

Applied Mathematics

by

R. Jesse Phagan

Woodstock Academy
Woodstock, Connecticut

Publisher
THE GOODHEART-WILLCOX COMPANY, INC.
Tinley Park, Illinois

Copyright 1997

by

THE GOODHEART-WILLCOX COMPANY, INC.

Previous edition copyright 1992

All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of The Goodheart-Willcox Company, Inc. Manufactured in the United States of America.

Library of Congress Catalog Card Number 94-30267 International Standard Book Number 1-56637-117-1

2 3 4 5 6 7 8 9 10 97 00 99 98

Library of Congress Cataloging in Publication Data

Phagan, R. Jesse.

Applied mathematics / by R. Jesse Phagan.
Rev. ed.

p. cm. Includes index. ISBN 1-56637-117-1 1. Mathematics. I. Title. QA39.2.P55 1997 513'.14--dc20

94-30267 CIP

R. Jesse Phagan has an extensive teaching and writing background in areas of math and electronics. He is the author of two electronics math books and an electronics book. He holds degrees in Industrial Arts/Technology Education, Vocational Education, and Electronics Technology. In addition to his teaching experience, he has worked in manufacturing and for the federal government.

Introduction

Mathematics is considered a basic life skill—a skill as important as the ability to read and write. **Applied Mathematics** has been developed as a complete math course. It is particularly relevant to the needs of students in vocational and technical programs. Through the use of realistic applications, an understanding of the importance of math is developed. Skills in problem solving are also developed. This textbook answers the question, "Where will I use this math concept?"

Applied Mathematics contains an abundance of Sample Problems, Practice Problems, and Problem-Solving Activities. With this combination, there should be no difficulty mastering objectives and developing sound math skills. Other features of this textbook include:

- Clear, readable text in a well-organized, easy-to-follow format, allowing students with diverse abilities to understand basic math.
- Coverage of the basics that builds understanding of math principles presented in later chapters.
- Exercises and activities that develop problem-solving abilities and provide examples
 of applications.
- Principles tied to real-life applications and current technologies.
- Key terms presented in boldface type and defined in context when introduced. In addition to this textbook, a workbook is available. The workbook consists of Practice Problems and Problem-Solving Activities that follow the format of the text to ensure continuity.

Skill in mathematics greatly simplifies the learning of vocational and technical subject matter. This, in turn, builds enthusiasm for learning and promotes successful achievement in technical areas. Sound math skills can foster confidence, which will contribute to success both in the classroom and on the job, as well as in meeting the challenges of everyday life.

Contents =

DADT	BASIC	MATH
PADI	DAOIL	

1 5	Place Values of Whole Numbers; General Procedures for Solving Word Problems; Word Problems using Addition; Word Problems using Subtraction; Word Problems using Multiplication; Word Problems using Division; Word Problems using Combination Arithmetic
2 F	Fractions
3 [Value of Decimal Numbers; Significant Figures and Rounding; Addition and Subtraction of Decimal Numbers; Multiplication and Division of Decimal Numbers; Converting Decimals and Fractions
4 1	Percentage
PART	II APPLIED BASIC MATH
5	Personal Finances
6	Graphs
7	Units of Measure
8	Reading Tools of Measurement
9	Perimeter, Area, and Volume

PART III BASIC AND APPLIED ALGEBRA
10 Signed Numbers and Operations
11 Introduction to Algebra. 207 Basic Terminology; Addition and Subtraction with Monomials; The Laws of Exponents; Multiplication and Division of Monomials; Addition and Subtraction of Polynomials; Multiplication of Polynomials; Division of Polynomials; Factoring
12 Algebraic Equations and Inequalities
13 Graphs of Linear Equations and Inequalities
14 Solving Problems with Algebra
PART IV ADVANCED APPLIED MATH
15 Applied Trigonometry
16 Scientific and Engineering Notation
17 Introduction to Estimating
Reference Section
Glossary
Index
Answers to Practice Problems and Odd-Numbered Test-Your-Skills Questions

Part I Basic Math

- 1. Solving Word Problems with Whole Numbers
- 2. Fractions
- 3. Decimal Fractions
- 4. Percentage







Solving Word Problems with Whole Numbers

OBJECTIVES

After studying this chapter, you will be able to:

- Identify operations needed to solve word problems.
- Set up word problems and perform the required arithmetic.
- By checking, state whether or not answers to word problems are correct.

