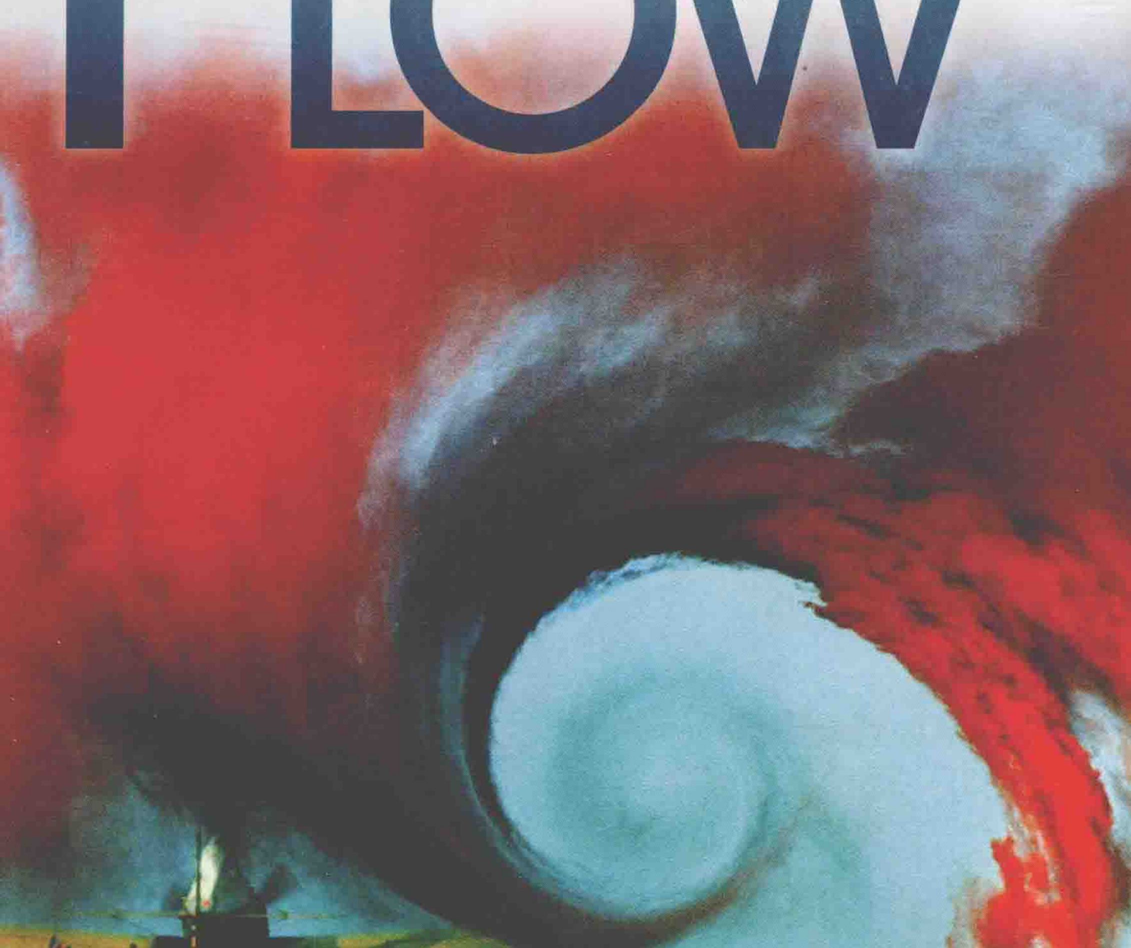
ROTATING FLOW



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Butterworth-Heinemann is an imprint of Elsevier
30 Corporate Drive, Suite 400, Burlington, MA 01803, USA
Linacre House, Jordan Hill, Oxford, OX2 8DP, UK

Library of Congress Cataloging-in-Publication Data
Childs, Peter R. N.
Rotating flow / Peter R. N. Childs.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-12-382098-3
1. Vortex-motion. 2. Rotational motion. I. Title.
QC159.C47 2011
532'.0595–dc22
2010028787

