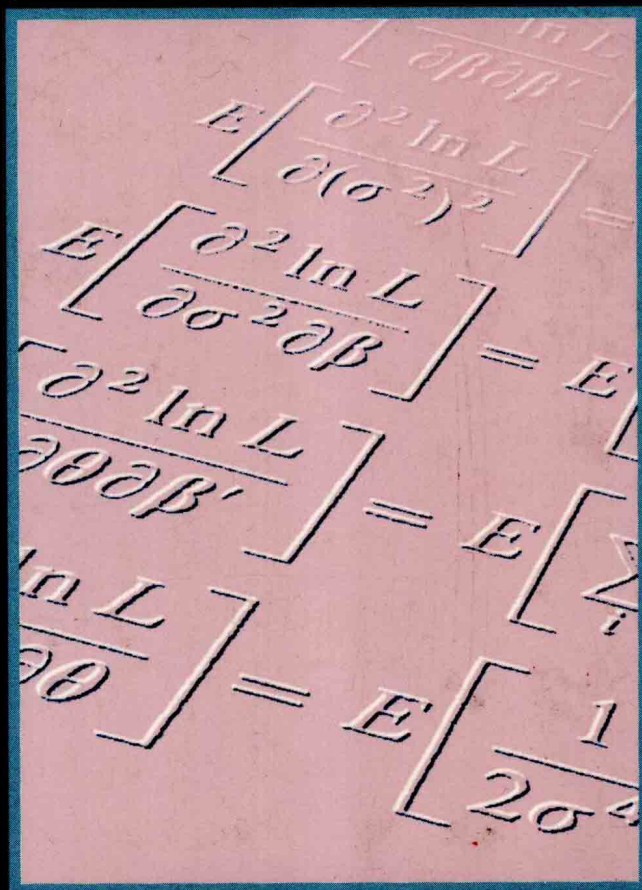


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# ECONOMETRIC ANALYSIS

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Second Edition



William H. Greene

# **ECONOMETRIC ANALYSIS**

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**William H. Greene**

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# PREFACE

This book is intended for a one-year graduate course in econometrics. The prerequisites for that course would include calculus, basic mathematical statistics, and an introduction to econometrics at the level of, say, Gujarati's *Basic Econometrics* (McGraw-Hill, 1988) or Maddala's *Introduction to Econometrics* (Macmillan, 1992). I have included in Chapters 2 through 4 self-contained summaries of the matrix algebra and statistical theory used later in the book. The remainder of the book is intended to provide an up-to-date summary of econometric methods. This includes the traditional treatment of the multiple linear regression model as well as some recent developments in estimation and hypothesis testing. The latter include GMM estimation, Lagrange multiplier and conditional moment tests, testing for unit roots in macroeconomic data, the analysis of panel data, and limited dependent variables.

Readers may wonder what has motivated a second edition so soon (three years) after the first. One consideration was some second thoughts about some of the presentations. But with this edition I hope to offer students an accessible treatment of some important topics which, surprisingly, still remain absent from even the most recently published texts. These include estimation by the method of moments (GMM), conditional moment testing, and models of duration. The latter was deliberately and, I now believe, mistakenly omitted from my first edition. The former has simply become too widespread to neglect. Also, my earlier disclaimer notwithstanding, I hope that Chapter 19 has satisfactorily updated the treatment of at least some topics in time-series analysis.

I have attempted to keep the mathematical level consistent throughout. This has meant liberal use of matrix algebra but has required relatively little advanced distribution theory. Still, purists may prefer more in the way of detailed mathematical proofs. I give formal proofs only when they are particularly revealing about some underlying principle that will reappear in other contexts or provide students with a useful tool for their work. White's proof of the limiting distribution of the Wald statistic in Chapter 10 is an example. In contrast, a proof of the central limit theorem, while obviously of great utility on its own, is a one-shot deal. For those who are teaching at a relatively high level and who desire more of the theorem/proof format, I would suggest Peter Schmidt's *Econometrics* (Marcel Dekker, 1976) as a very handy adjunct.

One feature that distinguishes this work from its predecessors is its greater attention to nonlinear models, including full chapters on nonlinear regression and nonlinear optimization. Computer software now in wide use has made estimation of nonlinear models as simple as estimation of linear ones, and the recent literature reflects that progression. The purpose of these chapters is to bring the textbook treatment up to the level of common practice. I have also included two long chapters on limited dependent-variable models. These models are becoming ever more common in the applied literature. I have written

these chapters because there is no other source that presents these topics at a level elementary enough to initiate the newcomer but complete enough to enable a diligent student to use the information to undertake a serious piece of empirical analysis.

