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Rudi Zagst
RiskLab GmbH
Arabellastraße 4
D-81925 München
e-mail: zagst@risklab.de
and
Zentrum Mathematik
Technische Universität München
D-80290 München
e-mail: zagst@ma.tum.de

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Meinen Eltern !

Als ich klein und hilflos war,
gabt Ihr mir Geborgenheit und Schutz.
Als ich größer wurde und Weisheit suchte,
halft Ihr mir, sie zu finden.
Als ich meinen Weg suchen mußte,
habt Ihr mich gehen lassen.
Als ich ihn fand,
halft Ihr mir, meine Ziele zu erreichen.
Als ich dieses Buch schrieb,
habe ich gemerkt, daß ich mich viel zu selten bei Euch bedankt habe.

Vielen Dank dafür,
daß ich mich stets auf Eure Liebe und Treue verlassen konnte.
Es war und ist ein Glück, daß es Euch gibt !

Preface

Who gains all his ends did set the level too low.

Although the history of trading on financial markets started a long and possibly not exactly definable time ago, most financial analysts agree that the core of mathematical finance dates back to the year 1973. Not only did the world's first option exchange open its doors in Chicago in that year but Black and Scholes published their pioneering paper [BS73] on the pricing and hedging of contingent claims. Since then their explicit pricing formula has become the market standard for pricing European stock options and related financial derivatives. In contrast to the equity market, no comparable model is accepted as standard for the interest-rate market as a whole. One of the reasons is that interest-rate derivatives usually depend on the change of a complete yield curve rather than only one single interest rate. This complicates the pricing of these products as well as the process of managing their market risk in an essential way. Consequently, a large number of interest-rate models have appeared in the literature using one or more factors to explain the potential changes of the yield curve. Beside the Black ([Bla76]) and the Heath-Jarrow-Morton model ([HJM92]) which are widely used in practice, the LIBOR and swap market models introduced by Brace, Gątarek, and Musiela [BGM97], Miltersen, Sandmann, and Sondermann [MSS97], and Jamshidian [Jam98] are among the most promising ones. However, up to now, none of the existing models can be considered as more than a standard for a sub-market such as the cap or swap market.

Inconsistencies usually appear once these models are to be used for pricing other interest-rate derivatives jointly.

To understand all the different interest-rate models, and to be able to develop new models, one needs a thorough background in stochastic calculus and financial mathematics. Excellent books for the advanced reader in this field are, e.g., Lamberton and Lapeyre [LL97], Musiela and Rutkowski [MR97], or Øksendal [Øks98]. On the other hand, there are also books written for a more economics oriented readership. Very good representatives, e.g., are Hull [Hul00] or Baxter and Rennie [BR96]. Books aiming for a middle way between these two species are, for instance, the excellent texts of Bingham and Kiesel [BK98] or Korn and Korn [KK99]. However, none of these books addresses the complete financial engineering process, i.e. modelling, pricing, hedging as well as medium and long-term risk and asset management. And indeed, this is the main reason

...why I have written this book.

In many discussions with my students at the universities of Ulm, Augsburg and Munich, as well as during my courses and consulting activities for banks, insurance companies, and other financial institutions, the question appeared of whether there is a book describing the whole process - from mathematical modelling and pricing to the risk and asset management of a complete portfolio or trading book. A list of different books has been the best advice I could give. Then some years ago, when we were discussing this very topic during a car ride from Munich to Ulm, a good friend of mine, Dr. Gerhard Scheuenstuhl, encouraged me to close this gap by writing a book about both sides of the coin, the mathematical modelling and the risk management. Of course, covering the whole story would have been a daunting task and would have resulted in many more pages than you hold in your hand. The background material of stochastic calculus had to be restricted, as well as the number of models and derivatives being discussed and the topics covering risk management issues. However, it was the aim of the author to give an insight into the long road of modelling an interest-rate market, mark-to-market a selection of interest-rate derivatives and simulate their future value using the market model (mark-to-future), as well as deriving valuable risk numbers applied within a reliable risk management process. So after all this,

...what is this book about?

