MATHEMATICS: *with Applications in the Management, Natural, and Social Sciences*

Lial & Miller

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MATHEMATICS

Preface

This text introduces the mathematical ideas needed by students of management, social science, and biology. We assume that the students using this text had a course in intermediate algebra, but that many of these students do not have a complete mastery of algebra fundamentals. Thus, the first three chapters (part I) provide a review, reinforcement, and extension of those ideas from algebra needed for a successful study of the text. Well-prepared students can skim, or even skip, much of these chapters.

The book is divided into four parts of approximately equal length. Part I, the introductory chapters, is explained above. Part II covers matrix theory and linear programming. Part III discusses probability, statistics, and decision theory. Part IV is a discussion of the elements of calculus. Each part is virtually self-contained, so that the parts can be studied in the permutation best fitting your curriculum needs.

A unique feature of this text is the many examples and cases we have chosen to illustrate the mathematical ideas discussed. Being able to say to the student, "Here is an example of that result as used by McDonald's, or Montgomery Ward, or Levi Strauss," makes the concepts of the course much more alive. Biological examples are chosen from the important areas of genetics, cancer research, ecology, and pollution control, among others.

The mathematical aspects of the examples and cases have been emphasized, rather than the business or biological details. Throughout the entire text we assume no special knowledge of business, biology, or economics on the part of the reader. The few necessary special terms from these fields are all explained whenever used.

This text has been designed so that it can be used for a large variety of teaching approaches. Any section titled "Applications of . . ." is optional, in the sense that nothing discussed in that section is needed for further mathematical development. These sections can be discussed as time and curriculum needs require. Some typical courses based on this book include the following:

One-year course in finite mathematics with calculus—Use the entire book. For complete coverage in one year, it may be necessary to treat some of the applications selectively. In this way, there should be enough time to cover the essential mathematical topics.

One-semester, or one- or two-quarter course in finite mathematics—Use parts II and III of the text. Some students may need a review of the methods of graphing straight lines, discussed in Chapter 2.

One-semester, or one- or two-quarter course in calculus—Use parts I and IV. Treat part I lightly and part IV heavily, say in the ratio of 3 to 5.

Survey of mathematics—Much of this material can be used quite well for students in a survey course for liberal-arts majors. Such students like the applications. To them it makes the mathematics seem much more alive.

Many people helped us to write this book. Vern Heeren of American River College gave his usual excellent advice. Chris Siragusa, of the Philco-Ford Corporation, provided useful help with the business examples. The examples and cases in this text were obtained by sending out over 200 letters to leading people in business and biology. We asked these people to send us mathematical examples in their fields that would be suitable in a beginning text. The response to our request was both substantial and gratifying. Some people sent us actual examples that they had worked on, while others suggested interesting articles in journals. In any event, this text would not contain the wealth of practical examples that it does without their help.

Kathleen Seifert and Zallia Todd typed much of the text. We would also like to thank two people at Scott, Foresman: Nat Weintraub is an editor who demands the best from his authors; and Robert Runck is a directing editor who fully understands the art of producing a book.

Margaret L. Lial
Charles D. Miller

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Part One

Relations and Sets
1

The one idea that occurs throughout every main area of contemporary mathematics is that of a relation, and especially the relation called a function. Relations and functions occur often in applications. Thus, we shall consider examples in which the supply of a commodity is a function of the price, the number of people in a city is related to time, and so on. We shall discuss relations and functions at the end of this chapter, and then use these ideas again and again throughout the book. However, before we can discuss relations, we need to look at some topics necessary to an understanding of relations. A brief treatment of sets and set notation begins the chapter, followed by a discussion of several sets of numbers and their properties. A section reviewing methods of finding solution sets of equations then leads to the work on relations.

1.1 Sets

Sets are perhaps the one mathematical idea most commonly associated in the public mind with the "new math" of the late 1950's and early 1960's. While proponents of the "new math" may have gone a bit overboard in making claims about sets, a brief treatment of sets is very beneficial, for the following reasons. (1) Sets help clarify and classify the ideas under discussion. (2) Sets make some mathematical ideas (such as the idea of a relation, discussed later) easier to understand. (3) When discussing probability, sets are useful in discussing events and the number of elements in an event.