It is now generally accepted that training in econometrics must include some exposure to the mechanics of computation. This, it seems to me, should be self-evident. Computers and computer software have come a long way from the days when one trudged across campus in the snow to hand a deck of cards to the operator of a hostile mainframe (only to find out the next day that the entire job had aborted because of a comma punched in the wrong column). The greatest advance has been the appearance of microcomputers, which have changed both teaching and research. One now has on a desktop as much computer power as was once contained in a room-sized mainframe. What this means for teaching is that students can be given realistic data sets and challenging empirical analyses as routinely as theoretical exercises. To this end, I have included in this book a large number of data sets, many of which have been used in studies already in the literature. In addition, the appendix to Chapter 20 contains a yearly data set on a number of macroeconomic variables. (Quarterly data are available from the same sources.) These could be used, for example, to update Klein's model I or, for the more ambitious, to estimate a new model.

There are many computer programs that students can use in an econometrics course. Their most important features are ease of use and flexibility; the same program can easily be used for many different types of analyses. A partial list of the general econometrics programs now in use is as follows:

ET*	Econometric Software, Bellport, New York (general econometrics, tobit, probit, matrix algebra)
GAUSS*	Aptech Systems, Kent, Washington (matrix programming language, maximum likelihood)
LIMDEP	Econometric Software, Bellport, New York (limited and qualitative dependent variables and general econometrics)
MicroTSP*	Quantitative Micro Software, Irvine, California (forecasting and time-series models, nonlinear regression)
RATS	VAR Econometrics, Minneapolis, Minnesota (regression analysis of time series)
SAS	SAS Institute, Cary, North Carolina (general econometrics and modeling)
SHAZAM	Professor Ken White, University of British Columbia (general econometrics, multiple-equations models, tobit, probit)
SORITEC	Sorities Group, Springfield, Virginia (large-scale multiple-equation modeling)
SPSS	SPSS, Inc., Chicago, Illinois (general statistical analysis for social scientists)
SST	Dubin Rivers Research, Pasadena, California (regression, maximum likelihood, limited dependent variables)
TSP	TSP International, Palo Alto, California (general econometrics, linear and nonlinear multiple-equation models, time-series analysis)

Most of these programs are available in both mainframe and microcomputer versions. Those marked with an asterisk were written especially for personal computers. All are general-purpose programs. Their primary differences (apart from price) are their range of techniques (suggested in the listing), the amount of programming required of the user, and the level of difficulty of command entry. For the last of these, programs vary from those with commands that are very low level, such as Gauss, which is a programming language,

to those that use powerful single commands to invoke large processors that automate many complex computations, such as TSP's LSQ procedures. Since the tastes and needs of users will vary, prospective users should contact the authors for information about the programs. Journals such as *The American Statistician* and the *Journal of Applied Econometrics* also publish reviews of particular programs. Finally, a clearing house for information about software is the Centre for Computing in Economics at the University of Bristol in the United Kingdom.

It is a pleasure to express my appreciation to those who have influenced this work (some inadvertently). I would add my name to the long list of practitioners, teachers, and authors who have thanked Arthur Goldberger for his contribution to their education. Dennis Aigner and Laurits Christensen were also influential in shaping my views on econometrics. The number of students and colleagues whose input has helped produce what you find here is far too large to allow me to thank them individually. I do owe a debt to Aline Quester, whose persistent questioning and encouragement strongly influenced Chapters 21 and 22, to David Hensher and Donald Waldman, who allowed me to cite their unpublished work in these chapters, and to Martin Evans and Paul Wachtel, whose suggestions helped to shape Chapter 19.

This book has benefited at several stages from the careful reading of many reviewers, including Badi H. Baltagi, University of Houston; Leonard A. Carlson, Emory University; Chris Cornwell, University of Georgia; Michael Ellis, Wesleyan University; K. Rao Kadiyala, Purdue University; William Lott, University of Connecticut; Edward J. Mathis, Villanova University; Thad Mirer, State University of New York at Albany; Peter J. Schmidt, Michigan State University; Terry G. Seaks, University of North Carolina at Greensboro; Donald Snyder, California State University, Los Angeles; Houston H. Stokes, University of Illinois at Chicago; Mark Watson, Harvard University; and Kenneth D. West, Princeton University. The empirical work has been improved by a thorough review by Ken White of the University of British Columbia. This edition will also reflect many of the suggestions of those too numerous to thank individually who wrote, called, and e-mailed (a new verb?) to comment on the first edition. I would also like to thank Jack Repcheck, who initiated my first edition; Ron Harris and Jill Lectka, of Macmillan; and Diane Belleville, of NYU, for their contribution to the completion of this book. I owe special thanks to Terry Seaks, whose painstaking review went far beyond the call of duty. I owe my greatest debt to my wife, Lynne, and to my daughters, Lesley, Allison, Elizabeth, and Juliana.

