed by a rubber band regardless of how it is stretched or bent. All of the forms the rubber band takes on in Figure 2.2 are called *equivalent.*

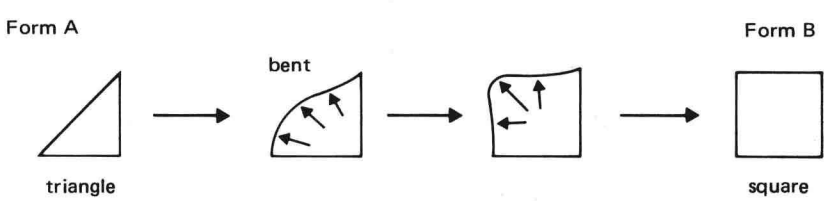


Figure 2.2

In topology a triangle is equivalent to a square because one can be transformed into the other without tearing the boundary. The only change occurs as the hypotenuse is stretched sufficiently and realigned to form the square.

Many theorists have shown that children's first geometric work is from a topological standpoint. Later, around age $3\frac{1}{2}$ to 4 years, the child begins to discriminate shapes or to view the geometric world from a Euclidean frame of reference. These topological concepts do not diminish in stature as Euclidean geometry develops but, rather, are refined and form the foundation for later geometric work.

In developing topological properties, the important properties are proximity, separation, enclosure, surrounding on a boundary, and betweenness. The activities in this chapter attempt to develop the child's topological perspectives. These properties form the basis for work with sets, number, mathematical vocabulary, and spatial relations.

2.2 Proximity (near, far)

The spatial perception of proximity is the earliest topological relationship developed by the child. The child distinguishes between those objects near and those farther away; for instance, the child could differentiate between the ball next to his chair and the one by the door.

Proximity is a relative relationship. The judgment of whether something is near or far is relative to what is used as a measure or guideline. Children discriminate proximity on two levels. On the first level the child compares the proximity of two objects both of which are in the same line of sight. On the second level the child compares the proximity of two objects that are not lying in the same direction. The second level is more difficult as

"Place the horse in the grassy area."

the child must retain a visual, mental image of the location of one object and then compare that image with the position of the other object. Remember that mathematical discriminations rely on a correlation of the child's maturation with his or her ability to utilize mental images when representing a comparison.

Proximity and order are apparent in drawing — looks like my teacher to me!

THE HOT SEAT

Concept: Identify nearest item given a choice between two.

Materials: Chair ("hot seat").

Classroom objects.

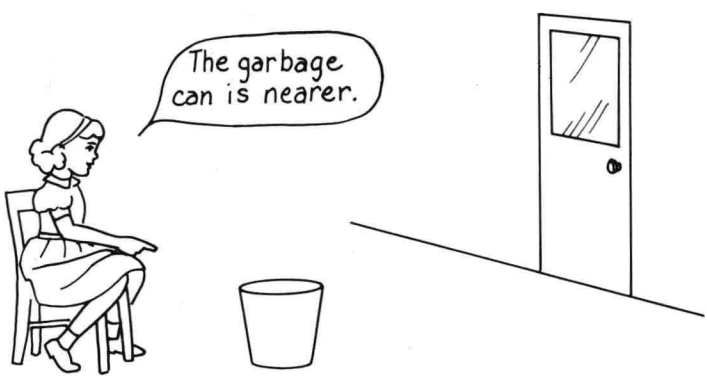
Group size: Any size group.

Procedure: Designate a chair the "hot seat" and place it at any point in the room. Have children take turns sitting in the "hot seat." Name two objects in the room; the child must tell which is nearer to him or her and which is farther from him or her.

The "hot seat" can be moved from time to time.

Variation: Name one object and have the child tell you whether it is near or far.

Questions: Can you name an object near you? Far from you?



BEANBAG BUDDIES

Concept: Manipulate objects in near and far situations.

Materials: Beanbags or pairs of similar objects.

Group size: Two, small group, or class.

Procedure: Call the names of two children and state "near" or "far." They both put their beanbags as near to each other or as far away from each other as they can.