PHYSICS TEXTBOOK

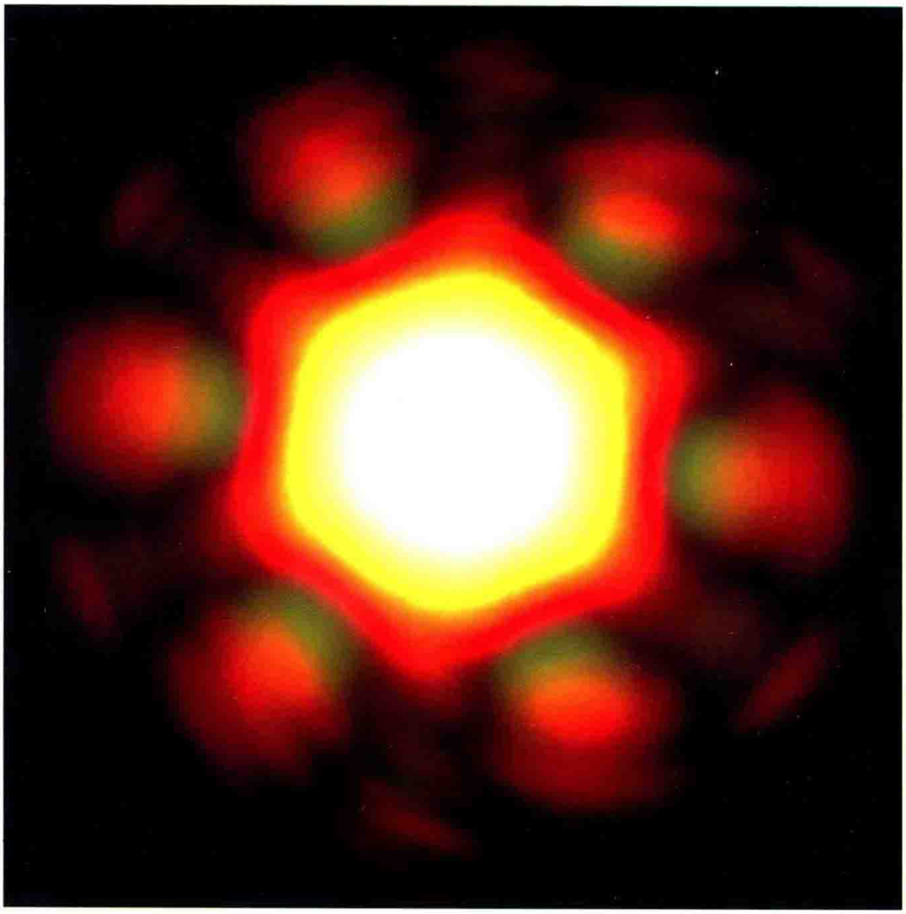
Dieter Meschede

WILEY-VCH

Optics, Light and Lasers

The Practical Approach to Modern Aspects of Photonics and Laser Physics

Second, Revised and Enlarged Edition



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Cover
The picture shows the light field emerging from a photonic crystal fibre excited with red and green light. The fibre which consisted of a solid core and a cladding with a periodic array of 300 nm holes spaced by about 2-3 μ is shown in Fig. 3.23, upper left. The fibre shows striking single mode behaviour no matter how short the wavelength is. In the fibre, red and green light propagate in a common single transverse lobe which appears white due to superposition of red and green. Details on photonic fibres are found in Sect. 3.4.6.

Courtesy of Professor Philip Russell, Max-Planck Research Group, University of Erlangen, Germany. The PCF used was fabricated at the University of Bath, U.K.

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Dieter Meschede

Optics, Light and Lasers

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Preface

Though being taught as a traditional subfield of classical electrodynamics, the field of optics is now once again considered to be an important branch of the physical sciences. Some even say that the 21st century will be the century of the photon, following the era of the electron.

In teaching physics, wave optics and interferometry are important topics with beneficial propaedeutic contributions to the theory of classical fields and quantum mechanics. In lecture halls today we can easily demonstrate wave, i.e., coherence phenomena with laser light sources. It is hence appropriate also in lecturing to devote more room to the concepts of optics created since the 1960s.