W. H. G.

# CONTENTS

<b>1</b>	<b>INTRODUCTION</b>	<b>1</b>
1.1.	Econometrics	1
1.2.	Econometric Modeling	1
1.3.	Theoretical and Applied Econometrics	3
1.4.	Plan of the Book	3
<b>2</b>	<b>MATRIX ALGEBRA</b>	<b>5</b>
2.1.	Introduction	5
2.2.	Some Terminology	5
2.3.	<b>Algebraic Manipulation of Matrices</b>	<b>6</b>
2.3.1.	Equality of Matrices	7
2.3.2.	Transposition	7
2.3.3.	Matrix Addition	7
2.3.4.	Matrix Multiplication	8
2.3.5.	Sums of Values	10
2.3.6.	A Useful Idempotent Matrix	11
2.4.	<b>Geometry of Matrices</b>	<b>13</b>
2.4.1.	Vector Spaces	13
2.4.2.	Linear Combinations of Vectors and Basis Vectors	14
2.4.3.	Linear Dependence	16
2.4.4.	Subspaces	16
2.4.5.	Rank of a Matrix	17
2.4.6.	Determinant of a Matrix	19
2.4.7.	A Least Squares Problem	21
2.5.	<b>Solution of a System of Equations</b>	<b>23</b>
2.5.1.	Systems of Linear Equations	23
2.5.2.	Inverse Matrices	24
2.5.3.	Nonhomogeneous Systems of Equations	26
2.6.	<b>Partitioned Matrices</b>	<b>26</b>
2.6.1.	Addition and Multiplication of Partitioned Matrices	26
2.6.2.	Determinants of Partitioned Matrices	27
2.6.3.	Inverses of Partitioned Matrices	27
2.6.4.	Deviations from Means	28
2.6.5.	Kronecker Products	28

<b>2.7.</b>	<b>Characteristic Roots and Vectors</b>	<b>29</b>
2.7.1.	The Characteristic Equation	29
2.7.2.	Characteristic Vectors	30
2.7.3.	General Results for Characteristic Roots and Vectors	31
2.7.4.	Diagonalization of a Matrix	31
2.7.5.	Rank of a Matrix	32
2.7.6.	Condition Number of a Matrix	32
2.7.7.	Trace of a Matrix	33
2.7.8.	Determinant of a Matrix	34
2.7.9.	Spectral Decomposition of a Matrix	34
2.7.10.	Powers of a Matrix	34
2.7.11.	Idempotent Matrices	36
2.7.12.	Factoring a Matrix	36
2.7.13.	The Generalized Inverse of a Matrix	37
<b>2.8.</b>	<b>Quadratic Forms and Definite Matrices</b>	<b>38</b>
2.8.1.	Nonnegative Definite Matrices	38
2.8.2.	Idempotent Quadratic Forms	39
2.8.3.	Ranking Matrices	39
<b>2.9.</b>	<b>Calculus and Matrix Algebra</b>	<b>40</b>
2.9.1.	Differentiation and the Taylor Series	41
2.9.2.	Optimization	44
2.9.3.	Constrained Optimization	46
2.9.4.	Transformations	48
<b>Exercises</b>		<b>49</b>

### 3 PROBABILITY AND DISTRIBUTION THEORY

53

<b>3.1.</b>	<b>Introduction</b>	<b>53</b>
<b>3.2.</b>	<b>Random Variables</b>	<b>53</b>
3.2.1.	Probability Distributions	53
3.2.2.	Cumulative Distribution Function	54
<b>3.3.</b>	<b>Expectations of a Random Variable</b>	<b>55</b>
<b>3.4.</b>	<b>Some Specific Probability Distributions</b>	<b>57</b>
3.4.1.	The Normal Distribution	58
3.4.2.	The Chi-Squared, $t$ , and $F$ Distributions	58
3.4.3.	Distributions with Large Degrees of Freedom	59
3.4.4.	Size Distributions—The Lognormal Distribution	60
3.4.5.	The Gamma Distribution	61
3.4.6.	The Beta Distribution	61
<b>3.5.</b>	<b>The Distribution of a Function of a Random Variable</b>	<b>61</b>
<b>3.6.</b>	<b>Joint Distributions</b>	<b>63</b>
3.6.1.	Marginal Distributions	64
3.6.2.	Expectations in a Joint Distribution	64
3.6.3.	Covariance and Correlation	65
3.6.4.	Distribution of Functions of Bivariate Random Variables	66
<b>3.7.</b>	<b>Conditioning in a Bivariate Distribution</b>	<b>67</b>
3.7.1.	Regression—The Conditional Mean	67
3.7.2.	Conditional Variance	68