Think of a **set** as a collection of objects. Thus, we can speak of the set of all products manufactured by General Motors, or the set of all species of bacteria in a given culture, or the set of all families in a clan. Sets are written using **set braces**, $\{\ \}$. Thus, the set whose objects are the numbers 5, 6, and 7 can be written

$$\{5, 6, 7\}.$$

The objects belonging to a set are called the **elements**, or **members**, of the set. The numbers 5, 6, and 7 are the elements of the set above. We can write

$$5 \in \{5, 6, 7\}$$

to express the fact that 5 is an element of the set $\{5, 6, 7\}$; while

$$8 \notin \{5, 6, 7\}$$

expresses the fact that 8 is not an element of the set $\{5, 6, 7\}$. We often designate sets by capital letters, so that if

$$B = \{5, 6, 7\}$$

we can write, for example, $6 \in B$, $7 \in B$, and $3 \notin B$.

Two sets are **equal** if they contain exactly the same elements. The sets $\{5, 6, 7\}$, $\{7, 6, 5\}$, and $\{6, 5, 7\}$ all contain exactly the same elements and hence are equal. This can be expressed by writing

$$\{5, 6, 7\} = \{7, 6, 5\} = \{6, 5, 7\}.$$

Sets which do not contain exactly the same elements are **not equal**. For example, the sets $\{5, 6, 7\}$ and $\{5, 6, 7, 8\}$ do not contain exactly the same elements and thus are not equal. We can write this as follows:

$$\{5, 6, 7\} \neq \{5, 6, 7, 8\}.$$

When discussing a particular situation or problem, we can usually identify a **universal set** (whether expressed or implied), which contains all the elements appearing in any set used in the given situation or problem. By convention, the symbol U is used to represent the universal set. For example, when discussing the set of employees of a company who favor a certain pension proposal, we might choose the universal set to be the set of all company employees. In discussing the types of animals found by Charles Darwin on the Galápagos Islands, the universal set might be the set of all species on all Pacific islands. Choice of a universal set is often arbitrary, and depends on the problem under discussion.

Sometimes, every element of one set also belongs to another set. For example, if

$$A = \{3, 4, 5, 6\}$$

and

$$B = \{2, 3, 4, 5, 6, 7, 8\},$$

then A is a **subset** of B, written $A \subset B$. For example, the set of all presidents of corporations is a subset of the set of all executives of corporations. Use the definition of subset to verify that if $E = \{3, 4, 5\}$, then $E \subset E$. In fact, using the definition of subset, it can be shown that every set is a subset of itself, or, in symbols, if A is any set, then

$$A \subset A.$$

Figure 1.1 shows a drawing which represents a set A which is a subset of a set B. The rectangle of the drawing represents the universal set. Diagrams like this are called **Venn diagrams**. We shall use Venn diagrams as an aid in clarifying and discussing sets and set ideas.

The set of all bacteria of diameter greater than two centimeters contains no elements. Such a set is called the **empty set** or **null set**, and symbolized $\varnothing$. To see that the empty set, $\varnothing$, satisfies the statement made above that every set is a subset of itself, first rephrase the definition of subset as follows: $A \subset B$ means that there is no element of A that is not an element of B. Using this rephrased definition of subset, verify that since $\varnothing$ contains no elements, it contains no elements that are not in $\varnothing$. Hence, $\varnothing \subset \varnothing$. In the same way, verify that if B is any set, then

$$\varnothing \subset B.$$

Given a set A, and a universal set U, we can form the set of all elements of U that do not belong to A. This set is called the **complement** of A, and is denoted A' (read "A prime"). For example, if $A = \{2, 4\}$ and $U = \{1, 2, 3, 4, 5\}$, then we can find set A' by listing all the elements of U that are not in A. Here $A' = \{1, 3, 5\}$. The Venn diagram of Figure 1.2 represents a set B and its complement B'.