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

ISBN: 978-0-12-382098-3

For information on all Butterworth–Heinemann publications visit our Website at www.books.elsevier.com

Printed and bound in China
11 12 13 10 9 8 7 6 5 4 3 2 1

Foreword

The subject of rotating flows is important for many engineers, mathematicians and physicists. The earth's weather system is controlled by the combined effects of solar radiation and rotation. The jet stream and ocean circulations occur as a direct result of the earth's rotation; hurricanes and tornadoes are extreme phenomena that owe their existence to rotation-induced swirl.

There are many examples of flow near rotating machines, the most important of which is the gas turbine. Rotating-disc systems are used to model—theoretically, experimentally and computationally—the flow and heat transfer associated with the internal-air systems of gas turbines, where discs rotate close to a rotating or a stationary surface. The engine designer uses compressed air to cool the turbine discs: too little air could result in catastrophic failure; too much is wasteful and increases the fuel consumption and CO_2 production of the engine. Small improvements to the cooling system can result in significant savings, but the optimum design requires an understanding of the principles of rotating flows and the development and solution of the appropriate equations.

In this book, Professor Childs draws on his extensive experience to cover the basic theory of rotating flows. He shows how the Navier-Stokes equations can be used to derive the boundary-layer and Ekman-layer equations for rotating surfaces, and how—with appropriate assumptions—these equations can be applied to a disc rotating in a stationary fluid (the classic free disc) and to discs rotating close to stationary surfaces (rotor-stator systems) or close to another rotating disc (rotating cavities). The solutions of the equations are compared with experimental measurements, and the numerical examples should be helpful to students and practising engineers alike.

The student familiar with conventional non-rotating flows has, like Alice, to go through the looking glass into a strange world that is counter-intuitive to the uninitiated. This book provides a guide to this important world, and the reader who is prepared to put in the effort and follow the sign-posted paths will emerge better informed and a lot wiser. It's a book that I would recommend to postgraduate students, practising engineers and scientists who wish to know more about this fascinating subject.

J Michael Owen

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Preface

Swirling, whirling, and rotating flow has proved fascinating and challenging throughout the ages. From sink vortices to the swirling motion seen in a cornfield as the wind blows across it, the subject provides a talking point and a level of complexity that often defies simple explanation. Atmospheric and oceanic flow is also significantly affected by rotation, including cyclonic and anticyclonic flow circulations, intense atmospheric vortices, and ocean circulations. The observed flow phenomena often do not match our expectations and intuition, and it is through the insight revealed by detailed observation and modeling that significant satisfaction in the study of rotating flow can result.

Modeling of rotating flow is critically important across a wide range of scientific, engineering, and product design applications, providing design capability for products such as jet engines, pumps, and vacuum cleaners and modeling capability for geophysical flows. Even for applications where rotation is not initially evident, the subject is often fundamental to understanding and modeling the details of the flow physics. Examples include the vorticity produced in flow along a channel, the secondary flow produced for flow around a bend, and the wing-tip vortices produced downstream of a wing.

There are fundamental differences between rotating and linear flows, and this text explores these differences, their physical manifestations, and modeling. The text includes both anecdotal explanations and in-depth development of modeling and analytical methods. This book introduces examples of rotating flow in technology, science, and nature. The fundamental methods of modeling such flows are explored in conjunction with simplified techniques that enable approximate solutions to practical applications. Simple explanations of complex flows can be very useful in developing understanding and flow models, even though they sometimes neglect important details. The in-depth proofs and advanced models provide an indication of the validity of approach and additional analytical capability.