- 3.7.3. Relationships Among Marginal and Conditional Moments 69
- 3.7.4. The Analysis of Variance 71
- 3.8. The Bivariate Normal Distribution 72**
- 3.9. Multivariate Distributions 73**
  - 3.9.1. Moments 73
  - 3.9.2. Sets of Linear Functions 74
  - 3.9.3. Nonlinear Functions 75
- 3.10. The Multivariate Normal Distribution 75**
  - 3.10.1. Marginal and Conditional Distributions 76
  - 3.10.2. Linear Functions of a Normal Vector 77
  - 3.10.3. Quadratic Forms in a Standard Normal Vector 77
    - 3.10.3a. Idempotent Quadratic Forms 77
    - 3.10.3b. Independence of Idempotent Quadratic Forms 79
  - 3.10.4. The  $F$  Distribution 79
  - 3.10.5. A Full Rank Quadratic Form 79
  - 3.10.6. Independence of a Linear and a Quadratic Form 80
- Exercises 81**
- Appendix: Integration—The Gamma Function 85**

## **4 STATISTICAL INFERENCE**

**87**

- 4.1. Introduction 87**
- 4.2. Samples and Sampling Distributions 87**
  - 4.2.1. Random Sampling 87
  - 4.2.2. Descriptive Statistics 88
  - 4.2.3. Sampling Distributions 90
- 4.3. Point Estimation of Parameters 91**
  - 4.3.1. Estimation in a Finite Sample 93
  - 4.3.2. Efficient Unbiased Estimation 95
- 4.4. Large-Sample Distribution Theory 99**
  - 4.4.1. Convergence in Probability 99
  - 4.4.2. Convergence in Distribution—Limiting Distributions 102
  - 4.4.3. Asymptotic Distributions 106
  - 4.4.4. Asymptotic Distribution of a Nonlinear Function 108
  - 4.4.5. Asymptotic Expectations 109
- 4.5. Efficient Estimation—Maximum Likelihood 110**
  - 4.5.1. Properties of Maximum Likelihood Estimators 114
  - 4.5.2. Estimating the Variance of the Maximum Likelihood Estimator 115
- 4.6. Consistent Estimation—The Method of Moments 117**
  - 4.6.1. Random Sampling and Estimating the Parameters of Distributions 117
  - 4.6.2. Computing the Variance of a Method of Moments Estimator 121
- 4.7. Interval Estimation 123**
- 4.8. Hypothesis Testing 125**
  - 4.8.1. Testing Procedures 126
  - 4.8.2. Tests Based on Confidence Intervals 128

4.8.3.	Three Asymptotically Equivalent Test Procedures	129
4.8.3a.	The Likelihood Ratio Test	130
4.8.3b.	The Wald Test	131
4.8.3c.	The Lagrange Multiplier Test	133
4.8.4.	An Example of the Test Procedures	134
4.8.4a.	Confidence Interval Test	135
4.8.4b.	Likelihood Ratio Test	135
4.8.4c.	Wald Test	135
4.8.4d.	Lagrange Multiplier Test	136
<b>Exercises</b>	<b>136</b>	

## **5 THE CLASSICAL LINEAR REGRESSION MODEL 140**

<b>5.1.</b>	<b>Introduction</b>	<b>140</b>
<b>5.2.</b>	<b>Specifying the Regression Model—An Example</b>	<b>140</b>
<b>5.3.</b>	<b>The Assumptions of the Linear Regression Model</b>	<b>143</b>
5.3.1.	Functional Form and Nonlinear Models	144
5.3.2.	The Regressor	146
5.3.3.	The Disturbance	146
<b>5.4.</b>	<b>Least Squares</b>	<b>148</b>
5.4.1.	The Least Squares Coefficients	148
5.4.2.	Evaluating the Fit of the Regression	150
<b>5.5.</b>	<b>Statistical Properties of the Least Squares Estimator</b>	<b>155</b>
<b>5.6.</b>	<b>Statistical Inference</b>	<b>159</b>
5.6.1.	Estimating the Sampling Distribution	159
5.6.2.	Testing a Hypothesis About $\beta$	160
5.6.3.	Tests Based on the Fit of the Regression	162
<b>5.7.</b>	<b>Prediction</b>	<b>164</b>
<b>Exercises</b>	<b>166</b>	

## **6 MULTIPLE REGRESSION 170**

<b>6.1.</b>	<b>Introduction</b>	<b>170</b>
<b>6.2.</b>	<b>Assumptions of the Linear Model</b>	<b>170</b>
<b>6.3.</b>	<b>Least Squares Regression</b>	<b>172</b>
6.3.1.	The Least Squares Coefficient Vector	173
6.3.2.	Some Examples	174
6.3.3.	Algebraic Aspects of the Solution	177
6.3.4.	Partitioned Regression and Partial Regression	179
6.3.5.	Partial Regression and Partial Correlation Coefficients	180
6.3.6.	Deviations from Means—Regression on a Constant	181
<b>6.4.</b>	<b>Statistical Properties of the Least Squares Estimator</b>	<b>182</b>
6.4.1.	Nonstochastic Regressors	182
6.4.2.	Stochastic Regressors	183
<b>6.5.</b>	<b>Statistical Inference</b>	<b>184</b>
6.5.1.	Testing a Hypothesis About a Coefficient	184
6.5.2.	Testing a Linear Restriction	187
6.5.3.	Test Statistics with Stochastic $\mathbf{X}$ and Normal $\epsilon$	190