This textbook attempts to link the central topics of optics that were established 200 years ago to the most recent research topics such as nonlinear optics, laser cooling or photonic materials. To compromise between depth and breadth, it is assumed that the reader is familiar with the formal concepts of electrodynamics and also basic quantum mechanics. This new edition has not only grown by an entire new chapter introducing the field of quantum optics. It also presents new material describing the rapidly rising role of photonic materials and fibres. Last but not least about 100 problems with varying degrees of difficulty have been included.

In scientific education, this textbook may serve as a reference for the foundations of modern optics: classical optics, laser physics, laser spectroscopy, concepts of quantum optics, nonlinear optics as well as applied optics may profit. Teaching will be complemented through materials presented by new media such as the internet. Nevertheless, the author strongly believes that conventional textbooks will continue to be a prime source of learning. Novel materials and complements will be made available, however, through the following website: www.uni-bonn.de/iap/oll.

Bonn, October 2006 *Dieter Meschede*

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1 Light rays

1.1 Light rays in human experience

The formation of an image is one of our most fascinating emotional experiences. Even in ancient times it was realized that our 'vision' is the result of rectilinearly propagating light rays, because everybody was aware of the sharp shadows of illuminated objects. Indeed, rectilinear propagation may be influenced by certain optical instruments, e.g. by mirrors or lenses. Following the successes of Tycho Brahe (1546–1601), knowledge about *geometrical optics* made for the consequential design and construction of magnifiers, microscopes and telescopes. All these instruments serve as aids to vision. Through their assistance, 'insights' have been gained that added to our world picture of natural science, because they enabled observations of the world of both micro- and macro-cosmos.

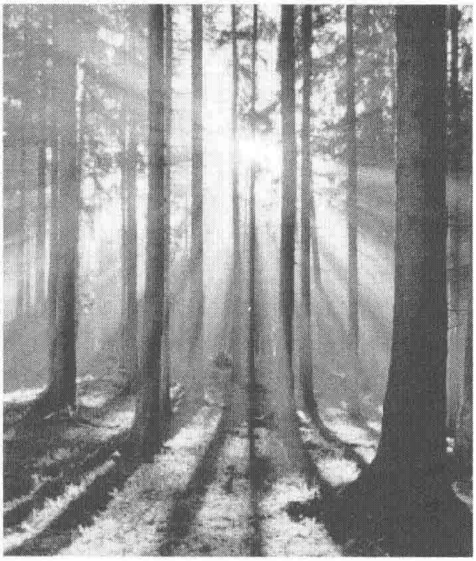

Fig. 1.1 Light rays.

Thus it is not surprising that the terms and concepts of optics had tremendous impact on many areas of natural science. Even such a giant instrument as the new Large Hadron Collider (LHC) particle accelerator in Geneva is basically nothing other than an admittedly very elaborate microscope, with which we are able to observe the world of elementary particles on a subnuclear length scale. Perhaps as important for the humanities is the wave theoretical description of optics, which spun off the development of quantum mechanics.

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In our human experience, rectilinear propagation of light rays – in a homogeneous medium – stands in the foreground. But it is a rather newer understanding that our ability to see pictures is caused by an optical image in the eye. Nevertheless, we can understand the formation of an image with the fundamentals of ray optics. That is why this textbook starts with a chapter on ray optics.

1.2 Ray optics

When light rays spread spherically into all regions of a homogeneous medium, in general we think of an idealized, point-like and isotropic luminous source at their origin. Usually light sources do not fulfil any of these criteria. Not until we reach a large distance from the observer may we cut out a nearly parallel beam of rays with an aperture. Therefore, with an ordinary light source, we have to make a compromise between intensity and parallelism, to achieve a beam with small divergence. Nowadays optical demonstration experiments are nearly always performed with laser light sources, which offer a nearly perfectly parallel, intense optical beam to the experimenter.

When the rays of a beam are confined within only a small angle with a common optical axis, then the mathematical treatment of the propagation of the beam of rays may be greatly simplified by linearization within the so-called 'paraxial approximation'. This situation is met so often in optics that properties such as those of a thin lens, which go beyond that situation, are called 'aberrations'.

The direction of propagation of light rays is changed by refraction and reflection. These are caused by metallic and dielectric interfaces. Ray optics describes their effect through simple phenomenological rules.

1.3 Reflection

We observe reflection of, or mirroring of light rays not only on smooth metallic surfaces, but also on glass plates and other dielectric interfaces. Modern mirrors may have many designs. In everyday life they mostly consist of a glass plate coated with a thin layer of evaporated aluminium. But if the application involves laser light, more often dielectric multi-layer mirrors are used; we will discuss these in more detail in the chapter on interferometry (Chap. 5). For ray optics, the type of design does not play any role.