We begin with an overview of the most important mathematical tools for describing financial markets, i.e. stochastic processes and martingales. These methods are applied to modelling a financial market and, in particular, to modelling an interest-rate market. We will learn about different interest-rate models driving the prices of financial assets, as well as different methods for pricing interest-rate derivatives. These are a pure application of the martingale theory, an application of the theory of Green's functions, and an application of the important change-of-numéraire technique. Each of these methods is applied within a specific model showing the wide spectrum of possibilities we have in the evaluation of financial products. However, it was not possible to cover all or even most of the existing interest-rate models available in the literature. Also it was not the intention of the author to compete with the books of Lamberton and Lapeyre, Musiela and Rutkowski, or Øksendal which go much further into the mathematical details than this book. Rather, we follow the books of Bingham and Kiesel or Korn and Korn on their middle way through mathematical finance before we leave their path to aim for the measuring and management of market risk. Short- and long-term risk measures will be discussed, as well as a selection of optimization problems, which are solved to maximize the performance of a portfolio under limited downside risk. Now you may ask

... for whom have I written this book?

This book is written for students, researchers, and practitioners who want to get an insight into the modelling of interest-rate markets as well as the pricing and management of interest-rate derivatives. Chapters 2 and 3 of the book give a rigorous overview of the mathematics of financial markets. They present the most important tools needed to describe the movement of market prices and define the theoretical framework for the pricing and hedging of contingent claims. A basic knowledge of probability theory and a certain quantitative background are recommended as prerequisites. Those looking for a crash course in stochastic processes and the modelling of financial markets will hopefully find this part to be a valuable source. However, if you are already familiar with the Itô calculus and the methods for pricing and hedging financial derivatives, you could immediately start with Chapter 4. Here and in Section 5 we focus on the modelling and pricing in interest-rate markets. If you already know most of the specific interest-rate models you may just glance over Section 4, which shows how an interest-rate market can be embedded in the financial market framework of Section 3. In Chapter 5 we describe the most popular interest-rate derivatives, and show how they can be priced using a specific interest-rate model. Because the trading and risk management of derivatives are dominated mainly by the application of specific models and techniques, the style of the book will also change gradually to a more economics oriented one. Real-world applications have to take care of market conventions such as daycounts or special rates which are, from a mathematical point of view, not too much of a deal but which may vary between different markets and result in misleading risk numbers and prices once they are ignored. Chapters 6 and 7 are intended to give an insight into the practical application of interest-rate models to the risk and portfolio management of interest-rate derivatives. They cover a selection of short- and long-term-oriented risk measures as well as comprehensive case studies based on real market data. We hope those interested in mark-to-future simulations, specific risk numbers, and their use for risk management will enjoy reading these chapters. Should you be more practice-oriented, you will not need a full understanding of stochastic calculus and martingale theory for Sections 5 to 7. A basic understanding of the main results of Chapters 2 to 4 will be sufficient. However, those getting fascinated with the potential of financial modelling may look for the "math necessities" within these chapters. Since all parts of the book have been used in teaching mathematical finance, financial engineering and risk management at different universities, this textbook may also serve as a valuable source for graduate and PhD students in mathematics or finance who want to acquire some knowledge of financial markets and risk management.

A final word!

Satisfying the needs of both practitioners and researchers is always a hard and sometimes too hard a problem to solve. A gap still exists between these two worlds, and accordingly, there remains a gap between the corresponding parts of the book. I have tried to make this gap as small as possible. I also had to restrict myself to a brief overview of stochastic calculus, where a lot more could have been said and proved. On account of the idea and limited size of the book, I had to select a small variety of interest-rate models and discuss their pricing effects rather than show for which market which model works best. The reader interested in this question may, for instance, refer to Brigo and Mercurio [BM01]. I also could not cover all risk-management topics, since this is a boundless field in its own right. Therefore I surely didn't gain all my intended ends. Nevertheless, I hope that I have succeeded in finding a good middle way to describe the process from mathematical modelling and pricing to the risk and asset management of interest-rate derivatives portfolios. Whether I have reached this target will be for the reader to judge.

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Munich, January 2002 Rudi Zagst

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Introduction

Financial markets at all times fascinated people trying to predict price movements and making money out of it. Fortune-tellers and self-made market prophets often tried to influence the market participants by their forecasts, very often not to their own disadvantage. However, following such a forecast and putting all one's eggs in one basket is a very risky thing to do. Of course, if prices move in the investor's favour, he might get rich. But unfortunately, bad things happen. On Black Friday, October 25, 1929 a tremendous market crash finished ten bullish years of increasing stock prices, leaving millions of people with empty pockets and starting one of the most significant depressions of the modern age. On Black Monday, October 19, 1987 the Dow Jones Index lost 23% or 500 points within minutes while it increased by only 1700 points over the previous five years. Could traders have known before about the forthcoming worst-case events?