Specific subjects covered include:

- an introduction to rotating flow
- fundamental equations
- vorticity and vortices
- rotating disc flow
- flow around rotating cylinders and in a rotating annulus
- flow in rotating cavities
- an introduction to atmospheric and oceanic circulations

This text has been written principally to assist engineers, technologists, physicists, and meteorologists in developing their understanding and modeling of the fluid mechanics associated with rotating flow. The text has been developed to be of value to practitioners, researchers, and students. It has been assumed that the reader will have already developed some skills in the field of fluid mechanics, typically compatible with first- and second-year undergraduate level. The text has been used for both short and extended courses in rotating flow and for augmenting undergraduate, graduate, and continuing professional development courses in fluid mechanics.

The book has principally arisen from over 20 years of research into rotating fluids and associated heat transfer at the Thermo-Fluid Mechanics Research Centre, University of Sussex, the author's former affiliation. Much of this work has been focused on the internal air system for gas turbine engines where rotating discs, cavities, and channels are common features, as well as flow visualization experiments. The research has been sponsored by a wide range of industrial companies and funding bodies, including Rolls-Royce plc, Alstom, Siemens, Ruston Gas Turbines, Turbomeca, Motor-Turbieren Union, Volvo, Fiat, Snecma, Industrial Turbinas Propulsores, ABB, Turbocam, the European Union, and the Engineering and Physical Sciences Research Council. Their support and partnership in the research programs is gratefully acknowledged. I have also been fortunate to have worked in the same laboratory as Professor Fred Bayley, Professor Mike Owen, Professor Alan Turner, and Dr. Christopher Long, all of whom have been influential in developing the science of rotating flow.

The book is organized into eight principal chapters. Chapter 1 provides an overview of rotating flow phenomena, setting the context for subsequent development of the subject. Chapter 2 provides an overview and detailed introduction to the fundamental mathematical means of modeling fluid flow, with a specific emphasis on the conservation equations of continuity, momentum, and energy. These enable detailed modeling of fluid flow and are presented in forms directly applicable to rotating flow. The subjects of vorticity and vortices are introduced and developed in Chapter 3.

Chapters 4, 5, 6, and 7 concentrate on rotating flow in specific geometric configurations. Chapters 4 and 5 focus on the flow in rotating disc applications important to a range of engineering applications, such as gas turbine engines and pumps, and provide a relatively simple geometric configuration for the modeling of, in some cases, complex three-dimensional and time-dependent flow. The subject of a plain rotating disc is considered in detail in Chapter 4, serving to provide an in-depth development of understanding of the flow physics and modeling approach for both laminar and turbulent flow. Chapter 5 builds on this treatment, extending consideration to the cavity formed between a stationary and a rotating disc. Chapter 6 is concerned with flows associated with rotating cylinders and spheres. This chapter also addresses the flow in a fluid-filled annulus where the inner or outer cylinder rotates and there may also be an axial flow through the annulus. The flow in a rotating annulus can, under certain conditions, result in the production of toroidal vortices called Taylor vortices, which arise from instabilities in the flow field. The flow in a cavity formed between two co-rotating discs is considered in Chapter 7. This subject is particularly challenging as the flow physics depends on the thermal and geometric boundary conditions and tends to be three-dimensional and time dependent and sometimes intractable even to the most sophisticated of current modeling techniques. The subjects of vortex flow and of rotating disc and rotating cavity applications have particular elements of commonality with geophysical flows.

Flow in the Earth's atmosphere and its oceans is significantly influenced by the Earth's rotation. Examples include the major circulations in the atmosphere and oceans, cyclones and anticyclones, and large-scale vortices. Chapter 8 provides a brief description of the atmosphere and oceans and the principal flow structures, with a specific focus on the associated flow circulations and the influence of the Earth's rotation.