<b>6.6.</b>	<b>Goodness of Fit and the Analysis of Variance</b>	<b>191</b>
<b>6.7.</b>	<b>Testing the Significance of the Regression</b>	<b>194</b>
<b>6.8.</b>	<b>Prediction</b>	<b>195</b>
6.8.1.	A Convenient Method of Computing the Forecasts	196
6.8.2.	Measuring the Accuracy of Forecasts	197
<b>Exercises</b>		<b>198</b>

## **7 HYPOTHESIS TESTS WITH THE MULTIPLE REGRESSION MODEL 203**

<b>7.1.</b>	<b>Introduction</b>	<b>203</b>
<b>7.2.</b>	<b>Testing Restrictions</b>	<b>203</b>
7.2.1.	Two Approaches to Testing Hypotheses	203
7.2.2.	Testing a Set of Linear Restrictions	204
7.2.3.	The Restricted Least Squares Estimator	205
7.2.4.	Testing the Restrictions	206
7.2.5.	Examples and Some General Procedures	206
<b>7.3.</b>	<b>Tests of Structural Change</b>	<b>211</b>
7.3.1.	Different Parameter Vectors	211
7.3.2.	Different Constant Terms	212
7.3.3.	Change in a Subset of Coefficients	213
7.3.4.	Insufficient Observations	214
<b>7.4.</b>	<b>Tests of Structural Change with Unequal Variances</b>	<b>215</b>
<b>7.5.</b>	<b>Alternative Tests of Model Stability</b>	<b>216</b>
<b>7.6.</b>	<b>Testing Nonlinear Restrictions</b>	<b>218</b>
<b>7.7.</b>	<b>Choosing Between Nonnested Models</b>	<b>222</b>
<b>Exercises</b>		<b>225</b>

## **8 FUNCTIONAL FORM, NONLINEARITY, AND SPECIFICATION 229**

<b>8.1.</b>	<b>Introduction</b>	<b>229</b>
<b>8.2.</b>	<b>Dummy Variables</b>	<b>229</b>
8.2.1.	Comparing Two Means	229
8.2.2.	Binary Variables in Regression	231
8.2.3.	Several Categories	232
8.2.4.	Several Groupings	233
8.2.5.	Threshold Effects	234
8.2.6.	Interactions and Spline Regression	235
<b>8.3.</b>	<b>Nonlinearity in the Variables</b>	<b>238</b>
8.3.1.	Functional Forms	238
8.3.2.	Identifying Nonlinearity	240
8.3.3.	Intrinsic Linearity and Identification	242
<b>8.4.</b>	<b>Specification Analysis</b>	<b>244</b>
8.4.1.	Selection of Variables	244
8.4.2.	Omission of Relevant Variables	245
8.4.3.	Inclusion of Irrelevant Variables	248

<b>8.5.</b>	<b>Biased Estimators and Pretest Estimators</b>	<b>248</b>
8.5.1.	The Mean-Squared-Error Test	249
8.5.2.	Pretest Estimators	251
8.5.3.	Inequality Restrictions	253
<b>8.6.</b>	<b>Bayesian Estimation</b>	<b>255</b>
8.6.1.	Bayesian Analysis of the Classical Regression Model	255
8.6.2.	Estimation with an Informative Prior Density	258
	<b>Exercises</b>	<b>260</b>
<b>9</b>	<b>DATA PROBLEMS</b>	<b>266</b>
<b>9.1.</b>	<b>Introduction</b>	<b>266</b>
<b>9.2.</b>	<b>Multicollinearity</b>	<b>266</b>
9.2.1.	Perfect Collinearity	267
9.2.2.	Near Multicollinearity	267
9.2.3.	The Symptoms of Multicollinearity	267
9.2.4.	Suggested Remedies for the Multicollinearity Problem	270
<b>9.3.</b>	<b>Missing Observations</b>	<b>273</b>
<b>9.4.</b>	<b>Grouped Data</b>	<b>277</b>
<b>9.5.</b>	<b>Measurement Error and Proxy Variables</b>	<b>279</b>
9.5.1.	One Badly Measured Variable	280
9.5.2.	Multiple Regression with Measurement Error	283
9.5.3.	The Method of Instrumental Variables	284
9.5.4.	Proxy Variables	286
9.5.5.	A Specification Test for Measurement Error	287
<b>9.6.</b>	<b>Regression Diagnostics and Influential Data Points</b>	<b>287</b>
	<b>Exercises</b>	<b>289</b>
<b>10</b>	<b>LARGE-SAMPLE RESULTS FOR THE CLASSICAL REGRESSION MODEL</b>	<b>292</b>
<b>10.1.</b>	<b>Introduction</b>	<b>292</b>
<b>10.2.</b>	<b>The Finite-Sample Properties of Least Squares</b>	<b>292</b>
<b>10.3.</b>	<b>Asymptotic Distribution Theory for the Classical Regression Model</b>	<b>293</b>
10.3.1.	Consistency of the Least Squares Coefficient Vector	293
10.3.2.	Asymptotic Normality of the Least Squares Estimator	295
10.3.3.	Asymptotic Distribution of a Function of $\mathbf{b}$ —The Delta Method	297
10.3.4.	Asymptotic Behavior of the Standard Test Statistics	299
10.3.5.	Consistency of $s^2$ and the Estimator of $\text{Asy.Var}[\mathbf{b}]$	301
<b>10.4.</b>	<b>Stochastic Regressors and Lagged Dependent Variables</b>	<b>302</b>
<b>10.5.</b>	<b>Normally Distributed Disturbances</b>	<b>305</b>
10.5.1.	Asymptotic Efficiency—Maximum Likelihood Estimation	305
10.5.2.	Cases in Which Least Squares Is Inefficient	307