When mathematics is applied to everyday life, seldom do the problems come simply in number form. Usually math problems come in forms of either worded questions or statements. The first chapter in this textbook of APPLIED MATHEMATICS is logically, then, an explanation in the techniques used to solve word problems.

To best serve the purpose of this chapter, the given problems deal only with *whole numbers*. These are numbers that do not contain decimals or fractions. The numbers in the problems are all positive. No negative numbers are used. All of the problems are **arithmetic operations**. That is, they deal with basic addition, subtraction, multiplication, and division.

PLACE VALUES OF WHOLE NUMBERS

The location of a digit in a number establishes the value of the digit. The location of a digit is called its **place value**. Refer to Fig. 1-1 for the place value of each location in a six-digit number. A digit is multiplied by its place value to obtain its value in the number. The complete number is formed by adding the products obtained by multiplying each digit by its place value.

Example	Place value	Multiply digit by	Value
314,562			
	ones	1	2 × 1 = 2
	tens	10	6 × 10 = 60
	hundreds	100	5 × 100 = 500
	thousands	1,000	4 × 1,000 = 4,000
	ten-thousands	10,000	1 × 10,000 = 10,000
	hundred-thousands	100,000	3 × 100,00 = 300,000
			Add: 314,562

Fig. 1-1. Place value chart.

Addition and place values

When performing addition, align the numbers so that digits with the same place value are located in the same column. Addition cannot be performed when digits with the same place value are located in different place value columns. When the addition of the numbers in any column results in a number greater than 9, a *carry* is brought to the next higher place value column.

Example: add
$$475 + 2,120 + 10,021$$

1 — carried from 10s column.

475

2,120

+10,021

12,616

Subtraction and place values

Subtraction requires place value columns to be aligned, as in addition. When subtracting, it is often necessary to *borrow* from the digit in the next higher place value column. The borrow reduces the digit it is borrowed from by 1 and adds 10 to the digit needing the borrow.

Example: subtract 3,517 - 243

$$1 \leftarrow$$
 borrowed from 100s column. The 1 in the 10s column 3,517 3,417 becomes 11.
$$\frac{-243}{3.274} = \frac{243}{3.274}$$

GENERAL PROCEDURES FOR SOLVING WORD PROBLEMS

Every word problem will have different techniques and procedures to follow in order to arrive at the final solution. It is possible, however, to establish a set of basic steps to follow. These steps will apply to every problem encountered. (You may not always see each of these steps included in the presentation of material in chapters that follow, but you can be sure the steps were performed.)

The first step is to identify the operation. Addition, subtraction, multiplication, and division are the operations of arithmetic. These operations will be used to solve the problem. At times, you can determine which operation to perform by first skimming the problem for key words. Of course, it is not enough merely to look for key words. You must think about what the question is asking. Also, problems do not *always* have key words, but the required operation can be determined by giving extra thought to the question. Making a sketch of the problem can sometimes be helpful.

The next step is to perform the arithmetic, always including the units. Each type of problem requires a standard format by which the numbers should be arranged. This is laid out in the sample problems of this chapter. Writing the numbers in the standard format is part of performing the arithmetic operation. Using this format will make the task much easier and will reduce mistakes. Units must be included with the final answer. They help to make it clear if the question asked in the word problem has been answered.

The final step is to check the answer. The answer should be checked in reference to the problem to see if it seems correct. This usually requires some knowledge of the subject of the problem, which comes with experience. Also, you can quickly check your arithmetic by using approximate figures.