About the Author

Peter Childs is the Professorial Lead of Engineering Design in the Faculty of Engineering at Imperial College, having taken up this chair in 2008. He was formerly the director of the Rolls-Royce-supported University Technology Centre for Aero-Thermal Systems, director of InQbate, the HEFCE funded Centre of Excellence in Teaching and Learning in Creativity, and a professor at the University of Sussex where he worked for 21 years. His general interests include creativity; the application of creative methods; sustainable energy component, concept, and system design; fluid flow in rotating applications; and heat transfer. He has undertaken extensive research activities principally focused on the internal air system for gas turbine engines. He has written several books on mechanical design, fluid flow, and temperature measurement and is a former winner of the American Society of Mechanical Engineers—International Gas Turbine Institute John P. Davis award for exceptional contribution to the literature of gas turbine technology.

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Notation and Units

An attempt has been made to use a nomenclature system that is consistent throughout the text. This is challenging in a subject dealing with a variety of applications and specific geometries. The letter u has been selected for velocity and components identified by subscripts. This has the advantage that the reader can readily identify the direction concerned, but it is more complex and prone to error than, say, a separate letter being used for each. The selection of u for velocity allows the choice for volume to be V. A distinction has been made where possible between velocity components in a stationary frame of reference denoted, in the case of cylindrical coordinates, by u_r, u_ϕ, and u_z, and velocity components in a rotating frame of reference denoted by u, v, and w.

Where possible emboldened letters have been used for vectors.

Many rotating flow applications involve consideration of inner and outer radii. It is common practice in disc geometries to use the letter a to denote an inner radius and the letter b to denote the outer radius. Similarly, for an annulus the letter a can be used for the radius of the inner cylindrical surface and the letter b for the radius of the outer cylindrical surface. The letter r can then be used for the local radius.

The relationship between fluid flow and heat transfer has required consideration of notation relevant to both fields. The letter t has been selected for time and the capital T for temperature.

There are many dimensionless groups that are useful in fluid mechanics. It is possible for ambiguities to arise between, for example, the Rossby number, Ro, and the multiplication between two variables R and o. It is hoped that the context will resolve such an ambiguity if it arises.

In accordance with the SI convention, a space has been left between numerals and units and between units.

a	radius (m)
A	area (m^2), coefficient (dimensionless)
A_{gap}	cross-sectional area of annular gap (m^2)
a_z	acceleration in z direction (m/s^2)
b	outer radius (m)

b_o	characteristic length (m)
B	coefficient (dimensionless)
c	clearance (m), swirl ratio (dimensionless)
c_{eff}	effective swirl ratio (dimensionless)
c_p	specific heat capacity at constant pressure (J/kg K)
c_{p*}	dimensionless specific heat capacity at constant pressure (dimensionless)
c_v	specific heat capacity at constant volume (J/kg K)
c_{wave}	wave speed (m/s)
C	vortex strength, coefficient
C_1	constant (1/s), constant (dimensionless), first radiation constant ($W \cdot m^2$)
C_2	constant (m^2/s), constant (dimensionless), second radiation constant (m K)
C_d	discharge coefficient (dimensionless)
C_D	drag coefficient (dimensionless)
C_f	skin friction coefficient (dimensionless)
C_L	lift coefficient (dimensionless)
C_m	moment coefficient (dimensionless)
C_{mc}	moment coefficient for a rotating cylinder (dimensionless)
C_p	pressure coefficient (dimensionless)
C_{rs}	empirical constant
Cw	nondimensional flow rate (dimensionless)
$Cw_{free\ disc}$	nondimensional flow rate entrained by a free disc (dimensionless)
Cw_{local}	local nondimensional flow rate (dimensionless)
Cw_{min}	minimum nondimensional flow rate to prevent ingress (dimensionless)
d	diameter (m)
d_{bolt}	exposed bolt head diameter (m)
d_h	hydraulic diameter (m)
D	journal diameter (m)
e	roughness grain size (m), eccentricity (m)
E	energy (J)
$E_{\lambda,b}$	spectral emissive power for a blackbody (W/m^3)
Ek	Ekman number (dimensionless)
Eu	Euler number (dimensionless)
f	friction factor (dimensionless), coefficient of friction (dimensionless), Coriolis parameter (1/s)
F	force magnitude (N)
$\mathbf{F}$	body force (N/m^3)
$F_{centrifugal}$	centrifugal force (N)
$F_{Coriolis}$	Coriolis force (N)
F_g	geometrical factor (dimensionless)
F_L	lift force magnitude (N)
F_r	body force in the r direction (N/m^3)
Fr	Froude number (dimensionless)
F_r'	body force in the r direction (N/m^3)
F_x	body force component in the x direction (N/m^3)
F_y	body force component in the y direction (N/m^3)
F_z	body force component in the z direction (N/m^3)
F_z'	body force component in the z direction (N/m^3)