A lot of people came up with more or less sophisticated statistical examinations and correlations, trying to find some regularities in the market. For example, they found empirical evidence for the hypothesis that *the shorter the skirts, the higher the stock prices.* While, from a real life perspective, this seems to be a little implausible, rules of thumb were set up such as *buy on bad news, sell on good news* or *sell in May and go away,* considering the flow of money or the nervousness of the people. Other rules which are very popular at the trading desks are *the trend is your friend* or *never catch a falling knife,* telling traders not to invest in a falling market. But even the famous Sir Isaac Newton (1643-1727), who lost a fortune at the time of the South Sea Bubble in 1720, sighed: "*I can measure the motion of bodies, but I cannot measure human folly.*"

Nevertheless, researchers tried to model financial markets and build a theoretical framework to explain market price behaviour. In 1829, the botanist R. Brown watched pollen particles under a microscope, observing that they may move according to a so-called Brownian motion. In 1900, L. Bachelier [Bac00] was the first to consider Brownian motion as a tool to describe the behaviour of stock prices. But not until 1923 was this process rigorously defined and constructed by N. Wiener [Wie23]. We will follow his way and give a brief overview of the basic *mathematical concepts* used to describe market price movements in Chapter 2. The definition and characteristics of stochastic processes are given in Sections 2.1 and 2.2. One of the central tools in stochastic calculus is Itô's lemma, which is described in Section 2.4 and can be applied to determine the stochastic differential equation for the prices of financial derivatives. This concept is closely related to the stochastic integral, which is defined in Section 2.3. Since martingales are one of the most important elements for evaluating the prices of financial instruments, they are defined in Section 2.5 and used to describe the prices of financial derivatives in terms of conditional expectations. The Feynman-Kac formula provides the gateway between such conditional expectations and the differential equations which can be solved numerically. This is discussed in Section 2.6.

In Chapter 3 we introduce the basic building blocks and assumptions for setting up a consistent framework to describe a *financial market* using stochastic processes. The primary traded assets and the basic trading principles are defined in Section 3.1. We also give conditions under which the normalized or discounted market prices can be described by martingales. In Section 3.2 we show under which conditions there are no arbitrage opportunities in the financial market. Another important characteristic of a financial market is its completeness, which will be discussed in Section 3.3. In Section 3.4 we show that financial derivatives can be uniquely priced if the financial market is complete. One of the most famous complete market models, the Black-Scholes model, is discussed in Section 3.5. It is used to derive the prices for (European) options on contingent claims. To price financial derivatives it might sometimes be more comfortable to use a numéraire other than the cash account. We could, for example, change the numéraire and use the so-called T−forward measure for describing the discounted market prices. This technique is described in Sections 3.6 and 3.7.

Chapter 4 deals with *interest-rate markets* and the zero-coupon bonds as primary traded assets. Because this market is a specific financial market, we will apply the results of Chapter 3 to embed it in the general framework of Section 3.1. We start by defining the general interest-rate market model in Section 4.1. No-arbitrage and completeness conditions in the interest-rate market model are given in Section 4.2, while Section 4.3 deals with the pricing of interest rate related contingent claims. Since there are infinitely many zero-coupon bonds corresponding to different maturity dates it has been one of the main challenges in interest rate theory to find the driving factors of the zero-coupon bond prices. One of the most general models or frameworks was introduced by Heath, Jarrow, and Morton [HJM92] and will be discussed in Section 4.4. One-factor models, such as the famous short-rate models and the Gaussian models, are discussed in Sections 4.5. A brief overview of multi-factor models can be found in Section 4.6. The LIBOR market models, a new class of models describing the behaviour of market rates rather than that of the short or forward short rates, is presented in Section 4.7. None of these models deal with the possibility of zero-rate changes because of defaults in the financial market. Therefore, we give an overview of credit risk models in Section 4.8.