RULES FOR SOLVING WORD PROBLEMS

- 1. Identify the operation.
- 2. Perform the arithmetic. Include the units.
- 3. Check the answer.

WORD PROBLEMS USING ADDITION

Although it may seem obvious, keep in mind there will be an increase in a quantity whenever addition of positive numbers is involved. Use the following key words to help identify word problems of addition.

- Sum—the result of adding together two or more numbers. Example: The *sum* of the numbers 1, 2, and 3 is 6. (1 + 2 + 3 = 6)
- Total suggests the complete amount or the result of adding amounts together. Example: A crew of 2 welders, 2 pipefitters, and 1 laborer makes a *total* of 5 workers. (2 + 2 + 1 = 5)
- In addition to—says that something has been added. Example: A quantity of 5, in addition to an existing quantity of 2, makes a quantity of 7. (5 + 2 = 7)
- Plus—similar to the word "bonus," meaning an additional amount is added. Example: Most students know 2 plus 3 is 5. (2 + 3 = 5)
- Increase signifies that the original amount has been made larger. Example: An *increase* of 5 students to the class of 20 brought the count to 25. (20 + 5 = 25)
- More than shows a larger number than the original amount.
 Example: If there were 3 before, and now there are 12 more than that, then there are now 15. (3 + 12 = 15)
- And—states that two numbers are combined. Example: Most students know 2 and 2 are 4. (2 + 2 = 4)

Sample Problem 1-1.

A supply of lumber costs a customer \$15, and nails \$9. What is the sum of the purchase?

Step 1. Identify the operation.

The key word is sum, which means addition.

Step 2. Perform the arithmetic. Include the units. In addition, the best format is for the numbers to be in columns, one above the other.

$$\frac{15}{+9}$$

Step 3. Check the answer.

The figure of \$24 seems to be the correct answer because it is higher than both individual numbers. Double-check the arithmetic.

Sample Problem 1-2.

When the lights in an automobile are on, the two headlights each use 120 watts of power. Each of the two taillights uses 9 watts of power. What is the total power used for the lights?

Step 1. Identify the operation.

The key word is total, which means addition.

Step 2. Perform the arithmetic. Include the units.

$$\begin{array}{r}
 120 \\
 120 \\
 9 \\
 \hline
 + 9 \\
 \hline
 258 W
 \end{array}$$

Step 3. Check the answer.

It is sometimes helpful to estimate the answer using numbers that are easy to add. In this case, each 9 can be changed to a 10. Doing so, the approximate answer would be 260. Notice that the numbers appearing in the problem are used twice. They are doubled because the problem states the wattage for "each" light.

Sample Problem 1-3.

A carpenter laid 375 roofing shingles one day, 520 the next day, and 460 the third day to finish the job. How many shingles were used for the job, not including waste?

Step 1. Identify the operation.

The problem does not have a specific key word. However, it should make sense that each day, more shingles were *added* to the roof.

Step 2. Perform the arithmetic. Include the units.

Step 3. Check the answer.

The easiest way to check this answer is by using numbers that are easy to add. Round the numbers to the nearest multiple of 50 as follows:

$$375 \rightarrow 400$$
 $520 \rightarrow 500$
 $460 \rightarrow 450$

Now, add the numbers together.

The estimated figure is approximately equal to the actual answer.

Practice Problems

Complete the following problems on a separate sheet of paper. Include units of measure (inches, feet, minutes, dollars, etc.).