1.3.1
Planar mirrors

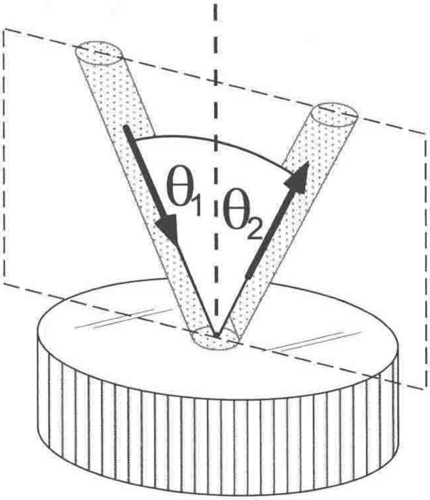

Fig. 1.2 Reflection at a planar mirror.

We know intuitively that at a planar mirror like in Fig. 1.2 the *angle of incidence* θ_1 is identical with the *angle of reflection* θ_2 of the reflected beam,

$$\theta_1 = \theta_2, \tag{1.1}$$

and that incident and reflected beams lie within a plane together with the surface normal. Wave optics finally gives us a more rigid reason for the laws of reflection. Thereby also details like, for example, the intensity ratios for dielectric reflection (Fig. 1.3) are explained, which cannot be derived by means of ray optics.

1.4
Refraction

At a planar dielectric surface, like e.g. a glass plate, reflection and transmission occur concurrently. Thereby the transmitted part of the incident beam is 'refracted'. Its change of direction can be described by a single physical quantity, the 'index of refraction' (also: refractive index). It is higher in an optically 'dense' medium than in a 'thinner' one.

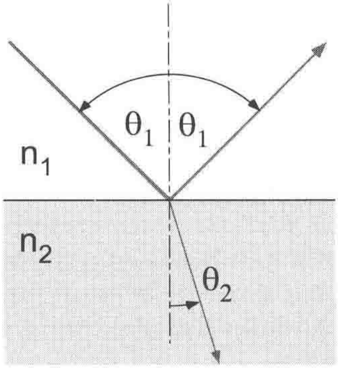

Fig. 1.3 Refraction and reflection at a dielectric surface.

In ray optics a general description in terms of these quantities is sufficient to understand the action of important optical components. But the refractive index plays a key role in the context of the macroscopic physical properties of dielectric matter and their influence on the propagation of macroscopic optical waves as well. This interaction is discussed in more detail in the chapter on light and matter (Chap. 6).

1.4.1
Law of refraction

At the interface between an optical medium '1' with refractive index n_1 and a medium '2' with index n_2 (Fig. 1.3) Snell's law of refraction (Willebrord Snell, 1580–1626) is valid,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{1.2}$$

where θ_1 and θ_2 are called the angle of incidence and angle of emergence at the interface. It is a bit artificial to define two absolute, material-specific refractive indices, because according to Eq. (1.2) only their ratio $n_{12} = n_1/n_2$ is determined at first. But considering the transition from medium '1' into a third material '3' with n_{13}, we realize that, since $n_{23} = n_{21}n_{13}$, we also know the properties of refraction at the transition from '2' to '3' . We can prove this relation, for example by inserting a thin sheet of material '3' between '1' and '2'. Finally, fixing the refractive index of vacuum to $n_{\text{vac}} = 1$ – which is argued within the context of wave optics – the specific and absolute values for all dielectric media are determined.

In Tab. 1.1 on p. 11 we collect some physical properties of selected glasses. The refractive index of most glasses is close to $n_{\text{glass}} = 1.5$. Under usual atmospheric conditions the refractive index in air varies between 1.000 02 and 1.000 05. Therefore, using $n_{\text{air}} = 1$, the refraction properties of the most important optical interface, i.e. the glass–air interface, may be described adequately in terms of ray optics. Nevertheless, small deviations and variations of the refractive index may play an important role in everyday optical phenomena in the atmosphere (for example, a mirage, p. 7).