10.5.3.	Alternative Estimation Criteria	308
10.5.4.	Detecting Departures from Normality	309
<b>Exercises</b>		<b>311</b>

## **11 NONLINEAR REGRESSION MODELS 314**

<b>11.1.</b>	<b>Introduction</b>	<b>314</b>
<b>11.2.</b>	<b>Nonlinear Regression Models</b>	<b>314</b>
11.2.1.	The Linearized Regression	315
11.2.2.	Nonlinear Least Squares Estimation	316
11.2.3.	A Specification Test for Nonlinear Regressions: Testing for Linear Versus Log-Linear Specification	321
<b>11.3.</b>	<b>Parametric Transformations of the Dependent Variable</b>	<b>324</b>
<b>11.4.</b>	<b>The Box–Cox Transformation</b>	<b>329</b>
11.4.1.	Transforming the Independent Variables	329
11.4.2.	Transforming the Model	331
11.4.3.	A Test for (Log-) Linearity	334
<b>11.5.</b>	<b>Hypothesis Testing and Parametric Restrictions</b>	<b>335</b>
11.5.1.	An Asymptotically Valid $F$ Test	336
11.5.2.	Wald Test	336
11.5.3.	Likelihood Ratio Test	336
11.5.4.	Lagrange Multiplier Test	337
<b>Exercises</b>		<b>341</b>

## **12 AN INTRODUCTION TO NONLINEAR OPTIMIZATION 343**

<b>12.1.</b>	<b>Introduction</b>	<b>343</b>
<b>12.2.</b>	<b>Optimization Problems</b>	<b>343</b>
<b>12.3.</b>	<b>Grid Search</b>	<b>344</b>
<b>12.4.</b>	<b>General Characteristics of Algorithms</b>	<b>344</b>
<b>12.5.</b>	<b>Gradient Methods</b>	<b>346</b>
12.5.1.	Steepest Ascent	346
12.5.2.	Newton's Method	347
12.5.3.	Maximum Likelihood Estimation	347
12.5.4.	Alternatives to Newton's Method	348
12.5.5.	Quasi-Newton Methods—Davidon–Fletcher–Powell	350
<b>12.6.</b>	<b>Some Practical Considerations</b>	<b>350</b>
<b>12.7.</b>	<b>Examples</b>	<b>352</b>
<b>12.8.</b>	<b>The Concentrated Log Likelihood</b>	<b>355</b>
<b>Exercises</b>		<b>356</b>

## **13 NONSPHERICAL DISTURBANCES 358**

<b>13.1.</b>	<b>Introduction</b>	<b>358</b>
<b>13.2.</b>	<b>Consequences for Ordinary Least Squares</b>	<b>359</b>
13.2.1.	Finite-Sample Properties	359
13.2.2.	Asymptotic Properties of Least Squares	360