Often, a description of a set gives more information than a list of the elements in the set. For example, it is perhaps

Figure 1.1 $A \subset B$

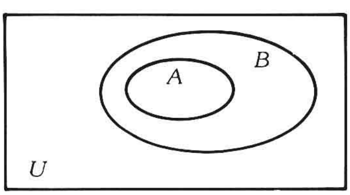
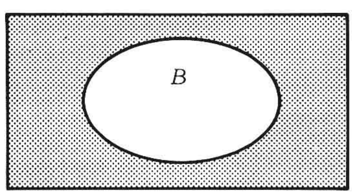


Figure 1.2 B' *is shaded*



difficult to see a common property of the elements in the set

$$\{\text{Alaska, Hawaii, California, Oregon, Washington}\},$$

but if we express the same set as

$$\{x \mid x \text{ is a state that touches the Pacific}\}$$

then the particular property of the elements that is intended is clear. This second set is read "the set of all elements x such that x is a state that touches the Pacific." A set expressed in this way is said to be written in **set builder notation**.

Given two sets A and B, the set of all elements belonging to both set A and set B is called the **intersection** of the two sets, written $A \cap B$. For example, the elements that belong to both $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 4, 5, 7, 9, 11\}$ are 2, 4, 5, and 7, so that

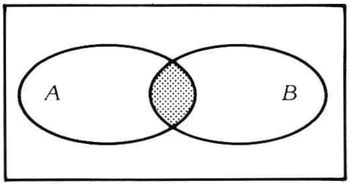
$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}.$$

In set builder notation,

$$A \cap B = \{x \mid x \in A \textit{ and } x \in B\}.$$

Note the use of the word *and*. The intersection of two sets requires all the elements that belong to *both* sets at the same time. The intersection of two sets is "no larger" than either of the two given sets. That is, for any two sets A and B, we have $(A \cap B) \subset A$ and $(A \cap B) \subset B$. The Venn diagram of Figure 1.3 represents two sets A and B, and their intersection, the set $A \cap B$.

Figure 1.3 *$A \cap B$ is shaded*



Two sets that have no elements in common are called **disjoint** sets. For example, there are no elements common to both $\{50, 51, 54\}$ and $\{52, 53, 55, 56\}$, so that these two sets are disjoint. Verify that

$$\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \varnothing.$$

The result of this example can be generalized: if A and B are any two disjoint sets, then $A \cap B = \varnothing$. (Also, if $A \cap B = \varnothing$, then A and B are disjoint sets.)

The set of all elements belonging to set A or set B is called the **union** of the two sets, written $A \cup B$. For example,

$$\{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}.$$

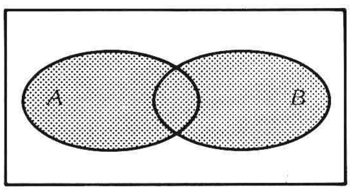
In set builder notation,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\},$$

where *or* means "one, or the other, or both."

The union of two sets is "no smaller" than either of the two given sets. That is, for any two sets A and B, we have $A \subset (A \cup B)$ and $B \subset (A \cup B)$. The Venn diagram of Figure 1.4 shows two sets A and B, and their union, $A \cup B$.

Figure 1.4 *A ∪ B is shaded*



Example 1 Find the intersection and union of the following pair of sets:

$$\{1, 3, 5, 7\} \qquad \text{and} \qquad \{2, 4, 6\}.$$

Since these two sets are disjoint,

$$\{1, 3, 5, 7\} \cap \{2, 4, 6\} = \varnothing.$$

For the union, we have

$$\{1, 3, 5, 7\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6, 7\}.$$

Example 2 For the following sets A and B, find $A \cap B$ and $A \cup B$:

$$A = \{x \mid x \text{ is a television set made in the U.S.A.}\},$$

$$B = \{x \mid x \text{ is a color television set}\}.$$

Using the definitions given above, we have

$$A \cap B = \{x \mid x \text{ is a color set made in the U. S.}\}$$

$$A \cup B = \{x \mid x \text{ is a television set made in the U. S. or } x \text{ is a color television}\}.$$