1.4.2
Total internal reflection

According to Snell's law, at the interface between a dense medium '1' and a thinner medium '2' ($n_1 > n_2$), the condition (1.2) can only be fulfilled for angles smaller than the critical angle θ_c,

$$\theta < \theta_c = \sin^{-1}(n_2/n_1). \tag{1.3}$$

For $\theta > \theta_c$ the incident intensity is totally reflected at the interface. We will see in the chapter on wave optics that light penetrates into the thinner medium for a distance of about one wavelength with the so-called 'evanescent' wave, and that the point of reflection does not lie exactly at the interface (Fig. 1.4). The existence of the evanescent wave enables the application of the so-called 'frustrated' total internal reflection, e.g. for the design of polarizers (Sect. 3.7.4).

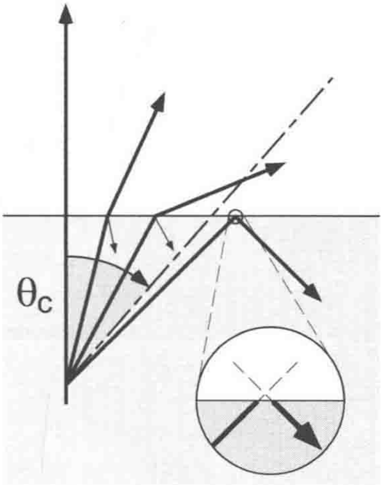

Fig. 1.4 Total internal reflection at a dielectric surface occurs for angles $\theta > \theta_c$. The point of reflection of the rays does not lie exactly within the interface, but slightly beyond (the Goos–Haenchen effect [68, 146]).

1.5 Fermat's principle: the optical path length

As long as light rays propagate in a homogeneous medium, they seem to follow the shortest geometric path from the source to a point, making their way in the shortest possible time. If refraction occurs along this route, then the light ray obviously no longer moves on the geometrically shortest path.

The French mathematician Pierre de Fermat (1601–1665) postulated in 1658 that in this case the light ray should obey a *minimum principle*, moving from the source to another point along the path that is *shortest in time*.

For an explanation of this principle, one cannot imagine a better one than that given by the American physicist Richard P. Feynman (1918–1988), who visualized Fermat's principle with a human example: One may imagine Romeo discovering his great love Juliet at some distance from the shore of a shallow, leisurely flowing river, struggling for her life in the water. Without thinking, he runs straight towards his goal – although he might have saved valuable time if he had taken the longer route, running the greater part of the distance on dry land, where he would have achieved a much higher speed than in the water.

Considering this more formally, we determine the time required from the point of observation to the point of the drowning maiden as a function of the geometric path length. Thereby we find that the shortest time is achieved exactly when a path is chosen that is refracted at the water–land boundary. It fulfils the refraction law (1.2) exactly, if we substitute the indices of refraction

n_1 and n_2 by the inverse velocities in water and on land, i.e.

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}.$$

According to Fermat's minimum principle, we have to demand the following. The propagation velocity of light in a dielectric c_n is reduced in comparison with the velocity in vacuum c by the refractive index n:

$$c_n = c/n.$$

Now the *optical path length* along a trajectory C, where the refractive index n depends on the position $\mathbf{r}$, can be defined in general as

$$\mathcal{L}_{\text{opt}} = c \int_C \frac{ds}{c/n(\mathbf{r})} = \int_C n(\mathbf{r})\, ds. \tag{1.4}$$

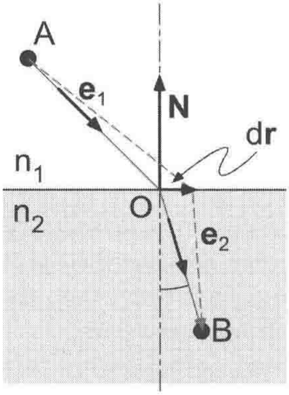

Fig. 1.5 Fermat's principle and refraction at a dielectric surface.

With the tangential unit vector $\mathbf{e}_t$, the path element $ds = \mathbf{e}_t \cdot d\mathbf{r}$ along the path can be calculated.