- 1. During a snowstorm, a snow plow operator cleared roads with lengths of 3 miles, 5 miles, 1 mile, 2 miles, and 6 miles. How many miles of road were cleared by this operator?
- 2. For a particular construction job, the contractor totaled his costs as follows: \$42 for 2 × 4s, \$122 for drywall, \$38 for paint, \$95 for miscellaneous supplies, and \$680 for labor. What were the contractor's total costs?
- 3. In a given week, an automotive technician worked 12 hours on Monday, 8 hours on Tuesday, 9 hours on Wednesday, 6 hours on Thursday, 11 hours on Friday, and 4 hours on Saturday. How many total hours did the technician work during the week?
- 4. To determine the needs for a circuit, an electrician estimated the current requirements for the following loads: computer = 5 amps, television = 4 amps, light bulb = 1 amp, and stereo = 6 amps. If all these loads were operated at the same time on one circuit, what is the total estimated current?
- 5. What is the full length of the wall illustrated in Fig. 1-2?

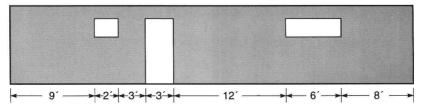


Fig. 1-2. Calculate the total length of the wall shown above.

6. The total resistance of a series electrical circuit is found by adding the individual resistances. Find the total resistance of the series circuit illustrated in Fig. 1-3.

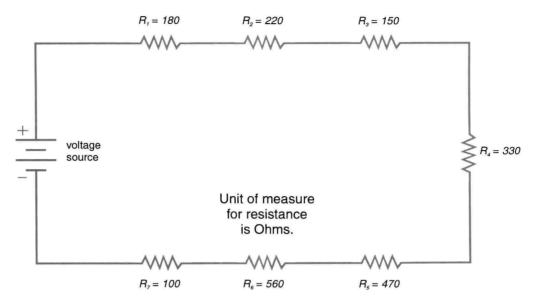


Fig. 1-3. Calculate the total resistance in this series circuit.

- 7. A quality-control technician performs destructive testing on a support beam by placing a weight of 1500 pounds on the beam. The weight is increased as follows until the beam breaks: 500 pounds, 875 pounds, 350 pounds, and 240 pounds. How many pounds made the beam break?
- 8. During a month-long blood drive, Red Cross volunteers visited area high schools and community colleges. How many pints of blood were donated during the drive if the first week brought in 150 pints, the second week brought in 238 pints, the third week brought in 484 pints, and the fourth week brought in 329 pints?
- 9. What length of fence is needed to enclose a play area with sides of 55 meters, 30 meters, 60 meters, and 28 meters?
- 10. A flooring contractor placed 45 red tiles, 68 green tiles, 71 yellow tiles, and 150 white tiles. How many tiles were placed?

WORD PROBLEMS USING SUBTRACTION

With subtraction, numbers are deducted. Since all numbers in this chapter will be positive, the larger number will be decreased by the smaller number. Only two numbers can be subtracted at a time, unlike addition, where many numbers can be added together in one operation. The number that is being deducted from is called the minuend. The number that is deducted is called the subtrahend. The answer is called the difference.

Example: 5 (minuend) - 3 (subtrahend) 2 (difference)

Ouite often, word problems state the total or end result of having done something. The question is to find the "missing step" of what was originally an addition problem. (This will become clearer as you continue to read.)

Use the following key words to help identify word problems of subtraction:

- Difference—the amount between two numbers. Example: The difference between 12 and 5 is 7. (12 - 5 = 7)
- Fewer means the total amount has been lowered. Example: With 5 fewer bolts in a box that originally had 12, there are 7 left. (12 - 5 = 7)

- Less than—suggests that something has been taken away from the original amount. Example: If there were 8 before, and now there are 6 less than that, then there are now 2. (8 6 = 2)
- Reduce—the process of making something smaller.

Example: The class began with 20 students but was *reduced* by 5, leaving 15 students. (20 - 5 = 15)

• **Decrease**—also makes the amount smaller.

Example: After a sale, a supply of 50 tires *decreased* by 10, so that 40 tires were left. (50 - 10 = 40)

• Minus - something has been removed.

Example: A carton once had 6 cans but now has 4 because it is *minus* 2. (6 - 2 = 4)

Sample Problem 1-4.