F_ϕ	body force in the azimuthal direction (N/m^3)
F_ϕ'	body force in the azimuthal direction (N/m^3)
g	acceleration due to gravity (m/s^2)
G	gap ratio (dimensionless)
G_1	coefficient
G_2	coefficient
G_3	coefficient
G_4	coefficient
G_5	coefficient
G_c	shroud clearance ratio (dimensionless)
Gr	Grashof number (dimensionless)
Gr_b	Grashof number (dimensionless)
G_λ	spectral radiation
h	film thickness (m)
h_o	minimum film thickness (m)
H	total head (m), height of bolt head (m)
I	moment of inertia (kg m^2)
k	thermal conductivity (W/m K)
k_*	dimensionless thermal conductivity (dimensionless)
k_1	constant (dimensionless)
k_2	constant (dimensionless)
k_3	constant
k_4	constant
k_5	constant
k_o	characteristic thermal conductivity (W/m K)
k_s	sand-grain roughness (m)
k_s'	sand-grain size (m)
K	coefficient (dimensionless)
K_1	velocity shape factor (dimensionless)
K_2	velocity shape factor (dimensionless)
K_{bl}	boundary layer constant (dimensionless)
K_f	form drag correction factor (dimensionless)
K_L	loss coefficient (dimensionless)
K_u	velocity shape factor (dimensionless)
K_v	velocity shape factor (dimensionless)
K_w	stress flow rate parameter (dimensionless)
l_o	characteristic length (m)
l_{pitch}	pitch (m)
L	characteristic length, length (m)
M	Mach number (dimensionless), molecular mass (kg/mol)
m	mass (kg)
$m_{element}$	mass of an arbitrary fluid element (kg)
$\dot{m}$	mass flow rate (kg/s)
$\dot{m}_d$	mass flow rate in the boundary layer (kg/s)
$\dot{m}_{in}$	inflow mass flow rate (kg/s)
$\dot{m}_o$	mass flow rate in the rotor boundary layer (kg/s)
$\dot{m}_{out}$	outflow mass flow rate (kg/s)
$\dot{m}_{ref}$	reference mass flow rate (kg/s)