Example: Fermat's principle and refraction
As an example of the use of the integral principle, we will again consider refraction at a dielectric surface and this time vary the length of the optical path between the points A and B in Fig. 1.5 ($\mathbf{r}_{AO}$ = vector from A to O etc., $\mathbf{e}_{1,2}$ = unit vectors). Since the path must be minimal it cannot change with small modifications $\delta\mathbf{r}' = \mathbf{r}'_{OB} - \mathbf{r}_{OB} = \mathbf{r}_{OA} - \mathbf{r}'_{OA}$. Thus

$$\begin{aligned} \mathcal{L}_{\text{opt}} &= n_1\mathbf{e}_1 \cdot \mathbf{r}_{AO} + n_2\mathbf{e}_2 \cdot \mathbf{r}_{OB}, \\ \delta\mathcal{L}_{\text{opt}} &= (n_1\mathbf{e}_1 - n_2\mathbf{e}_2) \cdot \delta\mathbf{r}'. \end{aligned}$$

In the homogeneous regions light has to follow a line, thus variations can only occur along the surface with the normal $\mathbf{N}$, i.e. $\delta\mathbf{r}' = \mathbf{N}\times\delta\mathbf{r}$. We use the commutativity of the triple product,

$$(n_1\mathbf{e}_1 - n_2\mathbf{e}_2) \cdot \delta\mathbf{r}' = (n_1\mathbf{e}_1 - n_2\mathbf{e}_2) \cdot (\mathbf{N}\times\delta\mathbf{r}) = ((n_1\mathbf{e}_1 - n_2\mathbf{e}_2)\times\mathbf{N}) \cdot \delta\mathbf{r},$$

and find minimal variation for

$$(n_1\mathbf{e}_1 - n_2\mathbf{e}_2) \times \mathbf{N} = 0.$$

This relation is a vectorial formulation of Snell's law (1.2), reproducing it immediately.

1.5.1 Inhomogeneous refractive index

In general, the index of refraction of a body is not spatially homogeneous, but has underlying, continuous, even though small, fluctuations like the material itself, which affect the propagation of light rays: $n = n(\mathbf{r})$. We observe such fluctuations in, for example, the flickering of hot air above a flame. From the phenomenon of mirages, we know that efficient reflection may arise like in the case of grazing incidence at a glass plate, even though the refractive index decreases only a little bit towards the hot bottom.

Again using the idea of the integral principle, this case of propagation of a light ray may also be treated by applying Fermat's principle. The contribution of a path element ds to the optical path length is $d\mathcal{L}_{\text{opt}} = n\,ds = n\mathbf{e}_{\text{t}} \cdot d\mathbf{r}$, where $\mathbf{e}_{\text{t}} = d\mathbf{r}/ds$ is the tangential unit vector of the trajectory. On the other hand $d\mathcal{L}_{\text{opt}} = \nabla\mathcal{L}_{\text{opt}} \cdot d\mathbf{r}$ is valid in accordance with Eq. (1.4), which yields the relation

$$n\mathbf{e}_{\text{t}} = n\frac{d\mathbf{r}}{ds} = \nabla\mathcal{L}_{\text{opt}} \quad \text{and} \quad n^2 = (\nabla\mathcal{L}_{\text{opt}})^2,$$

which is known as the *eikonal equation* in optics. We get the important *ray equation* of optics, by differentiating the eikonal equation after the path,[1]

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n. \tag{1.5}$$

A linear equation may be reproduced for homogeneous materials ($\nabla n = 0$) from (1.5) without difficulty.

Example: Mirage

As a short example we will treat reflection at a hot film of air near the ground, which induces a decrease in air density and thereby a reduction of the refractive index. (Another example is the propagation of light rays in a gradient wave guide – Sect. 1.7.3.) We may assume in good approximation that for calm air the index of refraction increases with distance y from the bottom, e.g. $n(y) = n_0(1 - \varepsilon e^{-\alpha y})$. Since the effect is small, $\varepsilon \ll 1$ is valid in general, while the scale length α is of the order $\alpha = 1\,\text{m}^{-1}$. We look at Eq. (1.5) for $\mathbf{r} = (y(x), x)$ for all individual components and find for the x coordinate with constant C

$$n\frac{dx}{ds} = C.$$

1) Thereby we apply $d/ds = \mathbf{e}_{\text{t}} \cdot \nabla$ and

$$\frac{d}{ds}\nabla\mathcal{L} = (\mathbf{e}_{\text{t}} \cdot \nabla)\nabla\mathcal{L} = \frac{1}{n}(\nabla\mathcal{L} \cdot \nabla)\nabla\mathcal{L} = \frac{1}{2n}\nabla(\nabla\mathcal{L})^2 = \frac{1}{2n}\nabla n^2.$$