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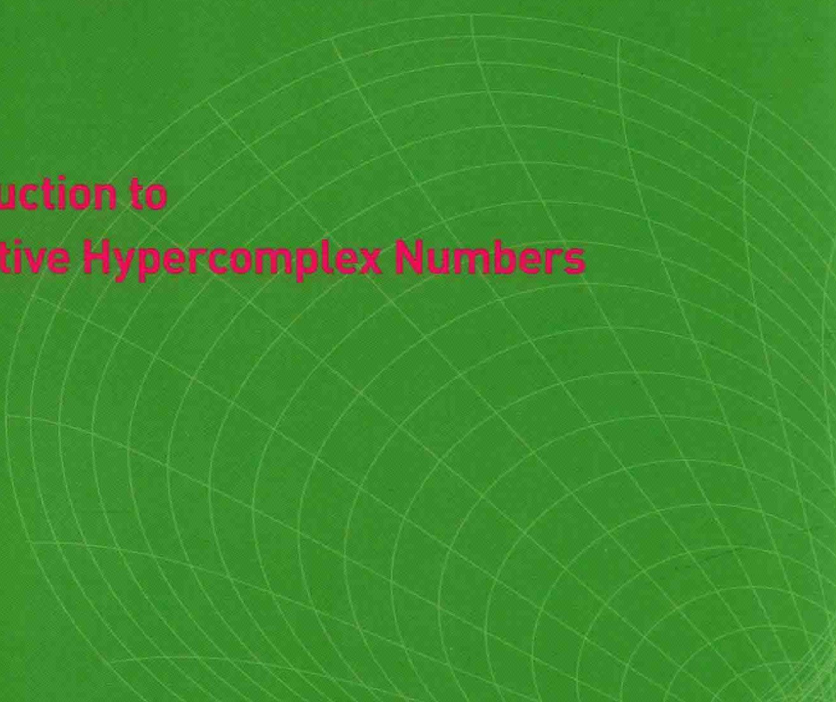
The Mathematics of Minkowski Space-Time

With an Introduction to Commutative Hypercomplex Numbers



Frontiers in Mathematics

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Non omnes arbusta iuvant humilesque myricae

To our friend and colleague Mario Lizzi,
who always strove for a non-humdrum life,
for his fundamental help in all the steps of our research.

Preface

This book arises from original research of the authors on hypercomplex numbers and their applications ([8] and [15]–[23]). Their research concerns extensions to more general number systems of both well-established applications of complex numbers and of functions of a complex variable.

Before introducing the contents of the book, we briefly recall the epistemological relevance of *Number* in the development of Western Science. In his "Metaphysics of number", Pythagoras considered reality, at its deepest level, as mathematical in nature. Following Pythagoras, Plato (Timaeus) explained the world by the regular polygons and solids of Euclidean geometry, laying a link between Number, Geometry and Physical World that represents the foundation of Modern Science. Accordingly, Galileo (Il Saggiatore § 6) took geometry as the language of Nature. These ideas that may appear trivial to modern rationalism, still have their own validity. For example, imaginary numbers make sense of algebraic equations which, from a geometrical point of view, could represent problems that admit no solutions. Despite such an introduction, complex numbers are strictly related to Euclidean geometry (Chap. 3) and allow formalizing Euclidean trigonometry (Chap. 4). Moreover, their functions are the means of representing the surface of the Earth on a plane (Chap. 8). In more recent times, another astonishing coincidence has been added to the previous ones: the space-time symmetry of two-dimensional Special Relativity, which, after Minkowski, is called Minkowski geometry, has been formalized [18] by means of *hyperbolic numbers*, a number system which represents the simplest extension of complex numbers [81].

Finally, N-dimensional Euclidean geometries and number theory have found a unified language by means of "Clifford algebra" [14], [42] and [45], which has allowed a unified formalization of many physical theories.

In this book, we expose first the same thread of association between *numbers* and geometries; secondly we show how the applications of the functions of a complex variable can be extended. In particular, after providing the basics of the classical theory of hypercomplex numbers, we show that with the commutative systems of hypercomplex numbers, a geometry can be associated. All these geometries except the one associated with complex numbers, are different from the Euclidean ones. Moreover the geometry associated with hyperbolic numbers is as distinctive as the Euclidean one since it matches the two-dimensional space-time geometry. This correspondence allows us to formalize space-time geometry and trigonometry with the same rigor as the Euclidean ones. As a simple application, we obtain an exhaustive solution of the "twin paradox". We suggest that, together with the introductory Sect. 2.2, these topics could be used as background for a university course in two-dimensional hyperbolic numbers and their application to space-time geometry and physics, such as *the mathematics of two-dimensional Special Relativity*.

After such algebraic applications of hyperbolic numbers, we broaden the study of space-time symmetry by introducing the functions of a hyperbolic variable. These functions allow us to extend the studies usually performed in Euclidean space, by means of functions of a complex variable, to two-dimensional space-time varieties. In addition, to offer to a larger audience the opportunity of appreciating these topics, we provide a brief discussion of both the introductory elements of Gauss' differential geometry and the classical treatment of constant curvature surfaces in Euclidean space. Nevertheless, for a better understanding, the reader should have a good knowledge of advanced mathematics, such as the theory of functions of a complex variable and elements of differential geometry.

The applications of hyperbolic numbers to Special Relativity may increase interest in multidimensional commutative hypercomplex systems. For these systems, functions can be introduced in the same way as for complex and hyperbolic variables. Therefore, we introduce in three appendices an outline of a research field that should be further developed for both a more complete mathematical formalization and an examination of physical applications.
In Appendices A and B, we begin the study of commutative hypercomplex systems with the four-unit system that has two relevant properties.

- Four unities closely recall the four-dimensional space-time.
- Their two-dimensional subsystems are given by complex and hyperbolic numbers whose applicative relevance is shown in the book.

Coming back to hypercomplex number systems, their algebraic theory was completed at the beginning of the XXth century [76] and concluded, in our view, with the article *Théorie des nombres* written by E. Cartan for the French edition of the *Enciclopédie des sciences mathématiques* [13]. This article is an extensive revision of E. Study's article for the German edition of the Encyclopedia (*Enzyklopädie der Mathematischen Wissenschaften*). Both these authors made contributions to the development of the theory of hypercomplex numbers. Today these numbers are included as a part of abstract algebra [46], and only a few uncorrelated papers introduce their functions. Therefore, to give new insights and inspiration to scientists interested in other fields (not abstract algebra), in Appendix C we give a rigorous and self-consistent exposition of algebra and function theory for commutative hypercomplex numbers by means of matrix formalism, a mathematical apparatus well known to the scientific community.

As a final observation, we remark that in this book many different mathematical fields converge as a confirmation of David Hilbert's assertion: *Mathematics is an organism that keeps its vital energy from the indissoluble ties between its various parts*, and — we shall add following Klein, who refers to Riemann's ideas — *from the indissoluble ties of Mathematics with Physics and, more generally, with Applied Sciences.*

Since the content involves different fields, this book is addressed to a larger audience than the community of mathematicians. As a consequence, also the language employed is aimed at this larger audience.

It is a pleasant task for us to thank Prof. Stefano Marchiafava of Rome University "La Sapienza", for useful discussions and encouragement in many steps of our work.

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