The electricity used in a house on the coast of Maine was 1538 kilowatts (kW) during the month of September. During that month, a windmill supplied 925 kilowatts to the house. How many kilowatts were supplied by the power company?

Step 1. Identify the operation.

This is an example of the "missing step" in an addition problem. The power company *plus* the windmill supplied the total electricity. The *difference* between the total and the windmill is what the power company supplied.

Step 2. Perform the arithmetic. Include the units.

$$\frac{1538}{-925}$$

$$\frac{613 \ kW}{}$$
 from the power company

Step 3. Check the answer.

The answer can be checked two ways: first, by comparing to what makes sense and, second, by addition. First, when 613 is compared to the question, it makes sense that the amount should be less than the total kilowatts. Second, in subtraction problems, add the difference (answer) to the subtrahend. The answer you get should be the same number as the minuend.

Note: The addition method is an exact check to see if the subtraction has been performed correctly. It is always a good idea, however, to also see if the answer makes sense in answering the question.

Sample Problem 1-5.

A lumber yard delivers $144-2 \times 4s$. After a day of framing walls, the contractor counts the 2 x 4s and finds 29 remaining. How many boards were used?

Step 1. Identify the operation.

This problem suggests subtraction because the total amount was *reduced* by what was used.

Step 2. Perform the arithmetic. Include the units.

$$\begin{array}{r}
144 \\
- 29 \\
\hline
115 boards used
\end{array}$$

Step 3. Check the answer.

Add the difference to the subtrahend.

Sample Problem 1-6.

A 55-gallon drum of paint thinner is in an auto body shop. The shop used the following amounts during the month: 3 gallons, 9 gallons, 4 gallons, 14 gallons, 2 gallons. How much is remaining in the drum?

Step 1. Identify the operation.

There are two methods of solving this type of problem. One is to start with the total and subtract each amount as it is taken. The other is to add all the amounts taken, then subtract from the original total. The end result with either method will be the amount remaining in the drum at the end of the month.

Step 2. Perform the arithmetic. Include the units.

Second method:

$$\begin{array}{r}
 3 \\
 9 \\
 4 \\
 14 \\
 + 2 \\
 \hline
 32 \ gal. \ used \\
 \hline
 55 \\
 -32 \\
 \hline
 23 \ gal. \ remaining \\
 \end{array}$$

Step 3. Check the answer.

Using two methods to solve the same problem is a good way to check your work. Also, the answer makes sense because if gallons are removed from the drum, the amount remaining should be less than the starting amount.

Practice Problems

Complete the following problems on a separate sheet of paper. Include units of measure (inches, feet, minutes, dollars, etc.).

- 1. An electric meter read 14,082 kilowatt hours at the end of January. At the end of February, it read 15,287 kilowatt hours. How many kilowatt-hours of electricity were used during the month of February?
- 2. An appliance warehouse delivered 35 refrigerators in one week. How many refrigerators remain in the warehouse if there were 87 in stock at the beginning of the week?
- 3. A tree farm had 150 blue spruce trees ready for its annual three-day sale. On the first day of the sale, 27 trees were sold. On the second day, 18 trees were sold. On the third day, a wholesaler offered to buy all the remaining trees. How many trees were left for the wholesaler to buy?
- 4. Determine the thickness of the material used for the base in Fig. 1-4.

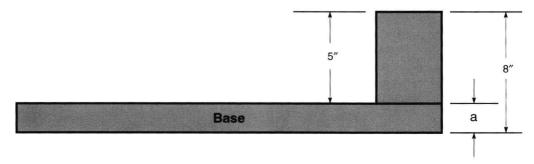


Fig. 1-4. What is the thickness of the base material shown above?