$\dot{m}_s$	mass flow rate in the stator boundary layer (kg/s)
$\dot{m}_{\text{superposed}}$	superposed mass flow rate (kg/s)
M_z	axial Mach number (dimensionless)
n	exponent (dimensionless)
$\boldsymbol{n}$	outward normal unit vector (m)
N	number of blades (integer), speed (rpm)
N_s	rotational speed (revolutions per second)
P	load capacity (N/m^2)
p	static pressure (Pa)
p'	fluctuating component of static pressure (Pa)
p_∞	static pressure in free stream (Pa)
p_*	dimensionless pressure (dimensionless)
p_a	external static pressure (Pa)
p_{cav}	rotor-stator cavity static pressure (N/m^2)
p_{max}	maximum annulus static pressure (N/m^2)
p_{min}	minimum annulus static pressure (N/m^2)
p_o	characteristic pressure (Pa)
$p_{reduced}$	reduced pressure (Pa)
p_{sat}	saturation vapor pressure (Pa)
Pr	Prandtl number (dimensionless)
q	heat flux (W/m^2)
q_x	volumetric flow rate per unit width in the x direction (m^2/s)
q_y	volumetric flow rate per unit width in the y direction (m^2/s)
$\dot{q}$	flow rate (m^3/s)
Q	flow rate (m^3/s), heat flux (W)
Q_s	side flow rate (m^3/s)
R	common radius (m), radius (m), specific gas constant (J/kg K)
r	distance along the r-axis (m)
$\boldsymbol{r}$	position vector (m)
r_e	radial extent of source region (m)
r_f	final radius (m)
r_i	inner radius (m)
r_m	mean radius (m)
r_o	outer radius (m)
r_s	shaft radius (m)
R_a	centre line average roughness (m)
$\mathfrak{R}$	molar gas constant (J/mol K)
Ra	Rayleigh number (dimensionless)
Re	Reynolds number (dimensionless)
$Re_{critical}$	critical Reynolds number (dimensionless)
Re_d	Reynolds number based on pipe diameter (dimensionless)
Re_x	Reynolds number based on x component of velocity (dimensionless)
Re_y	Reynolds number based on y component of velocity (dimensionless)
Re_z	external flow Reynolds number (dimensionless), Reynolds number based on z component of velocity (dimensionless)
Re_ϕ	rotational Reynolds number (dimensionless)
Re_ϕ'	modified rotational Reynolds number (dimensionless)
$Re_{\phi,local}$	local rotational Reynolds number (dimensionless)

$Re'_{\phi m}$	rotational Reynolds number based on annulus gap (dimensionless)
Ro	Rossby number (dimensionless)
s	gap (m)
s_c	clearance between shroud and rotor (m)
S	Taylor vortex critical speed factor (dimensionless), Sommerfield number (dimensionless)
S_o	solar constant (W/m^2)
St	Stanton number (dimensionless)
t	time (s)
t_o	characteristic timescale (s)
t_*	dimensionless time ratio (dimensionless)
T	temperature (K)
T_{av}	average temperature (K)
T_e	emission temperature (K)
T_o	characteristic temperature (K)
T_q	moment, torque (N m)
T_w	wall temperature (K)
T_*	dimensionless temperature (dimensionless)
Ta	Taylor number (dimensionless)
Ta_m	Taylor number based on mean radius (dimensionless)
$Ta_{m,cr}$	critical Taylor number based on mean annulus radius (dimensionless)
u	relative radial velocity (m/s), relative velocity in the x direction (m/s)
$\boldsymbol{u}$	velocity vector in a stationary frame of reference (m/s)
$\boldsymbol{u}'$	fluctuating component of the velocity vector in a stationary frame of reference (m/s)
u_∞	free-stream velocity component (m/s)
u_{dm}	mixed out radial velocity (m/s)
u_{max}	maximum velocity (m/s)
u_r	velocity component in the r direction (m/s)
u_r'	instantaneous radial velocity component (m/s)
u_{r*}	dimensionless radial velocity component (dimensionless)
$u_{resultant}$	resultant velocity magnitude (m/s)
u_x	velocity component in the x direction (m/s)
u_{x*}	dimensionless velocity component in the x direction (dimensionless)
$u_{x,o}$	characteristic velocity component in the x direction (m/s)
u_y	velocity component in the y direction (m/s)
u_{y*}	dimensionless velocity component in the y direction (dimensionless)
$u_{y,o}$	characteristic velocity component in the y direction (m/s)
u_z	velocity component in the z direction (m/s)
u_z'	instantaneous axial velocity component (m/s)
u_{z*}	dimensionless axial velocity component (dimensionless)
$u_{z,\infty}$	free-stream velocity component (m/s)
$u_{z,o}$	characteristic velocity component in the z direction (m/s)
$\bar{u}_z$	average axial component of velocity for the external flow (m/s)
u_τ	friction velocity (m/s)
u_ϕ	velocity component in the tangential direction (m/s)
$u_{\phi'}$	instantaneous tangential velocity component (m/s)
$u_{\phi*}$	dimensionless tangential velocity component (dimensionless)