<b>13.3. Efficient Estimation</b>	<b>361</b>
13.3.1. Generalized Least Squares	361
13.3.2. Maximum Likelihood Estimation	364
<b>13.4. Estimation When <math>\Omega</math> is Unknown</b>	<b>365</b>
13.4.1. Feasible Generalized Least Squares	365
13.4.2. Maximum Likelihood Estimation	366
<b>13.5. The Generalized Method of Moments (GMM) Estimator</b>	<b>370</b>
13.5.1. Method of Moments Estimators	370
13.5.2. Generalizing the Method of Moments	372
13.5.3. Testing the Validity of the Moment Restrictions	374
13.5.4. GMM Estimation of Econometric Models	375
13.5.5. Testing Restrictions	379
<b>Exercises</b>	<b>381</b>
<b>14 HETEROSCEDASTICITY</b>	<b>384</b>
<b>14.1. Introduction</b>	<b>384</b>
<b>14.2. Ordinary Least Squares Estimation</b>	<b>386</b>
14.2.1. Inefficiency of Least Squares	387
14.2.2. The Estimated Covariance Matrix of $\mathbf{b}$	388
14.2.3. Estimating the Appropriate Covariance Matrix for Ordinary Least Squares	391
<b>14.3. Testing for Heteroscedasticity</b>	<b>392</b>
14.3.1. White's General Test	392
14.3.2. The Goldfeld–Quandt Test	393
14.3.3. The Breusch–Pagan/Godfrey Test	394
14.3.4. Testing for Groupwise Heteroscedasticity	395
14.3.5. Tests Based on Regressions—Glesjer's Tests	396
<b>14.4. Generalized Least Squares When <math>\Omega</math> Is Known</b>	<b>397</b>
<b>14.5. Estimation When <math>\Omega</math> Contains Unknown Parameters</b>	<b>399</b>
14.5.1. Two-Step Estimation	399
14.5.2. Maximum Likelihood Estimation	402
<b>14.6. General Conclusions</b>	<b>407</b>
<b>Exercises</b>	<b>408</b>
<b>15 AUTOCORRELATED DISTURBANCES</b>	<b>411</b>
<b>15.1. Introduction</b>	<b>411</b>
<b>15.2. The Analysis of Time-Series Data</b>	<b>413</b>
<b>15.3. Disturbance Processes</b>	<b>415</b>
15.3.1. Characteristics of Disturbance Processes	415
15.3.2. AR(1) Disturbances	416
<b>15.4. Least Squares Estimation</b>	<b>418</b>
15.4.1. OLS Estimation with Lagged Dependent Variables	419
15.4.2. Efficiency of Least Squares	420
15.4.3. Estimating the Variance of the Least Squares Estimator	422
<b>15.5. Testing for Autocorrelation</b>	<b>423</b>
15.5.1. The Durbin–Watson Test	423

15.5.2.	Other Testing Procedures	426
15.5.3.	Testing in the Presence of Lagged Dependent Variables	428
<b>15.6.</b>	<b>Efficient Estimation When <math>\Omega</math> Is Known</b>	<b>428</b>
15.6.1.	Generalized Least Squares	428
15.6.2.	Maximum Likelihood Estimation	430
<b>15.7.</b>	<b>Estimation When <math>\Omega</math> Is Unknown</b>	<b>431</b>
15.7.1.	AR(1) Disturbances	431
15.7.2.	AR(2) Disturbances	435
15.7.3.	Estimation with a Lagged Dependent Variable	435
<b>15.8.</b>	<b>Forecasting in the Presence of Autocorrelation</b>	<b>437</b>
<b>15.9.</b>	<b>Autoregressive Conditional Heteroscedasticity</b>	<b>438</b>
	<b>Exercises</b>	<b>442</b>

## **16 MODELS THAT USE BOTH CROSS-SECTION AND TIME-SERIES DATA 444**

<b>16.1.</b>	<b>Introduction</b>	<b>444</b>
<b>16.2.</b>	<b>Time-Series–Cross-Section Data</b>	<b>444</b>
<b>16.3.</b>	<b>Models of Several Time Series</b>	<b>447</b>
16.3.1.	Cross-Sectional Heteroscedasticity	448
16.3.2.	Cross-Sectional Correlation	452
16.3.3.	Autocorrelation	455
16.3.4.	A Random Coefficients Model	459
16.3.5.	Summary	463
<b>16.4.</b>	<b>Longitudinal Data</b>	<b>464</b>
16.4.1.	A Basic Model of Heterogeneity	465
16.4.2.	Fixed Effects	466
16.4.3.	Random Effects	469
	16.4.3a. Generalized Least Squares	470
	16.4.3b. Feasible Generalized Least Squares When $\Omega$ Is Unknown	474
16.4.4.	Fixed or Random Effects?	479
	<b>Exercises</b>	<b>480</b>

## **17 SYSTEMS OF REGRESSION EQUATIONS 486**

<b>17.1.</b>	<b>Introduction</b>	<b>486</b>
<b>17.2.</b>	<b>The Seemingly Unrelated Regressions Model</b>	<b>487</b>
17.2.1.	Generalized Least Squares	488
17.2.2.	Feasible Generalized Least Squares	489
17.2.3.	Maximum Likelihood Estimation	493
	17.2.3a. Iterated FGLS	493
	17.2.3b. Direct Maximum Likelihood Estimation	493
17.2.4.	Autocorrelation	497
<b>17.3.</b>	<b>Systems of Demand Equations—Singular Systems</b>	<b>499</b>
<b>17.4.</b>	<b>Flexible Functional Forms—Translog Cost Function</b>	<b>503</b>
	<b>Exercises</b>	<b>508</b>