Many years were to pass between the first steps of Bachelier and Wiener and the introduction of standardized *financial products* and financial markets ready to support the exchange of these instruments on a common platform. Describing all of the existing financial products would have been a heroic task, and was not in the intention of this book. Nevertheless, Chapter 5 is dedicated to describing and pricing at least the most important financial instruments, which are all built on zero-coupon bonds as primary traded assets and may be enriched by additional conditions such as optionality or agreements with respect to future points in time. We start with a brief discussion on how the financial market defines the time between two specific dates in Section 5.1. Probably the simplest financial instrument derived from the primary traded assets is a portfolio of zero-coupon bonds which is, under special assumptions on their notional amount, called a coupon bond and described in Section 5.2. Coupon and zero-coupon bonds are the underlying instruments for the forward agreements and futures discussed in Sections 5.3 and 5.4. Zero bonds are also the main building block for another family of interest-rate instruments, the interest-rate swaps, which are presented in Section 5.5. Probably the easiest option in interest-rate markets is an option on a zero-coupon bond, which will be priced in Section 5.6.1. It is the basic tool for evaluating a great variety of optional interest-rate instruments. Examples are the caps and floors of Section 5.6.2, as well as the coupon bond options of Section 5.6.3. In Section 5.6.4 we show how options on interest-rate swaps can be priced. All the previous interest-rate options are considered to be market standard. Contingent claims with payoffs more complicated than that of standard (European) interest-rate call and put options are called exotic interest-rate options. An overview of some of these products is given in Section 5.7. All financial instruments are described from a practical point of view and priced according to the theoretical framework of the previous chapters. But the quality of pricing of all these instruments pretty much depends on the availability of good market or price information, especially with respect to yields and volatilities. Each model we use for pricing interest-rate derivatives has to be fitted to market data. Section 5.8 gives an overview of different sources of interest-rate information expressed by yield or zero-rate curves, market prices or volatilities. The latter are always quoted with respect to a benchmark model which is, most of the time, a version of the Black model. We will give a little more detail in Sections 5.8.2 and 5.8.3. A practical case study on how Black volatility information can be transformed to an implied volatility curve for the Hull-White model is shown in Section 5.8.4.

In 1991, the Basle Comittee on Banking Supervision set up directions for the risk management of derivatives and defined *market risk* to be the risk of a negative impact of changing market prices for the financial situation of an institution. For interest-rate instruments this risk is due to changes of the yield or zero-rate curve. Traditionally, it was measured by a parallel shift of the yield curve, but many risk and portfolio managers had to learn about the possibility of non-parallel movements of the yield curve, which is sometimes called shape risk. It is in the focus of this book to give an overview of how market risk can be measured and managed. In Section 6.1 we define the different risk measures based on small market movements and small periods of time, such as the first- and second-order sensitivities. Examples are the Black or key-rate deltas and gammas as well as duration and convexity. Beside this short-term sensitivity risk portfolio managers will change their portfolio if the risk of the portfolio return falling below a given benchmark is too high. This considers large market movements as well as longer time horizons and is also known as downside risk. It is usually carried out by a scenario analysis and involves calculating the portfolio's profit or loss over a specified period of time under a variety of different scenarios. The scenarios can either be chosen by the management or generated by a specific interest-rate model as it was discussed in Chapter 4. Using these scenarios, different measures of downside risk, such as the lower partial moments or the value at risk, can be calculated as discussed in Section 6.2. An interesting question is what properties a somehow "good" risk measure should have. We try to answer this question in Section 6.3. Because simulations are often very time-consuming, it may make sense to concentrate on just a few explanatory risk factors. This idea is discussed in Section 6.4.

Having defined the different possibilities for calculating market risk, what kind of risk does a fortune-teller consider? Well, she may look into her cards and see what will happen giving this scenario a probability of one. Unfortunately, we will have to work a little harder to find an adequate probability distribution or even a representative set of market scenarios. One special method, tailor-made for the Hull-White model, is described in the appendix but there is a whole variety of simulation methods available on modern computer systems and programs. So what should we do with these simulations and risk numbers? The answer is *risk management.*

Basically, risk management deals with the problem of protecting a portfolio or trading book against unexpected losses. It therefore expresses the desire of a portfolio manager or trader to guarantee a minimum holding period return or to create a portfolio which helps to cover specific liabilities over time. Done right, risk management may avoid extreme movements of the portfolio value, reduce the tracking error or even the trading costs. However, there are different possibilities to set up a risk-management or hedging process which are adressed in Chapter 7. If we are interested in controlling short-term risk or if we like to hedge against small movements in market prices, we may choose a sensitivity-based risk management. This method is described in Section 7.1. If we are dealing with a longer time horizon, or want to be safe against large market movements, we may prefer a downside risk management, as introduced and discussed in Section 7.2. We explicitly state the corresponding optimization programs and prove that they solve the given practical problems. The differences and possibilities of the methods described are documented by extensive practical case studies, showing that risk management is much more than setting some parameters to zero. These case studies may also help to emphasize that high quality risk management is one of the key requirements for a modern portfolio management.