- 5. A crop duster carries 1500 pounds of chemical fertilizer. The pilot releases 750 pounds of fertilizer on a field. How much fertilizer remains on the plane?
- 6. A plumber was paid \$2250 for a job that took one week. His expenses were as follows: \$350 for pipe, \$105 for fittings, \$780 for fixtures, \$53 for miscellaneous supplies, and \$285 for his helper's wages. How much money was left for the plumber?
- 7. A printing company employee opened a 55-gallon drum of cleaning fluid and removed 12 gallons. A month later, the employee removed another 12 gallons. Surprisingly, this emptied the drum—the balance of the fluid was lost through evaporation. How much fluid evaporated?
- 8. The total resistance in a series circuit is the sum of the resistances of the individual resistors. What is the value of resistor R_2 in Fig. 1-5?

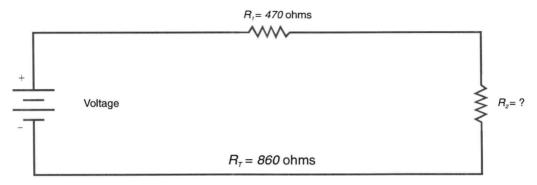


Fig. 1-5. Calculate the value of R2.

- 9. To determine the weight of a puppy, a person steps on a scale and determines her weight to be 150 pounds. Then, while holding the puppy, she steps on the scale again. This time, the scale reads 178 pounds. How much does the puppy weigh?
- 10. A sawmill is monitoring its supply of 6×6 timbers. The mill ships out the timbers as follows: 40 timbers on the first truck, 28 timbers on the second truck, 56 timbers on the third truck, 18 timbers on the fourth truck, and 29 timbers on the fifth truck. If there were 250 timbers in stock before the first shipment, how many timbers remain after the fifth truck leaves the mill?

WORD PROBLEMS USING MULTIPLICATION

Multiplication is a form of addition. It is like adding the same number many times. To recognize multiplication in a word problem, look for words that suggest a number is repeated. Keep in mind that multiplication (with whole numbers) will always result in an increase in the original number. The numbers being multiplied are **factors**; the answer is the **product**.

A multiplication problem can be set up in several formats. Each of the following formats are read "6 times 2 equals 12."

 $6 \times 2 = 12$

6 * 2 = 12

 $6 \cdot 2 = 12$

(6)(2) = 12 (adjacent quantities enclosed in parentheses)

Sometimes, as you will learn in upcoming chapters, letters will be used in equations and formulas to represent different numbers. When they are, you may also see a multiplication problem in another format. In this format, letters or numbers and letters are written side by side. For example:

ab = c is read "a times b equals c" or "a-b equals c"

2b = 12 is read "2 times b equals 12" or "2-b equals 12"

Use the following key words to help identify word problems of multiplication:

• **Product**—the result of multiplication.

Example: The *product* of 2 and 3 is 6. $(2 \times 3 = 6)$

• At – often used with pricing items and states the cost of the item.

Example: A case of oil (24 cans) at \$1 each costs \$24. $(1 \times 24 = 24)$

• Times – describes how often a number is repeated.

Example: If you swim 1 mile, 3 times today, you will swim a total of 3 miles. $(1 \times 3 = 3)$

• By – often used with dimensions to represent surface area, which is calculated with multiplication.

Example: The garage is 20 feet by 30 feet. The area is 600 square feet. $(20 \times 30 = 600)$

• Rate—an amount that is repeated.

Example: A worker, paid at the rate of \$6 for each hour of work, makes \$48 in an 8-hour day. $(6 \times 8 = 48)$

• Per – indicates how often an event is repeated.

Example: A car traveling 60 miles per hour for 3 hours will cover 180 miles. $(60 \times 3 = 180)$

Note: The word *per* is also used with division. Use multiplication when the individual numbers are given, and the question asks for the total.

Sample Problem 1-7.

A worker earns \$7 per hour. He works 40 hours each week. What is the weekly gross pay (pay before payroll deductions)?

Step 1. Identify the operation.

Each hour, \$7 is earned, which is repeated for 40 hours. Another way to interpret this problem is to recognize \$7 as a rate of pay. Rate becomes the key word for multiplication.