<b>18</b>	<b>REGRESSIONS WITH LAGGED VARIABLES</b>	<b>511</b>
18.1.	Introduction	511
18.2.	Distributed Lag Models	511
18.2.1.	Lagged Effects in a Regression Model	512
18.2.2.	The Lag and Difference Operators	514
18.3.	Unrestricted Finite Distributed Lag Models	515
18.4.	Polynomial Distributed Lag Models	519
18.4.1.	Estimation by Restricted Least Squares	523
18.4.2.	Determining the Degree of the Polynomial	523
18.4.3.	Determining the Lag Length	524
18.5.	The Geometric Lag Model	525
18.5.1.	Economic Models with Geometric Lags	526
18.5.2.	Stochastic Specifications in the Geometric Lag Models	528
18.5.3.	Estimating the Moving Average Form of the Geometric Lag Model	529
18.5.3a.	Uncorrelated Disturbances	529
18.5.3b.	Autocorrelated Disturbances	531
18.5.3c.	Estimating the MA Form of the Partial Adjustment Model	534
18.5.4.	Estimating the Autoregressive Form of the Geometric Lag Model	534
18.5.4a.	Uncorrelated Disturbances	534
18.5.4b.	Autocorrelated Disturbances—Instrumental Variables	535
18.5.4c.	Autoregressive Disturbances—Hatanaka's Estimator and the Maximum Likelihood Estimator	535
18.5.4d.	MA Disturbances—The Adaptive Expectations Model	536
18.5.5.	Disturbance Specifications in the Geometric Lag Model	537
18.6.	Dynamic Regression Models	538
18.6.1.	Nonlinear Least Squares Estimation of ARIMA and ARMAX Models	539
18.6.2.	Computation of the Lag Weights in the ARMAX Model	542
18.6.3.	Stability of a Dynamic Equation	543
18.6.4.	Forecasting	544
	<b>Appendix: Nonlinear Least Squares Estimation Exercises</b>	<b>545</b>
		<b>547</b>
<b>19</b>	<b>TIME-SERIES MODELS</b>	<b>549</b>
19.1.	Introduction	549
19.2.	Stationary Stochastic Processes	549
19.2.1.	Autoregressive–Moving Average Processes	550
19.2.2.	Vector Autoregressions	552



19.2.3.	Stationarity and Invertibility	554
19.2.4.	<i>Autocorrelations of a Stationary Process</i>	557
<b>19.3.</b>	<b>Integrated Processes and Differencing</b>	<b>559</b>
<b>19.4.</b>	<b>Random Walks, Trends, and Spurious Regressions</b>	<b>560</b>
<b>19.5.</b>	<b>Unit Roots in Economic Data</b>	<b>563</b>
<b>19.6.</b>	<b>Cointegration and Error Correction</b>	<b>566</b>
<b>19.7.</b>	<b>Generalized Autoregressive Conditional Heteroscedasticity</b>	<b>568</b>
19.7.1.	Maximum Likelihood Estimation of the GARCH Model	570
19.7.2.	Pseudo-Maximum Likelihood Estimation	574
19.7.3.	Testing for GARCH Effects	575
19.7.4.	An Example	576
<b>Exercises</b>		<b>577</b>

## **20 SIMULTANEOUS EQUATIONS MODELS 578**

<b>20.1.</b>	<b>Introduction</b>	<b>578</b>
<b>20.2.</b>	<b>Fundamental Issues in Simultaneous Equations Models</b>	<b>578</b>
20.2.1.	Illustrative Systems of Equations	579
20.2.2.	A General Notation for Simultaneous Equations Models	582
<b>20.3.</b>	<b>The Problem of Identification</b>	<b>585</b>
20.3.1.	The Rank and Order Conditions for Identification	589
20.3.2.	Identification Through Nonsample Information	595
<b>20.4.</b>	<b>Methods of Estimation</b>	<b>598</b>
20.4.1.	Ordinary Least Squares and Triangular Systems	599
20.4.2.	Indirect Least Squares	601
20.4.3.	Estimation by Instrumental Variables	601
20.4.4.	Single-Equation Instrumental Variable Methods	602
20.4.4a.	Estimating an Exactly Identified Equation	602
20.4.4b.	Two-Stage Least Squares	603
20.4.4c.	Limited Information Maximum Likelihood	605
20.4.4d.	Two-Stage Least Squares with Autocorrelation	608
20.4.4e.	Two-Stage Least Squares in Models That Are Nonlinear in Variables	609
20.4.5.	System Methods of Estimation	610
20.4.5a.	Three-Stage Least Squares	611
20.4.5b.	Full-Information Maximum Likelihood	612
20.4.6.	Comparison of Methods	615
<b>20.5.</b>	<b>Specification Tests</b>	<b>616</b>
<b>20.6.</b>	<b>Properties of Dynamic Models</b>	<b>619</b>
20.6.1.	Dynamic Models and Their Multipliers	619
20.6.2.	Stability	622
20.6.3.	Adjustment to Equilibrium	624
<b>Exercises</b>		<b>626</b>
<b>Appendix: Yearly Data on the U.S. Economy</b>		<b>628</b>