$u_{\phi,\infty}$	velocity component in the circumferential direction in the free stream (m/s)
$u_{\phi,1}$	velocity component in the tangential direction at the inner radius (m/s)
$u_{\phi,2}$	velocity component in the tangential direction at the outer radius (m/s)
$u_{\phi,c}$	velocity component in the circumferential direction in the inviscid core (m/s)
$u_{\phi,o}$	velocity component in the circumferential direction at the rotor (m/s)
$\bar{u}$	average velocity (m/s)
$\bar{\boldsymbol{u}}$	average velocity vector (m/s)
U	amplitude of velocity (m/s)
U_o	characteristic velocity (m/s), reference velocity (m/s)
v	relative tangential velocity (m/s)
v_c	inviscid rotating core relative tangential velocity (m/s)
v_o	relative rotor tangential velocity (m/s)
v_∞	relative tangential velocity in the free-stream (m/s)
V	volume (m^3)
$V_{paraboloid}$	volume of the paraboloid (m^3)
w	relative axial velocity (m/s)
$\boldsymbol{w}$	relative velocity vector (m/s)
w_*	dimensionless relative velocity vector (dimensionless)
W	load (N)
x	distance along the x-axis (m), ratio of local to disc outer radius (dimensionless)
x_*	dimensionless location (dimensionless)
x_a	ratio of inner to outer radius (dimensionless)
x_e	ratio of local to disc outer radius defining extent of source region (dimensionless)
$x_{e,downstream}$	ratio of local to disc outer radius defining extent of source region (dimensionless)
$\boldsymbol{X}$	arbitrary vector (m)
y	distance along the y-axis (m), wall distance (m)
y_*	dimensionless location (dimensionless)
y_+	universal coordinate (dimensionless)
$y_{sublayer}$	sublayer thickness (m)
z	distance along the z-axis (m)
z_*	dimensionless axial location (dimensionless)
$\boldsymbol{z_p}$	upward unit vector in pressure coordinates
α	ratio of radial and tangential components of stress (dimensionless), coefficient (dimensionless)
α_o	coefficient for the rotor (dimensionless)
α_r	absorptivity (dimensionless)
α_s	coefficient for the stator (dimensionless)
β	swirl fraction or velocity ratio (dimensionless), coefficient (dimensionless)
β_v	volumetric expansion coefficient (m^3/m^3K)
β_ω	relative vorticity (1/s)
$\beta*$	velocity ratio when the superposed flow rate is zero (dimensionless)
δ	boundary layer thickness (m)
ε	eccentricity ratio (dimensionless), emissivity
ε_m	coefficient (dimensionless)

$\varepsilon_m{}'$	coefficient (dimensionless)
ε_M	coefficient (dimensionless)
η	mass flow ratio (dimensionless)
ζ	dimensionless distance (dimensionless)
ϕ	azimuth angle (rad)
ϕ_{h_o}	angular position of minimum film thickness (^{o})
$\phi_{p_{max}}$	angular position of maximum pressure (^{o})
ϕ_{p_o}	film termination angle (^{o})
$\phi*$	nondimensional circumferential location (dimensionless)
ϕ'	relative azimuth angle (rad)
γ	coefficient (dimensionless), isentropic index (dimensionless)
γ_o	coefficient for the rotor (dimensionless)
γ_s	coefficient for the stator (dimensionless)
κ	thermal diffusivity (m^2/K)
λ	wavelength (m), longitude (^{o})
λ_t	turbulent flow parameter (dimensionless)
λ_m	rotation parameter (dimensionless)
μ	viscosity (Pa s)
μ_e	eddy viscosity (Pa s)
μ_o	reference viscosity (Pa s)
$\mu*$	dimensionless viscosity (dimensionless)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
ρ_d	partial density of dry air (kg/m^3)
ρ_o	reference density (kg/m^3)
ρ_r	reflectivity (dimensionless)
ρ_v	partial density of water vapour (kg/m^3)
$\tilde{\rho}$	density (kg/m^3)
$\rho*$	dimensionless density (dimensionless)
σ	normal stress (N/m^2), Stefan-Boltzmann constant ($Wm^{-2}K^{-4}$)
σ_{im}	ratio of mean axial and tangential components of flow (dimensionless)
σ_x	normal stress in the x direction (N/m^2)
σ_r	normal stress in the r direction (N/m^2)
σ_s	squeeze number (dimensionless)
σ_y	normal stress in the y direction (N/m^2)
σ_z	normal stress in the z direction (N/m^2)
σ_φ	normal stress in the φ direction (N/m^2)
τ	shear stress (N/m^2)
τ_e	wind shear stress in the direction of flow (N/m^2)
τ_o	shear stress at the rotor (N/m^2)
τ_r	radial component of shear stress (N/m^2)
$\tau_{r,o}$	radial shear stress at the rotor (N/m^2)
τ_{rz}	viscous shear stress acting in the z direction, on a plane normal to the r direction (N/m^2)
$\tau_{r\phi}$	viscous shear stress acting in the φ direction, on a plane normal to the r direction (N/m^2)
τ_s	surface shear stress (N/m^2)