Step 2. Perform the arithmetic. Include the units.

\$280 weekly pay

Step 3. Check the answer.

The best way to check this type of problem is to see if the answer makes sense in the problem. This answer does make sense based on an approximation of how much money should be earned.

Sample Problem 1-8.

A stack of lumber is 32 boards wide by 16 boards high. How many boards are in the stack?

Step 1. Identify the operation.

Multiplication is the indicated operation because the stack is 16 layers high, with 32 boards in each layer. Therefore, 32 is repeated 16 times. Another way to look at this problem is to recognize the key word by as an indication of multiplication.

Step 2. Perform the arithmetic. Include the units.

$$\frac{32}{\times 16} \times \frac{16}{512}$$
 boards in the stack

Step 3. Check the answer.

With any multiplication problem, it is possible to repeatedly add the basic number enough times. In this problem, 32 would be repeated by addition, 16 times. A quicker check is to estimate the answer by changing to numbers that are easy to multiply. Round to the nearest multiple of 5 as follows:

$$32 \rightarrow 30$$

$$16 \rightarrow 15$$

Multiply the estimated numbers together.

$$\begin{array}{r} 30 \\ \times 15 \\ \hline 450 \text{ estimated} \end{array}$$

Note: The estimated answer is not quite near the exact answer. However, it is close enough to indicate the answer is likely correct.

Sample Problem 1-9.

An 8-foot-long, pressure-treated, 6 x 8 post costs \$9. If 50 posts will be needed for a fence, what will be the total cost of the fence posts?

Step 1. Identify the operation.

Recognize the fact that a price indicates *rate*. This tells you to multiply. Also, notice that \$9 is to be repeated 50 *times*.

Step 2. Perform the arithmetic. Include the units.

50 \times 9 \$450 total cost for fence posts

Step 3. Check the answer.

The estimate method could be done by rounding 9 to 10 and multiplying to get an estimated 500.

Practice Problems

Complete the following problems on a separate sheet of paper. Include units of measure (inches, feet, minutes, dollars, etc.).

- 1. If a car is traveling at an average of 55 miles per hour, how many miles will it travel in 6 hours?
- 2. A certain gasoline-powered electric generator burns fuel at the rate of 4 gallons per hour. A contractor plans to use this generator for five 8-hour days. How much gasoline should the contractor plan to use?
- 3. A room is 14 feet wide and 16 feet long. If one floor tile measures 1 foot \times 1 foot, how many tiles would be needed to cover this floor?
- 4. A drafting job is available for \$8 per hour. What is the gross pay (before deductions) for a 40 hour week?
- 5. The veterinarian's instructions on a certain canine medicine read "5 mg per pound of body weight." What is the dosage for a 62 pound dog?
- 6. The formula for torque is as follows:

Torque = Length \times Force

How much torque is exerted if 30 pounds of force is applied to a 2-foot-long lever? (The unit of measure for torque is ft-lb.)

7. The formula for power in an electrical circuit is as follows:

Power = Current × Voltage

Power is measured in watts, current is measured in amps, and voltage is measured in volts. How much power is used by an electric drill drawing 8 amps of current when connected to 120 volts?

8. Using the screw placement shown in Fig. 1-6, determine the number of screws estimated for 15 sheets of plywood.

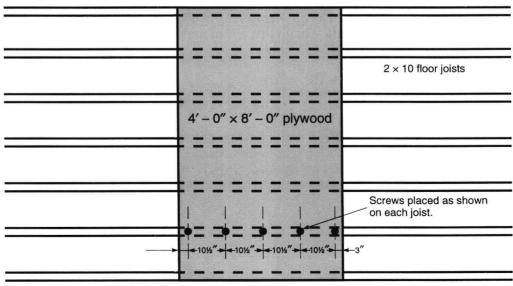


Fig. 1-6. Screw placement for a plywood subfloor. Note that placement for only one joist is shown. Screws must be placed in each joist.