τ_{xy}	viscous shear stress acting in the y direction, on a plane normal to the x direction (N/m^2)
τ_{xz}	viscous shear stress acting in the z direction, on a plane normal to the x direction (N/m^2)
τ_{yx}	viscous shear stress acting in the x direction, on a plane normal to the y direction (N/m^2)
τ_{yz}	viscous shear stress acting in the z direction, on a plane normal to the y direction (N/m^2)
τ_{zr}	viscous shear stress acting in the r direction, on a plane normal to the z direction (N/m^2)
τ_{zx}	viscous shear stress acting in the x direction, on a plane normal to the z direction (N/m^2)
τ_{zy}	viscous shear stress acting in the y direction, on a plane normal to the z direction (N/m^2)
$\tau_{z\phi}$	viscous shear stress acting in the φ direction, on a plane normal to the z direction (N/m^2)
τ_{ϕ}	tangential component of shear stress (N/m^2)
$\tau_{\phi,o}$	tangential shear stress at the rotor (N/m^2)
$\tau_{\phi r}$	viscous shear stress acting in the r direction, on a plane normal to the ϕ direction (N/m^2)
$\tau_{\phi z}$	viscous shear stress acting in the z direction, on a plane normal to the ϕ direction (N/m^2)
ω	vorticity (1/s), frequency (Hz), angular velocity (rad/s)
ω	vorticity vector (1/s)
ω_r	radial vorticity component (1/s)
ω_x	vorticity component about the x-axis (1/s)
ω_y	vorticity component about the y-axis (1/s)
ω_z	vorticity component about the z-axis (1/s)
ω_{φ}	azimuthal vorticity component (1/s)
ξ	nondimensional decay rate (dimensionless)
ψ	stream function (m^2/s)
$\Delta C_{m,form}$	form drag moment coefficient (dimensionless)
ΔC_p	nondimensional static pressure difference (Pa)
$\Delta H_{vaporisation}$	enthalpy of vaporization (J/mol)
Δp	static pressure difference (Pa)
Δp_s	circumferentially averaged pressure drop across the shroud (Pa)
ΔT	temperature difference (K)
Δz	depression of surface (m)
Φ	velocity potential function (m^2/s)
Γ	circulation (m^2/s)
Ω	angular velocity magnitude (rad/s)
$\boldsymbol{\Omega}$	angular velocity vector (rad/s)
Ω'	angular velocity of rotating frame of reference (rad/s)
Ω_{cr}	critical angular velocity magnitude (rad/s)
Ω_x	angular velocity component about the x-axis (rad/s)
Ω_y	angular velocity component about the y-axis (rad/s)
Ω_z	angular velocity component about the z-axis (rad/s)

Contents